Supply Chain Model of Finance Payment, Which Made up of Providers, Central Enterprises and Distributors

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Abstract: Supply chain finance payment is one of the most important factors of improving efficiency and supply chain enterprise. Supply chain system is based on the suppliers, enterprises and distributors. Respectively, taking the expected cash flow, dynamic bill payment amount known, dynamic bill flow, cash flow dynamic known and dynamic planning bill payment into account, these established the supply chain finance payment model. Numerical examples show the effectiveness of the model and algorithm provides a decision-making method to deal with the bill payment problems in supply chain.

Key words: Supply chain, supply chain financial, payment order, dynamic programming

INTRODUCTION

In a valid supply chain system, financial management and logistics management are both important. Being bound by the fund, they cannot pay suppliers bill payment correctly and reasonably, making the operation cost increased, what’s more some highly developing companies have higher yielding investment projects with short-term funds to use, but constrained by fund, let them to have missed for short-term high profit opportunities. In order to improve the efficiency of the whole supply chain and supply chain value within the enterprise, Qiu (2007) gives a discriminant method for optimal distributor in the deferred payment and payment discount policy payment time. Chen (2008) points out that supply chain finance credit payment policy can solve the difficult problem in funds and effective supply chain. Srinivasa Raghavan and Mishra (2011) studied on financial management problems of Supply Chain based on the third party (on short financial loans under financial constraints. Sushil (2011) studied the model based on suppliers, distributors and logistics enterprises in the supply chain finance. Zhang (2011) was investigated in batch and credit period joint decision problems in supply chain. Gangha and Carpinetti (2011) proposed a prediction of supply chain performance fuzzy logic based on SCOR model. Li (2011) analyzes the risk of the supply chain financial payment system. Ou and Zhang (2012) put option model into application to the supply chain profit and got the idea that the downstream enterprises, investment in fixed assets and income are nonlinear price.

Studies on the supply chain above are mostly from the performance and financial management point of view, either details about how the financial pay reasonable to maximize the interests of enterprises in supply chain or the specific implementation of the strategy has given. While considering the effects of expected cash amount and known dynamic bill payment amount, dynamic bill flow and known cash flow dynamic, dynamic planning of financial payment, case modeling, payment order and payment time node problem of multi bills for enterprises. Although, the calculation is complex, it can guarantee the accuracy of the optimal strategy.

BASIC ASSUMPTIONS OF THE MODEL AND EXPLANATION OF THE SYMBOL

Model parameters:

- \( \Gamma \) = All bills
- \( C_i \) = The i-th bills payables
- \( u_i \) = Represents the i-th bill payment paid to the discount rate (rebate)
- \( \alpha \) = At that time or before the i-th bill to pay the purchase price, rebate will be got
- \( d_i \) = Represents the agreed moment of paying for the i-th bill
- \( V_i \) = Said that after the agreed time, the i-th bill paying the purchase price, changes applied for late fees pay for the purchase price
- \( s_i \) = Mean the moment that the center is the i-th upstream’s, \( s_i < \alpha < d_i \)
- \( A_i \) = That represents the center enterprises paid to the actual payment of I statement in t time

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\( P_t = \) The \( t \) bill payment center enterprise payment at the initial moment of the discounted value \\
\( r = \) Rate of return as said before the payment time, central enterprises by capital investment \\
\( M_t = \) In the \( t \) time, total outstanding downstream distributors \\
\( Q = \) The total amount of cash that represents the center of enterprise at 0 \\

We will study the financial problems of the system with three level supply chain composed of upstream suppliers, downstream distributors, the central enterprises. We consider such a situation as central enterprises received many different suppliers of raw materials, production of products supply downstream distributors. A central enterprise received from downstream distributors has to return and then paid to the upstream suppliers. We hypothesized that \( M_t \) represents the total time in \( t \) downstream distributors to return. Here we assume that the expected return from the downstream distributors and paid to the upstream suppliers expected payment is known.

We define the \( C_i \), expressed in \( s \), moment of our total payment due to the upstream \( i \)-th bill. When the goods have been shipped to the central enterprises upstream suppliers, bill payables would arise. The goods sent to central enterprises at any time, are like there are many bill payment. Center of enterprise's goal is to arrange payment order, payment from distributors to return it to supplier, make the center of enterprise profit maximization in the process. If \( i \)-th bill is paid before a certain time, center of enterprises will get a loan discount rate (rebate) \( u_i \), this time we defined as \( a_i \). This rebate is the supplier to encourage early payment center enterprise. Of course, if the center of enterprise contract extension after time \( d_i \), payment will be charged as late fees, fine ratio is \( v_i \). From the downstream distributors, to return after the center of enterprise investment summary can obtain a certain profit, earnings ratio for \( r \). Central enterprise's goal is the minimum payment to the supplier's cash flow, to obtain maximum benefits with the rest of the money. Therefore, this study designed a minimum expected pay discount supply payment value model. Because the research problem is the payment problem in discrete time frame, to date as the unit of time we set time range of models by \( T \), payment date every bill in this time range. So, we have:

\[
X_s = \begin{cases} 
1 & \text{Note } i \text{ to be paid at time } t \\
0 & \text{Other} 
\end{cases}
\]

\( i \)-the actual payment of bills is denote by \( A_i \);

the present discounted due to pay the purchase price is as the value of the \( P_t = A_t(t/1+r)^t \). Where \( r \) is the investment return. In this way, center enterprise's goal is the discounted value of the minimum payment to supplier:

\[
\min(\sum_{t=1}^{T} P_t) = \min\left(\sum_{t=1}^{T} \frac{C_t(1-u_t)}{(1+r)^t}X_t + \sum_{t=2}^{T} \frac{C_t}{(1+r)^{t-1}}X_t + \sum_{t=T}^{T} X_t\right)
\]

Only once Every bill is to complete the payment in time, that is:

\[
\sum_{t=1}^{T} X_t = 1
\]

\( \forall t \in \mathbb{I} \) is, at the time here we need to clarify that the \( i \)-th bill cannot be paid before \( s \).

We assume that in each period of time, all the existing cash flow sufficient to pay all the bills payment and only at the end of each time interval for payment.

All cash = back section distributors + original capital investment income - the purchase price paid to suppliers.

Our day interval, with the following equation:

When \( t = 1 \):

\[
Q_1 = Q_0(1+r) + M_1 - \sum_{t=1}^{T} A_i
\]

When \( t = 2 \):

\[
Q_2 = Q_1(1+r) + M_2 - \sum_{t=1}^{T} A_i
= Q_0(1+r) + M_2 - \sum_{t=1}^{T} A_i(1+r) + M_1 - \sum_{t=1}^{T} A_i
\]

Thus, for \( \forall t \in \mathbb{T} \) there:

\[
Q_t = Q_{t-1}(1+r) + \sum_{t=1}^{T} (M_t, 0 + r + r - \sum_{t=1}^{T} X_t A_i(1+r)^{-t})
\]

Constraint conditions:

Constraint 1: At \( a_i \), in time before the payments to suppliers, in addition to the provisions of the purchase price discounts intermediate enterprise vendors do not give additional revenue. Therefore, in order to investment income, middle enterprises will not be \( a_i \) point in time before the advance payment.

Constraint 2: At time point \( a_i \), and point \( d_i \), between payments to suppliers, manufacturers will not give the intermediate enterprise payment discounts. Therefore, in
order to invest income, middle enterprises will not advance at time point \( d \), payment payment

**Constraint 3:** If the time at point \( d \), middle is not the sum of the cash paid to suppliers of the \( i \)-th bill payment, payment will not occur.

In the Results section cycle each time point \( t \), the sum of our existing funds:

\[
Q_{t}(1+r)^{t} + \sum_{i=t}^{\infty} M_{i}(1+r)^{i-t} = A_{t}(t)
\]

Obviously, the last day of the borrower cycle is the maximum amount of money. If a bill is paid, provided Centre companies need to have so much cash to have guitar models, namely:

\[
Q_{t}(1+r)^{t} + \sum_{i=t}^{\infty} M_{i}(1+r)^{i-t} = C_{t}(1-u_{t})
\]

Pay bills minimum required amount of money is \( C_{t}(1-u_{t}) \), If:

\[
Q_{t}(1+r)^{t} + \sum_{i=t}^{\infty} M_{i}(1+r)^{i-t} < C_{t}(1-u_{t})
\]

Bill \( i \) would certainly not be paid.

If:

\[
Q_{t}(1+r)^{t} + \sum_{i=t}^{\infty} M_{i}(1+r)^{i-t} > C_{t}(1-u_{t})
\]

Well before the bill \( i \) at \( d \), certainly cannot pay. In order to find an appropriate point in time, so this time we settled all suppliers of the discounted minimum payment value, we construct Lagrangian function:

\[
\min_{p} \sum_{t} P_{t} = \min_{t} \left( \sum_{i=t}^{\infty} C_{i}(1-u_{i}) \frac{X_{i}}{1+r} + \sum_{i=t}^{\infty} C_{i} \frac{X_{i}}{1+r} - \frac{X_{i}}{1+r} X_{i} + \sum_{i=t}^{\infty} X_{i} \right) + \lambda
\]

By seeking extreme conditions we get a point in time at which the results section is the most favourable to the central enterprises. The key issue here is to find the right time to make a relative value of discounted value of minimum payments. And this is not the appropriate time relative to a specific time, but a relative time range, the time value cannot be determined accurately.

**PAYMENT ISSUES UNDER THE KNOWN AMOUNT OF EXPECTED CASH AND DYNAMIC BILL PAYMENT**

In what has been discussed above, we consider it is a static problem, namely the number of bills which has been paid is known. When the order of bill payment is decided, the money received from the downstream distributors is also known. Here implies the assumption, that is to say when the bill has not been fully paid, no new bill appears; no new cash flow comes back from distributors either. The truth is, however, both the payment for goods to suppliers and the receivable from downstream distributors are dynamic, so we need to decide which bills in a given time should be paid in priority. Therefore we will only consider a case that the payment to suppliers is dynamic and the receivable is also dynamic. However, the receivable at the beginning of every paragraph cycle can guarantee the total amount of the payment for goods from the downstream recovery is \( M \).

Assuming that the bill has been paid over a year earlier and two new bills \( j \) and \( k \) need to be paid in \( t = t_{k} \), the middle enterprises will need to choose one of more advantageous to themselves to pay first. Suppose in moments of \( t = t_{j} \), the existing fund is \( m \). After the time of \( t_{j} \) the two bills need to be paid in turn. The first payment time for \( j \) is \( t_{j} \), then the time for \( k \) is \( t_{j} + t \), the first payment time for \( k \) is \( t_{j} + t \), then the time for \( j \) is \( t_{j} \). We assume after we receive the payment of one bill, the existing fund is still \( M \):

\[
A_{j}(t_{j}) = m(1+r)^{t_{j}} + M(1+r)^{t_{j}} - 1
\]

and \( A_{j}(t_{k}) = M((1+r)^{t_{k}} - 1) \).

Discounted present value is:

\[
P_{x} = \frac{A_{j}(t_{j})}{(1+r)^{t_{j}}} + \frac{A_{j}(t_{k})}{(1+r)^{t_{j}}} = \frac{m(1+r)^{t_{j}} + M(1+r)^{t_{j}} - 1}{(1+r)^{t_{j}}} + \frac{M((1+r)^{t_{k}} - 1)}{(1+r)^{t_{j}}}
\]

If we assume that we pay \( j \) first, \( k \) second, it is good for central enterprises. Therefore, we get \( P_{x} < P'_{x} \), namely:

\[
\frac{m(1+r)^{t_{j}} + M(1+r)^{t_{j}} - 1}{(1+r)^{t_{j}}} < \frac{m(1+r)^{t_{j}} + M(1+r)^{t_{j}} - 1}{(1+r)^{t_{j}}} + \frac{M((1+r)^{t_{k}} - 1)}{(1+r)^{t_{j}}}
\]

We get \( t'_{j} < t_{j} \).

From the derivation, we conclude that the payment order which makes the payment cycle shorter is the best choice.

Then, what we need to solve is how to calculate the cycle time of payment, namely the two kinds of payment.
time and compare the length of the two kinds payment time. By the following equation we can calculate the time it takes to pay \( j \) first. We can assume that at the moment \( t_k \), the bill of \( j \) and \( k \) has been produced:

\[
m(t+r)^{n-1} + M(t+r)^{n-1} - 1 = \begin{cases} C_i(t-u_i) & t_i \leq u_i \\ C_i & a_i < t_i \leq d_i \\ C_i(t+v_i) & t_i > d_i \end{cases}
\]

Use of the time to calculate \( t_i \), calculate to pay \( k \) needed the time \( t_k \):

\[
M(t+r)^{n-1} - 1 = \begin{cases} C_i(t-u_i) & t_i \leq u_i \\ C_i & a_i < t_i \leq d_i \\ C_i(t+v_i) & t_i > d_i \end{cases}
\]

In the same way, we change the payment order, also receive time \( t'_k \), \( t'_k \). We can compare the time \( t_k \) and \( t'_k \) to determine the size of the order of payment. There is a problem that the selection of three kinds of payment methods, this algorithm is in the form of prior agreement to pay down the premise of computing time, so as to effectively compare the kind of order is more conducive to business centers. Suppose now we have several bills \( N \) needs to be paid, then how do we determine the order to pay it? Fig. 1 illustrates the judgments to pass bill payment order process: \( N = N - \{j\}, S = \emptyset \).

**IN THE DYNAMIC FLOW AND DYNAMIC CASH FLOW BILL PAYMENT PROBLEMS**

The expected cash flow and billing streams are unknown, we will discuss the end of each time interval \( t \) and those bills need to be paid. We require:

- \( d_i \) final payment deadline has passed bills should be paid first.
- Right in the discount period then pay \( \alpha_i \) bill.
- And then just after the deadline to pay the bill on \( d_i \).
- Final payment discount period ranging between \( \alpha_i \) and \( d_i \) bill, will not pay.
- Over the deadline to pay overdue bills in accordance with the multi-priority of payment.
- In the discount period on \( \alpha_i \) bill in accordance with the priority of payment discounts and more.

Suppose \( H_i \) is exactly the time \( t \) for \( \alpha_i \) moment all the bills; \( H_i \) at time \( t \) is exactly \( d_i \) time for the entire bill; \( H_i \) at time \( t \) is outstripped \( d_i \) time for the entire bill; \( \alpha_i \) represents a time \( t \) after paying the bill for the total amount of cash.

The model process (Fig. 2) is based on the following assumptions: First pay the bill \( d_i \) after the deadline. However, if the punitive fine for delaying payment after the expiration is little, we cannot pay overdue bills with high punitive fine for delaying payment after the expiration. Then, it's meaningless to pay as the above process and we need to find other ways.

**DYNAMIC PROGRAMMING DECISION PROBLEM TO PAY BILLS**

We have already analyzed the problem of how to pay for multiple orders in previous pages and now we will discuss the most reasonable payment time for an order. Generally speaking, after a bill produces, if you don't pay immediately, you can use the money to invest and generate revenue. If companies don't pay the bills but have an investment, they are equivalent to have a loan from the bank. If the revenue from the investment is lower than bank interest, it is no use for the investment. In order to encourage downstream enterprises have the payments timely, enterprises will provide corresponding reimbursement policy at all reimbursement time points. For example, advanced prepayment has discounts, while overdue payment needs to pay the fine for delaying payment, etc.

The general idea for the payment of a single bill is: Under the condition of the benefit culvert \( u(t) \) of the known funds, interest expense culvert \( v(t) \) and repayment
At the end of the time $t$ pay the bills collection $N_i$; take $i \in N$.

If $t < a$, NO

If $a < t < d$, YES

If $t = d, C_i(t-d_i) \leq A_i$

$H_i = H_i(\cup \{k\})$

If $t > d, C_i(t+v_i)^{v_i} \leq A_i$

$H_i = H_i(\cup \{k\})$

$A = A_i - \sum C_i(t-d_i)$

$C_i(t+v_i)^{v_i} \leq A_i$

$H_i = \emptyset \land A > 0$

$A = A_i - \sum C_i$

$C_i \leq A$

$H_i = \emptyset \land A > 0$

Fig. 2: Model process

Table 1: Prepaid billing fund's year income, bank interest and reimbursement fee

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_i(t)$</td>
<td>0</td>
<td>5</td>
<td>4.5</td>
<td>3.75</td>
<td>2</td>
</tr>
<tr>
<td>$u_i(t)$</td>
<td>0</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$c_i(t)$</td>
<td>0.1</td>
<td>&lt;0.5</td>
<td>3.5</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

$c_k(t) = \text{The fund has been used for } t \text{ periods at the } k \text{ time node. The reimbursing fees when paying the bill.}$

$\alpha$ is the discount factor ($0 < \alpha < 1$): the unit income value one year later is equivalent to the current unit $\alpha$.

The following dynamic programming model is set up. Period of $k (k = 1, 2, \ldots, n)$ means the time period limit for planning to use the money. State variables $s_k$: At the first period of time $k$, the number of periods where funds have been used. Decision variables $x_k$: Having a repayment or continuing to use the funds at the beginning of time $k$, respectively remarked by R (payment) and K (occupancy).

The equation of state transition:

$$s_{k+1} = \begin{cases} s_k + 1 & \text{when } x_k = K \\ 1 & \text{when } x_k = R \end{cases}$$
Phase target is:

\[ v_j(s_k, x_k) = \begin{cases} 
q_k(s_k) - u_k(s_k) & \text{when } x_k = K \\
q_0(0) - u_k(0) - c_k(s_k) & \text{when } x_k = R 
\end{cases} \]

Index function is:

\[ V_k(n) = \sum_{j=1}^{n} v_j(s_k, x_k) (k = 1, 2, \ldots n) \]

An optimal index function \( f_k(s_k) \) stands for the largest benefits by using this batch of funds that has been used for \( s_k \) years from the time node of \( k \) to the time period of \( n \). Then, the following reverse dynamic programming equation is available.

That is:

\[ f_k(s_k) = \max_{x_k = K, R} \{ v_j(s_k, x_k) + \alpha f_{k+1}(s_{k+1}) \} \]

We get:

\[ f_k(s_k) = \max_{x_k = K, R} \{ q_k(s_k) - u_k(s_k) + \alpha f_{k+1}(s_{k+1}) \} \]

\[ q_0(0) - u_k(0) - c_k(s_k) + \alpha f_{k+1}(0) \]

Assume that a prepaid billing funds’ year income and bank interest, reimbursement fee are shown in the Table 1 below and try to determine when the bills have to be paid, the yield would be largest. (\( \alpha - 1 \))

According to the dynamic programming model, we can conclude the optimal strategy is: \{K, K, R, K\}, that is, the bills neither in discount node time 1 nor in equal pay node time 2 pay their bills on time, but pay bills on time in the first punishment node time 3.

The dynamic programming model can also be applied in the condition of invariable total amount of the payment bills. You pay bills and at the same time you will receive a new bills of equal value and the receivable from the downstream or lines of credit. Therefore, in each time node, how should the enterprise make a decision to make maximal profit in this payment cycles? If we use the above data, we can draw a conclusion that on the time node 1 and 2, we don’t pay and we pay on time node 3. We continue to take up new bills to pay money on node 4, so that in this payment cycle total revenue of the enterprise is maximization.

**CONCLUSION**

This study researches the thesis from the perspective of operational decisions and dynamic programming. This study studies the sequence of multiple bills payment and the payment time node decision-making of dynamic planning bill. It also develops and adds supply chain about financial management theory basis, helping the related enterprises in the supply chain to apply the theory and technology as a reference and practice guidance tool in the process of operation management. Therefore, we can solve the current common financial risk problems existing in some supply chain, so as to increase the value of each enterprise in the supply chain, to improve operational efficiency and to improve market competitiveness of the supply chain. This study can also have a further study, such as third-party financial institutions efficiency research, the third-party financial institutions risk measurement, supply chain finance risk measurement, etc.

**ACKNOWLEDGMENTS**

The study is subsidized by Founed project Ministry of Humanity and Social Sciences Planning Fund "The study on the factors of whether Chinese enterprises overseas mergers and acquisitions could succeed with the influence of financial crisis"(10YAZH053).

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