Path Planning for Urgent Repair of Transmission Line Based on Diamond-boundary Limit Algorithm

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Abstract: In this study, an improved diamond-boundary limit algorithm is proposed for optimizing the resource scheduling of transmission line and path planning. This algorithm is based on Dijkstra algorithm, establishes four boundary conditions in term of ellipse-circumscribed diamond boundary, makes a directional search for nodes only inside the diamond so that reduces nodes in the search process. This algorithm transfers the nonlinear boundary limit into linear one so to simplify the operation and raise the efficiency. By analyzing the shortest path and the consuming time of different limit algorithm in an example, it is proved that this algorithm is validity and practicability.

Key words: Transmission line, urgent repair, resource scheduling, diamond boundary algorithm, shortest path

INTRODUCTION

Electric Power is a pillar industry for economic development. As a kind of important infrastructure of power grid, transmission lines frequently layout in remote areas, so they are vulnerable to natural disasters, such as wildfire, fierce wind and icing etc., with serious failures consequently. In the emergency treatment process of transmission line, emergency management personnel would deploy resources promptly by using the power grid management system to analyze, estimate, respond and deal with the emergency in the best time. When serious natural disasters occur with the wide and long-lasting impact, divers and vast relief supplies are demanded. The resources are usually limited in each scattered supply site so that how to plan for emergency maintenance and for searching the shortest path in shortest time, have become foci.

At present, in respect of transmission line fault repair planning, the domestic and foreign scholars focus on the emergency maintenance time and the shortest path. In some references (Zhang et al., 2008; Wei et al., 2008; Wang and Zhou, 2009), parallelogram-, ellipse-, or rectangle-boundary limit algorithm is used to calculate the shortest path for a node-to-node supply and demand. They are all based on Dijkstra algorithm with different boundary conditions. The principle of ellipse-boundary limit algorithm is simple, but it is time-consuming with a lot of involution and evolution operation on computer. While, with simple boundary conditions, rectangle or parallelogram-boundary limit algorithm is time-saving. In some references (Wang et al., 2006, 2011), based on traditional genetic algorithm, an algorithm is improved on searching area, selecting nodes and crossover operator and creates a new coding method to converge quickly for the optimal solution. In a reference (Zhang et al., 2006), based on ant colony algorithm, the efficiency is improved by determining the coefficient of factors in terms of the importance of each node. Hu and He (2007) taking the advantages of both genetic algorithm and taboo search algorithm, it puts forward an optimized strategy of a hybrid topological-genetic algorithm to find the best path.

To improve the efficiency of path search for transmission line urgent repair, in this study, an Dijkstra-based diamond-boundary limit algorithm for optimal path is constructed which establishes four boundary conditions in term of ellipse-circumscribed diamond and makes a directional search for nodes only inside the diamond to reduce the amount of nodes in the search process.

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DIAMOND-BOUNDARY LIMIT ALGORITHM

Limit algorithm is mostly used in the transportation network to calculate the shortest path between two sites. Taking starting site and the destination as the foci, the Euclidean distance between the two sites is weighted to be the length of long axis and draw an ellipse. Then combining with Dijkstra algorithm, the ellipse-boundary limit algorithm is constructed.

Ellipse-boundary limit algorithm can reduce searched nodes for the best path. Since amount of involution and evolution operation are involved, the computer will take a long time to run this algorithm in a large system. In an emergency, local emergency personnel and GPS guide system should be promptly informed of the best path which requires efficient computation and quick response. By circumscribing ellipse with a diamond to limit, this algorithm can meet the above requirement very well and ensure the accuracy of the optimal path.

**General ellipse-boundary limit algorithm:** Firstly, starting sites' coordinates and the destination's coordinates in the network are set, then Euclidean distance of the two sites can be calculated which is usually less than the actual shortest path and should be improved. Generally, statistical analysis is carried out in the geographic area between the two sites and n pairs of nodes are picked out randomly. Then path L between nodes i and j is worked out and sample set C is established with $\lambda$-the ratio of L to Euclidean distance d. Based on sample set C, statistical analysis would be conducted to get a unilateral confidence interval with a confidence level of 95%. The value of interval is less than the value of $\lambda$. According to the above process, the shortest path of any two arbitrary nodes within the geographical area could be worked out. Taking the two nodes as two foci, the length of long axis equals to the product of the distance between two foci and the coefficient $\lambda$ and then a restricting ellipse is constructed.

**Ideal restricting diamond**

**Construction of restricting ellipse:** Assume that supply site S and demand site G are on the axis X, according to the ellipse-boundary limit algorithm, S and G would be used as two foci of the ellipse and $2a = \lambda |SG|$ would be the long axis of the ellipse. In the ellipse, the sum of two distances from any node to two foci is less than $2a$, i.e., $|SP| + |SG| < 2a$. To determine the coefficient $\lambda$, statistical analysis on typical section within the selected area of an electronic map is conducted as an example, 900 pairs of nodes are randomly selected to measure their Euclidean distance $E_i$ and the shortest path distance $D_i$ on the map, then to calculate the ratio of $D_i$ to $E_i$ and finally to get sample statistics of the 900 nodes. For higher confidence in result, the confidence level is 95% when unilateral confidence interval is taken. As the result of calculation, the value of sample mean is 1.37 and the value of $\lambda$ is 1.863 with the confidence level 95% (i.e., in the ellipse, the probability that the shortest path would not found is less than 5%). After then, the long axis could be calculated as $2a = 1.863 |SG|$ and the ellipse has been determined now.

**Construction of restricting diamond:** In Fig. 1, S and G are axisymmetric of axis Y on the axis X. Given that the coordinates of S are (-a, 0) and the ones of G are (a, 0). According to ellipse equation and the tangent of the ellipse in the second quadrant, equations are established as follows in Eq. 1. The solutions are below in Eq. 2:

$$\begin{align*}
\frac{x^2}{a^2} + \frac{y^2}{b^2} &= 1, \quad c, d > 0 \\
y &= cx + d
\end{align*}$$ (1)

$$\begin{align*}
x &= \frac{-a^2c}{\sqrt{b^2 + a^2c^2}} \\
y &= \frac{b^2}{\sqrt{b^2 + a^2c^2}}
\end{align*}$$ (2)

These solutions are the coordinates of P which is the tangent point on the ellipse. Plugging P into the tangent equation MN, the solution is $d = \sqrt{b^2 + a^2c^2}$, i.e. the coordinates of point M are $(a, \sqrt{b^2 + a^2c^2})$. Since the area of diamond is four times as the one of triangle, the formula would be elicited in Eq. 3:

$$S = \frac{2d}{c} = 2\left(\frac{b^2 + a^2c^2}{c}\right)$$ (3)
By derivation of S and making $S' = 0$, the minimum area can be calculated as $S_{min} = 4ab$ which shows that the value calculated by diamond algorithm is identical to the one by parallelogram algorithm.

For the circle-boundary limit algorithm, the supply site is always the center of the circle and the length of long axis is the radius of the circle's trajectory equation, i.e. $R = 2a$. The area of circle is $S'_C = 4\pi a^2$. Then, the ratio of the minimum area for circle to the one for diamond is worked out in Eq. 4:

$$\tau = \frac{S}{S_{min}} = \frac{4\pi a^2}{4ab} = \frac{\pi a}{b}$$

(4)

Since $\lambda = 1.863$ and $a = \frac{\sqrt{1.863^2 - 1}}{1.572} = 1.572b$, $\tau = 4.394$ is worked out. It can be seen that the area for restricting circle is almost 5 times as the one for restricting diamond and that the diamond-boundary limit algorithm greatly reduces the area to search.

**Construction of actual restricting diamond:** In the actual cases, supply site $S(x_s, y_s)$ and demand site $G(x_d, y_d)$ may not always be on the axis X in geographic coordinate system as shown in Fig. 2. Actual diamond-boundary conditions are discussed here below.

**Construction of an ellipse:** According to the relationship between the geographic coordinate system XOY and the elliptical coordinate system $X'O'Y'$ (Zhu et al., 2011), the transformed formula from the geographic coordinate system to elliptical one is shown in Eq. 5:

$$\begin{align*}
x' &= (x - x_s)\cos\alpha + (y - y_s)\sin\alpha \\
y' &= -(x - x_s)\sin\alpha + (y - y_s)\cos\alpha
\end{align*}$$

(5)

where, $\alpha$ is the angle between axis y and axis $y'$. ($x_s, y_s$) are the coordinates of O' in the geographic coordinate system transformed from the elliptic coordinate system and O' is the center point between S and G as calculated in Eq. 6:

$$x_o = \frac{x_s + x_d}{2}, \quad y_o = \frac{y_s + y_d}{2}$$

(6)

(x, y) are the coordinates in the geographic coordinate system, ($x', y'$) are the coordinates in the elliptic coordinate system (Tang et al., 2000). Plugging $x'$ and $y'$ into elliptical equation, new equation for the ellipse in the elliptic coordinate system is established in Eq. 7:

$$\frac{(x - x_o)^2}{a^2} + \frac{(-\alpha - x_o)^2}{b^2} = 1$$

(7)

Here in Eq. 7 are:

$$a = \frac{\sqrt{\left(\left(x_s - x_d\right)^2 + \left(y_s - y_d\right)^2\right)}}{2}$$

and:

$$b = \sqrt{a^2 - c^2} = \sqrt{\frac{\left(\left(x_s - x_d\right)^2 + \left(y_s - y_d\right)^2\right)}{4}}$$

Construction of restricting diamond is shown in Fig. 3.

**Solutions of boundary condition MII and JL of the restricting diamond:** In the elliptic coordinate system, it is hard to establish boundary equation of diamond due to
amount of involution and evolution operation. Therefore, to simplify here, two straight lines MH and JL are constructed with the maximum $x_{\text{max}}$ and the minimum $x_{\text{min}}$ of a point’s abscissa on the ellipse. $x = x_{\text{min}}$ and $x = x_{\text{max}}$ are two boundary conditions. To get the solution of $x_{\text{max}}$ and $x_{\text{min}}$, the partial derivative operation of elliptic equation to $X$ is conducted and make the result equal to zero, then the extremum of $x$ can be calculated in Eq. 8:

$$x_n = x + \sqrt{a^2 \cos^2 \alpha + b^2 \sin^2 \alpha}$$ \hspace{1cm} (8)

In terms of these two boundary lines, next step is to construct the diamond based on the ellipse.

**Solutions of boundary condition MJ and HL of the restricting diamond:** It is easy to build the equation for the straight line SG and then the equation for line MO is established here below in Eq. 9. Line MO is vertical to line SG through point $O'$:

$$y = \left(\frac{b}{a}x + x_n + \frac{x_n^3}{y_n}\right)$$ \hspace{1cm} (9)

Plugging $x_n = x_{\text{max}}, x_{\text{min}}$ into Eq. 9 to work out the vertical coordinates of points M and M as below in Eq. 10:

$$y_n = \left(\frac{b}{a}x + x_n + \frac{x_n^3}{y_n}\right)$$ \hspace{1cm} (10)

According to characteristics of diamond, it can be known that MH and MJ are symmetrical as for straight line MO. Given that there are arbitrary points Q ($x, y$) on line MH and Q' ($x', y'$) on line MJ, so equations are established in Eq. 11:

$$\begin{align*}
\frac{x'm + x + x}{2} + \frac{y' + y}{2} &= \frac{y'_2 + x'_2}{y' + y}
\frac{y' - y}{x' - x}
\frac{x_n^2 + y_n^2}{x_n^2 + y_n^2}
\end{align*}$$ \hspace{1cm} (11)

The formula of $x'$ is constructed by Eliminating $y'$ and then equations of lines MJ and HL are built by making $x' = x_n$ in Eq. 12:

$$x_n = 2x_i - \frac{(x_i^2 - y_i^2)}{x_i^2 + y_i^2} - \frac{2x_i y_i}{x_i^2 + y_i^2} y$$ \hspace{1cm} (12)

Until now, four equations of restricting diamond boundaries MH, JL, MJ and HL have been established.

**Description on diamond-boundary limit algorithm:** When classic Dijkstra algorithm traverses the adjacent nodes of any node, the judgment whether node $j$ is located in the diamond would be made by determining if the value of $d_j$ is zero or not. Then the shortest path of diamond-boundary limit algorithm comes out, the process is shown in Fig. 4 here below.
CASE STUDY

Unlimited Dijkstra algorithm, ellipse-boundary limit algorithm and diamond-boundary limit algorithm are all Dijkstra-based. With different geometric shape to limit, their boundary condition equations and the time consumed by computer to calculate the shortest path are different too. In this case below, using the basic Dijkstra algorithm, the influence on computing speed and the shortest path which is caused by different geometric boundary, is analyzed.

Assumed that 20 persons and 350 material are wanted at the fault site of transmission line, different supply sites have material of 180, 150 and 260 individually, different maintenance personnel rally centers have 7, 5, 10, 8 and 12 persons respectively. As for the above situation, emergency scheduling is launched to send supplies and dispatch maintenance personnel to the demand site in shortest time. The fault site and supply sites are located in Fig. 5.

First step is to pre-judge the whole situation and determine priorities. Those supply sites with a short distance from the fault site would be prior with less time consuming to reach the destination. Therefore, the order of priority is 1-2-3 for supply sites and 1-2-5-4-3 for maintenance personnel rally centers. According to the above priority of supply sites 1 and 2, it is given that (180+150)<350, so that material must be offered from all three supply sites for this scheme of emergency scheduling. However, as another scheme of emergency scheduling, when material are offered only from supply sites 1 and 3, (180+260)=350 meets the demand on material very well with the tiny difference on distance. In comparison, the latter scheme is better than the former one.

With the similar process above, technicians should be dispatched from maintenance personnel rally centers 1, 2 and 3 (Zhang et al., 2005).

To show the general operation of this algorithm, the amount of total nodes between supply site 1 to the fault site is determined by randomly taking an integral number in the interval of [20, 40]. Those nodes will follow Gaussian random distribution with the mean of μ = 7 and the standard deviation of σ = 3 and the distances of each two neighbor nodes are set according to the ones in Fig. 5 above. Different nodes are randomly connected with a practical and reasonable limitation of 20 kilometers. The distribution of path nodes between the fault site and the supply site 1 is shown in Fig. 6.

![Fig. 5: Geographic distribution for emergency scheduling](image)

![Fig. 6: Distribution of path nodes from supply site 1 to the fault site](image)
From the above data, it can be seen that the distances searched by ellipse algorithm and diamond algorithm are shorter than the one by unlimited algorithm. Besides, consumed time to search by diamond algorithm is apparently decreased comparing with the one by ellipse algorithm.

The comparison of distance for three algorithms is conducted as shown in Fig. 7. In this figure, it can be seen that both of ellipse limit algorithm and diamond limit algorithm will take advantages on the amount of total searched nodes and on total searched distance, since those two algorithms search nodes under a limitation of direction while the unlimited algorithm mainly focus on the shortest path to search nodes.

The comparison of consumed time for three algorithms is conducted and shown in Fig. 8 below. It can be seen that diamond limit algorithm takes advantage on consumed time of searching nodes with simplified boundary conditions. Besides, this superiority will extend with the increase of complexity and nodes.

Referring to above mentioned method to set the distribution of path nodes between supply site 1 and fault site, path node distributions from the left selected 4 sites to fault site are set. Then, the shortest distance and consumed time calculated by three algorithms are shown in Table 2.

From the data above, it can be seen that limit algorithms are obviously prior to unlimited algorithm in respect of the amount of searched nodes, optimal distance and computing time. The reason is that the unlimited algorithm searches the best path by comparing the distances from a node to all its adjacent nodes, i.e., extensive searching from start point as center point is conducted in a specific area. On the contrary, these limit algorithms are directional to search the adjacent nodes with boundary conditions. Once conditions are not met, the corresponding nodes would not be taken into account.

Diamond-boundary limit algorithm is a kind Dijkstra algorithm based on ellipse-boundary limit algorithm. In above tables, the optimal path is the same for two limit algorithms. What’s more, with simplified boundary conditions, diamond-boundary limit algorithm takes a significant advantage of time-saving search on computer. Although ellipse-circumscribed diamond enlarges searched area, the increment of nodes with the increment of area could be ignored without influence on efficiency of the algorithm which can be seen from consumed time to search for the optimal path.

**CONCLUSION**

According to characteristics of transmission line urgent repair, diamond-boundary limit algorithm for
optimal path in restricted searched area is selected for emergency scheduling and the shortest path planning in this study. This algorithm is based on traditional ellipse-boundary limit algorithm and improved to significantly reduce the computer running time and raise the efficiency. In the above case, validity and practicability of this algorithm are well tested by analyzing consumed time and the shortest path among different algorithms.

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