Probability Distribution Function of Wind Power Variations
Based on the MSTV-EGARCH Model

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Abstract: In this study, we establish a new MSTV-EGARCH model to study the wind power laws. By using the
new model, we study the probability distribution function of wind power Variations and propose a root
transformation method. The probability distribution of wind power can be transformed into a normal
distribution.

Key words: MSTV-EGARCH model, root transformation, wind power, normal distribution

INTRODUCTION

With the constraints of resources and the increasingly harsh of environment, energy development
pattern dominated by fossil energy must be changed. In recent years, a global upsurge of renewable energy
development has emerged. China has built 8 million kW class large-scale wind power base. By the end of 2012,
China's wind power installed capacity has more than 70000000 kW, ranking first in the world. In 2020 the
national wind power installed capacity will exceed 200000000 kW.

Since the wind power does not consume any fuel, it is a clean energy and wind comes from the atmospheric
motion, it is a kind of renewable energy and not exhausted because the development of wind power.

Power from wind turbines is mainly related to the wind speed. Because of the influence of the uncertainty
of the wind, intermittent and wind farm in units of the wake, making wind turbines do farm in units of the wake,
making wind turbines do not like conventional generator that according to the demand of electric power to
determine the power (Abdel-Aal et al., 2009; Cadenas and Rivera, 2007; Ewing et al., 2008; Ewing et al.,
2006; Guo et al., 2012; Jiang et al., 2013).

Large scale wind power base usually need access to the grid in order to realize the transmission and absorption
of wind power. The random fluctuation of wind power is considered to be the major factors adversely affect the
power grid. Fluctuations of wind power, whether or

overcome the disadvantageous impact of wind power integration on the grid to improve the wind power
prediction precision has important significance (Kavasseri et al., 2009; Li et al., 2011; Liu et al., 2011;
Payne, 2009; Tol, 1997; Tan et al., 2010; Vacearo et al., 2012; Zhang et al., 2012).

Wind farms typically have dozens, hundreds of wind
turbine. Therefore, the fluctuation of wind power has great differences of space-time.

Motivated by the above and (Shojaei et al., 2013;
Cenesizoglu and Timmermann, 2012; Hosseinpard et al.,
2009; Bollerslev, 1986; Engle, 1982; Granger and Joyeux,
1980; Hosking, 1981; Ling and Li, 1997), in this study, we
establish a new MSTV-EGARCH model to study the wind
power laws. By using the new model, we study the
probability distribution function of wind power
VARIations and propose a root transformation method.
The probability distribution of wind power can be
transformed into a normal distribution.

EGARCH MODEL

The EGARCH model is to reflect the time-Varying
characteristics of the wind power fluctuations in the
model, which can effectively capture the wind rate
volatility clustering phenomenon, but it does not solve
the leverage effect of volatility of wind power rate.
Therefore, we establish the asymmetric EGARCH model,
obeys a certain distribution, so it can be used to analysis
of wind power units.

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**VAR measure:** VAR is a very important content in the EGARCH model, it obeys a certain distribution, so it can be used to analysis of wind power units.

**Calculation of VAR:** VAR measure can estimate the probability distribution of wind power, the basic formula for VAR is:

$$\text{VaR}_\alpha = P\alpha \sigma_\alpha \sqrt{T}$$

**VAR test:** The VAR model of back testing accuracy test of the VAR model, refers to the calculation of VAR model results to actual loss coverage. The laboratory likelihood ratio tests the null hypothesis that the most appropriate, the test statistic for:

$$\text{LR}_{\alpha} = -2 \ln \left[ (1-p^*)^{(1-p^*)} + 2 \ln [(1-p^*)^{(1-p^*)} p^*] \right]$$

If the initial on LR statistics was established, $L_{\alpha}$ obeys $\chi^2$ distribution. The quantile chi square distribution of 95% is 3.841. When the statistics $\text{LR} > 3.841$, the initial hypothesis is refused.

The measurement of failure independence test probability likelihood ratio statistics:

$$\text{LR}_{\alpha} = -2 \ln \left[ (1-p^*_{\alpha})^{(1-p^*_{\alpha})} p^*_{\alpha} \right] + 2 \ln [(1-p^*_{\alpha})^{(1-p^*_{\alpha})} p^*_{\alpha}]$$

$\text{LR}_{\alpha}$ obeys $\chi^2$ distribution:

$$p_0 = \frac{n_0}{n_0 + n_1}$$

$$p_1 = \frac{n_1}{n_0 + n_1}$$

$$p_2 = \frac{n_0 + n_1}{n_0 + n_1 + n_2 + n_3}$$

where, $n_0$ means that wind power testing, these two statistics $\text{LR}_{\alpha}$ and $\text{LR}_{\alpha}$ Christoffersen constructed on the basis of the extent of coverage and time independent hybrid test. And the test statistic for the combination of $\text{LR}_{\alpha}$ and $\text{LR}_{\alpha}$:

$$\text{LR}_{\alpha} = \text{LR}_{\alpha} + \text{LR}_{\alpha}$$

where, $\text{LR}_{\alpha}$ obeys $\chi^2$ distribution. By using the statistics $\text{LR}_{\alpha}$, we can cover the extent and timing of independence for testing simultaneously.

Just after determining the parameters of the $\text{LR}_{\alpha}$, $\text{LR}_{\alpha}$, and $\text{LR}_{\alpha}$, we select the correct estimation method for calculation.

**Diagnostic and test of models:** The EGARCH model requires the following three tests:

- The parameter estimators of model must be with significant test
- The model must be smooth and reversible, namely the reciprocal of all characteristic roots are within the unit circle
- The residuals of model must be with Q-inspection and the residuals must be a white noise sequence

**Solution of the model**

**Data preprocessing:** The negative of data is bad data, which will be pre-set to zero in order to reduce the prediction error, improve forecast accuracy.

**Solution based on MSTV-EGARCH model:** Ergle (1982) first proposed conditional heteroskedasticity (ARCH(P)) model:

$$\eta = \eta_0 + \sum_{j=1}^{n} \alpha_j \eta_{t-j} + \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j \epsilon^2_{t-j}$$

Bollerslev (1986) extends the ARCH model, the generalized autoregressive conditional heteroskedasticity (GARCH (p,q)) model:

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j \epsilon^2_{t-j} + \sum_{j=1}^{q} \beta_j \sigma^2_{t-j}$$

where $\eta_t$ is wind power series, $\epsilon_t$ is residuals and $\sigma_t^2$ is the conditional variance.

Nelson and Cao (2009) found that the nonnegativity constraints in the linear GARCH model are too restrictive. It imposes the nonnegative constraints on the parameters $\alpha$ and $\beta$. But there are no restrictions on these parameters in the EGARCH model (11). In the EGARCH model:

$$\ln(\sigma_t^2) = \alpha_0 + \sum_{j=1}^{p} \alpha_j \ln(\left| \eta_{t-j} \right| / \sigma_{t-j}) + \sum_{j=1}^{q} \beta_j \ln(\sigma_{t-j})$$

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In this study, we propose the MST-EGARCH model as the following:

\[ \varepsilon_t = u + \sum_{j=1}^\infty \phi_j \varepsilon_{t-j} + \sum_{j=1}^\infty \phi_j' \varepsilon_{t-j} + e_t \]

\[ e_t = \sigma_t \varepsilon_t \]

\[ \sigma_t^2 = \alpha_0 + \sum_{j=1}^\infty \alpha_j \sigma_{t-j}^2 + \sum_{j=1}^\infty \beta_j \sigma_{t-j}^2 \]

Where the transformation is determined by the following first-order Markov process:

\[ p\left( \begin{pmatrix} S_t = 0 \\ S_{t+1} = 0 \end{pmatrix} \right) = 1 - p_0 \]

\[ p\left( \begin{pmatrix} S_t = 1 \\ S_{t+1} = 1 \end{pmatrix} \right) = p_1 \]

**GH skew student's t-distribution.** Under the conditions of GH restriction, GH skew Student's t-distribution follows the Prius distribution and satisfying:

\[ f_{\varepsilon}(x) = \left( \alpha^2 - \beta^2 \right)^{\frac{1}{4}} K_{\frac{1}{2}}(\alpha \sqrt{\delta^2 + (x-u)^2}) \cdot \exp(\beta(x-u))/\sqrt{2\pi \alpha^2 - \beta^2} K_{\frac{1}{2}}(\delta \sqrt{\alpha^2 - \beta^2}) \]

\[ (\sqrt{\delta^2 + (x-u)^2})^{\frac{1}{4}} \]

And the above equation, Kj modified Bessel functions and parameters can meet the necessary conditions:

\[ \delta > 0, |\beta| < \alpha \quad \text{if} \quad \lambda > 0 \]

\[ \delta > 0, |\beta| < \alpha \quad \text{if} \quad \lambda = 0 \]

\[ \delta > 0, |\beta| < \alpha \quad \text{if} \quad \lambda < 0 \]

GH can be proved at the end of the performance of all values:

\[ f_{\varepsilon}(x) = \text{const.} \left| x \right|^{\lambda-1} \exp(-\alpha |x| + \beta x) \]

As \( x \to \pm \infty \)

A generalized hyperbolic variable can be expressed as:

![Fig. 1: Root conversion flowchart](attachment:flowchart.png)

By Y-N (0, 1) and Z-QIG model, we found that wind power fluctuation characteristics in line with GH Student's t-distribution.

But measured by data found that wind power fluctuations sometimes does not meet the common probability distribution is easy to describe its features, for this situation, we propose a method with the conversion, the fluctuations of wind power of converted to normal probability distribution, to take advantage of the nature and root converted to normal probabilities describe the characteristics of wind power fluctuations distribution. In order to lower the amount of data in the case of the correctness of the prediction is still maintained at a high level of confidence that we be unified into a normal distribution. Compared with the traditional log, ln, 1/y, Cox-Box conversion method, the conversion method can avoid root loss of data information. Root conversion flowchart as Fig. 1.

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