Multi-objective Structural Optimization Base on Improved NSGA-II Algorithm

M.Z. Lai, Z.M. Duan, G.Y. Zhang and B.D
1College of Computer Science and Technology, Harbin University of Science and Technology, Harbin, Heilongjiang 150080, China
2HLJ Province Electronic and Information Products Supervision Inspection Institute, Harbin, Heilongjiang, 150090, China
3School of Astronautics, Harbin Institute of Technology, Heilongjiang 150001, China

Abstract: A kind of fast and elitist multi-objective genetic algorithm (nondominated sorting genetic algorithm-II) was presented to solve high dimension and multi-modal optimal problems. The fuzzy information could be converted into a mathematically well-structured problem based on fuzzy optimal theory. And the improved crossover operator of NSGA-II was applied to obtain the optimal solution. According to the test results on a typical test function and an application on the structural fuzzy multi-objective optimization of three-bar truss, more reasonable distributed solutions could be obtained and the diversity of the solutions could be maintained. It provides beneficial references for engineering application of fuzzy multi-objective structure optimization.

Key words: Structural optimization, fuzzy multi-objective, nondominated sorting genetic algorithm-II (NSGA-II)

INTRODUCTION

In the initial phase of engineering structures design, there contains a lot of uncertain information, such as random information and fuzzy information and so on and the optimal design of engineering structures is normally reflected in a multi-objective form. For the traditional multi-objective optimization problem, the most direct solution is to use weighted sum method to integrate the various objective functions into one single-objective optimization problem. Conceptually, the weighted sum method could be seen as multi-objective optimization methods used in the genetic algorithm. The method will assign different weight factors to each of the objective functions and then combine the weighted object to a single objective function. In fact, weighted sum methods used in the genetic algorithm are essentially different from weighted sum method used in the traditional multi-objective optimization algorithm (Forseea and Flemming, 1995; Rubenstein-Montano et al., 2000). In multi-objective optimization, weighted sum methods are used to obtain a compromise solution. The only operation requirement of this algorithm is the appropriate weight vector. But for a given problem it’s often difficult to obtain a group of suitable weight factors. In the genetic algorithm, the initial weighted sum methods are used for genetic search along to Pareto. With the process of the evolution, the adaptive weight factors need to readjust. So it does not necessarily require a good weight vector to ensure the genetic algorithm operation. In addition, the weights’ Shortcomings in the traditional multi-objective optimization could also be weakened based on the population search and revolutionary search. The research of multi-objective genetic algorithm is the mechanism using Pareto multi-objective optimization technology (Zitzler and Thiele, 1999), therefore Nondominated Sorting Genetic Algorithm-II (NSGA-II) could be used to solve the multi-objective fuzzy optimization problem.

CALCULATION PROGRESS OF FUZZY MULTI-OBJECTIVE OPTIMIZATION BASED ON NSGA-II

The Nondominated Sorting Genetic Algorithm (NSGA) was introduced by (Srinivas and Deb, 1994). The algorithm is realized by individual classification, so NSGA is widely used in solving practical problems. However it is not adequate for high-dimensional and multi-modal issues. In 2000, Deb NSGA improved NSGA as NSGA-II (Deb et al., 2000), so the computational speed and the robustness of the algorithm were enhanced greatly. In order to demarcate the fitness of small elements in the same level after fast-grade non-winners ranking, so the prospective Pareto jurisdictions elements can be extended to the entire Pareto domain as evenly as possible to fill the

Corresponding Author: M.Z. Lai, College of Computer Science and Technology, Harbin University of Science and Technology, Harbin, Heilongjiang 150080, China

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whole space, the concept of crowding distance was put forward, using more crowded distance operator to replace the need to share complex calculation parameters of appropriate sharing methods.

**Self-crossover operator design of NSGA-II:** Generally, real-coded genetic algorithm is adopted, cross operator adopts arithmetic crossover:

\[
C_{ik} = \frac{1}{2}[(1 - \beta_k)p_{ik} + (1 + \beta_k)p_{ik}]
\]

\[
C_{ik} = \frac{1}{2}[(1 + \beta_k)p_{ik} + (1 - \beta_k)p_{ik}]
\]

where, \(C_{ik}\) is the \(k\)th individual of the \(i\)th generation; \(p_{ik}\) and \(p_{ik}\) are the choice of the parent generation; \(\beta_k\) represents a random number of population density.

Now by improving the random number, we can combine this random number with the ranking grades of each individual in the populations, namely:

\[
\beta_k = \frac{\text{rankA} \times \text{rankB}}{\text{rankA} + \text{rankB}}
\]

where, \text{rankA} and \text{rankB} are, respectively the ordination of A and B populations.

So with the development of the evolution, the individuals from the population will tend to the same surface of Pareto, the sequence of A will be consistent with B and \(\beta_k\) tends to 0.5.

**Clearing fuzzy constraint function:** Linear membership function was utilized in constraint function to form different transformation forms from the non-Fuzzy to Fuzzy its multi-objective fuzzy optimization model can be converted into the following mathematical model:

\[
\begin{align*}
\text{Find} & \quad X = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n \\
\text{min} & \quad f_i(X) \quad i = 1, 2, \ldots, n \\
\text{s.t.} & \quad \mu_{e} \geq w_{e} \lambda^* \\
& \quad \mu_{u} \geq w_{u} \lambda^* \\
& \quad \mu_{d} \geq w_{d}
\end{align*}
\]

where, \(w_{e}, w_{u}, \text{and} \ w_{d}\) are the weight factors of performance constraints and design variable constraints; \(\mu_{e}\) is the memberships of performance constraints; \(\mu_{u}\) and \(\mu_{d}\) are the up and down limits of design variable constraints; \(\lambda^*\) is the level cut sets.

The Eq. (4) uses different constraints in different design level. The objective functions use linear weighted method so as to make the objective function and constraint function in different important degrees and meet the actual engineering requirements.

**Calculation flow of the improved NSGA-II:** Firstly, according to Eq. (4), multi-objective constraint function or design variable constraint is cleared by using linear membership functions, then making the optimization based on the improve NSGA-II. Finally, the global optimal solution is obtained, as shown in Fig. 1.

**TEST FUNCTIONS AND THE RESULTS**

After the linear membership function was utilized in constraint function, the results of test functions are given directly here:

\[
C_{ik} = \frac{1}{2}[(1 - \beta_k)p_{ik} + (1 + \beta_k)p_{ik}]
\]

Find \( \min f_i(x) \quad i = 1, 2, 3 \)

\[
\begin{align*}
 f_i(x) &= (1 + g(x)) \cos(0.5x_i) \cos(0.5x_i) \\
 f_i(x) &= (1 + g(x)) \cos(0.5x_i) \sin(0.5x_i) \\
 f_i(x) &= (1 + g(x)) \sin(0.5x_i)
\end{align*}
\]

s.t. \( g(x) = \sum_{i=1}^{n} (x_i - 0.5)^2 \quad 0 \leq x_i \leq 1 \quad i = 1, 2, \ldots, 12 \)

Programmed with MBASIC language, population initialization was shown in Table 1.

When hereditary algebra is 1000 and 2000, Perato space solution sets of NSGA-II were shown in Fig. 2 and 3.

The improved NSGA-II population parameters were shown in Table 2, NSGA-II Perato space solution sets of hereditary algebra 1000 were shown in Fig. 4.

![Fig. 1: Calculation flow of the improved NSGA-II](image-url)
Table 1: Initialized population parameters for NSGA-II

<table>
<thead>
<tr>
<th>Population size</th>
<th>Binary crossover operator</th>
<th>Mutation operator</th>
<th>Mating pool</th>
<th>Crossover probability</th>
<th>Variation probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>20</td>
<td>20</td>
<td>100</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 2: Initialized population parameters with the improved NSGA-II

<table>
<thead>
<tr>
<th>Population size</th>
<th>Hereditary algebra</th>
<th>Binary crossover operator</th>
<th>Mutation operator</th>
<th>Mating pool</th>
<th>Crossover probability</th>
<th>Variation probability</th>
</tr>
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<td>100</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

APPLICATION OF NSGA-II IN THE FUZZY STRUCTURAL OPTIMIZATION

A fuzzy optimization design for the three-bar truss’ minimum quality was shown in Fig. 5. The known parameters are as follows:

\[ \rho = 20, \lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1 \]

The allowable limitations and tolerance of each weight were shown in Table 3.

The vertical displacement function \( f_i(X) \) of the truss’ load location and the weight function \( f(X) \) are the minimum optimal goals. So it is a multi-objective fuzzy optimization problem.

Set \( A_1 = A_2 = x_n, A_3 = x_3 \). And the truss’ fuzzy optimization model is shown as follows:

According to the constraint conditions, we set the stress design standards as \( \lambda_1 = \lambda_2 = \lambda_3, \) the boundary design standards is \( \lambda_4 \). For calculating simple, setting \( \lambda_1 = \lambda_2 = \lambda_3 = 1 \). Solve the question with NSGA-II. Population parameters were shown in Table 4.

The perato solution space of NSGA-II with hereditary algebra 200 and 1000 were shown in Fig. 6 and 7, which reflects that distribution of the Perato space solution with hereditary generation 200 and 1000 is almost the same. So the solution almost could not be improved after hereditary generation 200 with NSGA-II algorithm.

\[
\begin{align*}
\text{Find} & \quad X=\{x_n, x_i\}^T \\
\text{min} & \quad f(X) = 10 f_1(X) + f_2(X) \quad \text{(The objective is the truss load location)} \\
\text{s.t.} & \quad 20 x_1 (x_1 + x_2 + x_3 + x_4) \leq 20 + 0 + \lambda_1 - \lambda_2 \quad \text{(Bar 1: stress, 20)} \\
& \quad 20 f_1(x) (f_1(x)^2 + 2 f_2(x)) \leq 20 + 0 + \lambda_3 - \lambda_4 \quad \text{(Bar 2: stress, 0.2)} \\
& \quad -20 f_1(x) (f_1(x)^2 + 2 f_2(x)) \leq 20 + 0 + \lambda_3 - \lambda_4 \quad \text{(Bar 3: stress, 0.2)} \\
& \quad 0 \leq 10 f_2(x) (x_i) \leq 10 f_2(x) \quad i = 1, 2, 3, 4 \\
& \quad x_i \in [0,1] \quad i = 1, 2, 3, 4
\end{align*}
\]

The improved multi-objective NSGA-II population parameters of fuzzy optimization design for the three-bar truss’ minimum quality were shown in Table 5, the Perato solution space of improved NSGA-II with hereditary algebra 200 were shown in Fig. 8. Compare the NSGA-II, the improved NSGA-II distributes
Table 3: Allowable limitations and tolerance of each weight factor

<table>
<thead>
<tr>
<th>$\sigma_0^m$</th>
<th>$\sigma_0^p$</th>
<th>$\sigma_1^m$</th>
<th>$\sigma_1^p$</th>
<th>$\Delta^m$</th>
<th>$\Delta^p$</th>
<th>$\mu^m$</th>
<th>$\mu^p$</th>
<th>$d_1^m$</th>
<th>$d_1^p$</th>
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<tbody>
<tr>
<td>20</td>
<td>4</td>
<td>-15</td>
<td>3</td>
<td>3</td>
<td>0.6</td>
<td>0.1</td>
<td>0.02</td>
<td>10/E</td>
<td>2/E</td>
</tr>
</tbody>
</table>

Table 4: Initialized population parameters for fuzzy multi-objective NSGA-II

<table>
<thead>
<tr>
<th>Population size</th>
<th>Binary crossover operator</th>
<th>Mutation operator</th>
<th>Mating pool</th>
<th>Crossover probability</th>
<th>Variation probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>20</td>
<td>20</td>
<td>100</td>
<td>0.95</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 5: Initialized population parameters for fuzzy multi-objective improved NSGA-II

<table>
<thead>
<tr>
<th>Population size</th>
<th>Hereditary algebra</th>
<th>Binary crossover operator</th>
<th>Mutation operator</th>
<th>Mating pool</th>
<th>Crossover probability</th>
<th>Variation probability</th>
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<td>200</td>
<td>20</td>
<td>100</td>
<td>0.95</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Numerical results with different methods

<table>
<thead>
<tr>
<th>Genetic algorithm</th>
<th>Hereditary algebra</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$f(x_1)$</th>
<th>$f(x_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSGA-II</td>
<td>200</td>
<td>0.5738</td>
<td>3.1153</td>
<td>3.9938</td>
<td>4.7583</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.5736</td>
<td>3.1157</td>
<td>3.9935</td>
<td>4.7581</td>
</tr>
<tr>
<td>Improved NSGA-II</td>
<td>200</td>
<td>0.5732</td>
<td>3.1158</td>
<td>3.9937</td>
<td>4.7571</td>
</tr>
</tbody>
</table>

Fig. 5: Three-bar truss

Fig. 6: Perato space solutions with hereditary

more reasonably and easily gets the optimal global solution. The results were shown in the Table 6.

Fig. 7: Perato space solutions with hereditary generation

200 generation 1000
ACKNOWLEDGMENTS

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REFERENCES


CONCLUSION

The traditional method to deal with fuzzy multi-objective optimization is more likely to lose the global optimal solution. Furthermore, the choice of the different levels of solution sets and weights needs more engineering experience.

NSGA-II solution is the optimal solution set from the beginning to the current population. The elitist solution is always believed as a better way to improve the algorithm efficiency. The application of improved multi-objective NSGA-II algorithm of this paper indicates that the elitist policy keeps the elitist non-inferior solution from the former generation to current and makes the function of the elitist solution continue. By improving cross operator, regulating the parameter and improving the genetic elitist policy to control the solution range in the genetic operation, some non-optimal solutions join the genetic operation. From the figures of the final Pareto solution sets, convergence property had changed greatly. There also obtained a better balance between diversity and convergence.

Fig. 8: Paret space solutions of improved NSGA-II with hereditary algebra 200