A Depositors' Pessimism Contagion Model Affected by External Cause

1,2 Jianxin Chen and 2 Yifan Li

1 School of Business Administration, South China University of Technology, 510641, Guangzhou, China
2 Faculty of Applied Mathematics, Guangdong University of Technology, 510520, Guangzhou, China

Abstract: In this study, based on transformation of the attitude between two kinds of depositors we built up the contagion model. Then equilibrium solves and unstable equilibrium solves of the model are discussed on the perspective of system dynamics stable. The critical conditions of pessimism contagion are obtained which were caused by withdraw in advance of depositors. Internal factors were also obtained for bank that was in a crisis caused by liquidity. It is significant to block and postpone the early withdrawal of depositors. At the same time, it can provide theoretical support for us to prevent the banks from the banking crisis and simultaneously it also can keep the bank system stable.

Key words: Bank runs, pessimism contagion model, affected the external cause

INTRODUCTION

Bank runs has been an important economic phenomenon recently. Seriously bank runs will bring destruction to the banks and even to the whole banking system. Especially in the last 20 years, with the relaxation of financial regulation and the acceleration of the economic globalization process, a series of banking crises and currency crises has happened and the bank crisis caused by bank runs is often seen. As the current development trend, even in countries, which has not happened bank runs, potential crisis of bank runs still plague the banks of all the world (such as: Countrywide Bank (U.S.), ICICI Bank (India), Northern Rock Bank (U.K.,)). How to take measures to prevent bank runs is still a difficulty problem, especially in the countries that is not perfect in banking system (Calvo, 2012). On account of that and the huge costs of the bank runs, the research of the trigger mechanism of the banks runs has important practical significance.

The in-depth reasons of the bank runs can be divided into the following three kinds according to the foreign scholars:

- Panic-based bank runs. As the representative model of the panic-based bank runs, the Diamond and Dybvig (1983) model (DD model) is famous. They assumed that the bank runs was spontaneous and random phenomenon. They thought that even the sunspot may lead to the panic-based bank runs. Including Chang and Velasco (2001), Jeitschko and Taylor (2001), Erris (2003), Macey (2006) and Martin (2006), a number of the economics held this view

- Fundamental-based bank runs. Some scholars quested the cause of panic-based bank runs (sunspot and some random factors) and they believed that the deterioration of macro economy under asymmetric information induced the bank runs (Allen and Gale, 1994; Gu, 2011a). The literatures in this fields are Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), Chen (1999) and Gu (2011b)

- Common function of panic-based bank runs and fundamental-based bank runs. In addition to the above views, some other scholars though that the bank runs had something to do with the panic-based bank runs and fundamental-based bank runs. Some empirical studies had been found by Calomiris and Mason (2003), Schumacher (2000), Starr and Yilmaz (2007), Nakata (2010) and Balkenborg et al. (2011). Based on the results of the empirical research, the theoretical work of Chen (1999) and Goldstein and Pauzner (2005) also held this view

At present, most studies are based on the perspective of the bank and the change of the macroeconomic variables to study the cause of the bank runs. To study the trigger mechanism of the bank runs, there are little literatures based on the mutual infection among the depositors. Thus ignoring the interaction between the individual depositors of a bank to research the bank runs is one-side. Iyer and Puri (2012) point out, even if the bank system was operating in well condition,

Corresponding Author: Jianxin Chen, Faculty of Applied Mathematics, Guangdong University of Technology, 510520, Guangzhou, China
which the mutual infection of the depositors might cause the bank runs. But the existing literatures of this respect are very lack and our study is about this.

**MODEL**

According to Lux (1995) we make the following assumptions

- The number of all depositors is $2N$, where $N$ is constant. The number of the pessimistic depositors is $n_1(t)$ and the number of not pessimistic depositors is $n_2(t)$. So we have $n_1(t) + n_2(t) = 2N$.
- Pessimistic depositors are infected by the not pessimistic depositors through contacting and the transfer probability of the pessimistic depositors is $\beta_1$. The not pessimistic depositors are infected by the pessimistic depositors through contacting and the transfer probability of the not pessimistic depositors is $\beta_2$.
- In unit time times of a pessimistic depositor who contact with the not pessimistic depositors is $C_1(N)$. Times of a not pessimistic depositor who contact with the pessimistic depositors is $C_2(N)$.
- If all the depositors are infected by the positive opinions from external factors (such as the speech of the financial authority, etc). Assuming effect rate is $\delta$.

The times of the transfer rate between the pessimistic depositors and the not pessimistic depositors in unit time can be written as follows:

$$\begin{cases}
\frac{dn_1}{dt} = \beta_1 C_1(N) \frac{n_1}{2N} n_2 - \beta_2 C_2(N) \frac{n_2}{2N} n_1 + \delta n_1 \\
\frac{dn_2}{dt} = -\beta_1 C_1(N) \frac{n_1}{2N} n_2 - \beta_2 C_2(N) \frac{n_2}{2N} n_1 - \delta n_2
\end{cases} \tag{1}$$

Let difference between the not pessimistic depositors and the pessimistic depositors is $n_1(t) + n_2(t) = 2n$ and the average attitude of the depositors is:

$$x_t = \frac{n}{N}, (-1 \leq x_t \leq 1)$$

then Eq. 1 can be written as:

$$\frac{dx}{dt} = \frac{1}{2} \left[ \beta_1 C_1(N) - \beta_2 C_2(N) \right] (1 + x)(1 - x) + \delta (1 - x)$$

Let $\beta_1(x) = ve^{-\alpha x}$, $\beta_2(x) = ve^{-\alpha x}$, where $\alpha$ is the infection intensity coefficient and $\alpha > 0$. $v$ is the change speed. Then Eq. 1 can be written:

**Fig.1.** When $v = 0.4$, $\alpha = 1$, 1.3, 1.6 and 1.9, the phase plane diagram of Eq. 2

$$\frac{dx}{dt} = \frac{1}{2} \left[ (1 - x)(1 + x)ve^{-\alpha x} - \frac{1}{2} (1 + x)(1 - x)ve^{-\alpha x} \right] + \delta (1 - x) \tag{2}$$

$x = 1$ is the equilibrium point of (2). When:

$$x = 1 \geq \ln \left( \frac{\delta}{2v} \right) + 1 - \frac{\delta}{2v}$$

$x = 1$ is the stable equilibrium point. Since:

$$0 < \ln \left( \frac{\delta}{2v} \right) + 1 - \frac{\delta}{2v} < 1$$

we have:

$$\ln \left( \frac{\delta}{2v} \right) + 1 - \frac{\delta}{2v} < 0$$

So $x = 1$ is always stable equilibrium point of Eq. 2.

Fixed $v = 0.4$, $\alpha = 1$, 1.3, 1.6 and 1.9, the phase plane diagram of Eq. 2 is Fig. 1. It can be seen from the Fig. 1 when the effect rate of the speech of the financial authority $\delta = 0.3$, all the depositors of the bank change into one kind-not pessimistic depositors. At this time the bank is stable and there are no bank runs.

Fixed $v = 0.4$, $\delta = 0.3$ and $\alpha = 1$, the trajectory of the Eq. 2 can be seen in Fig. 2. $x = 1$ is the stable equilibrium point. No matter how the change of $x(t)$, all depositors are not pessimistic depositors and they will not draw in advance.
Fig. 2: When $v = 0.4$, $\alpha = 1$ trajectory of Eq. 2

Fig. 3(a-b): When $\alpha = 1$, $x = -0.8$ and 0.1, the bar chart of Eq. 2

When $\alpha = 1$, the initial value of $x = -0.8$ and 0.1, the bar chart of Eq. 2 is Fig. 3. As $x = 1$ is the stable equilibrium point of the system, $x$ will converge to the steady state with the time $t$. The reason is that the positive speech of the financial authority cancels the pessimistic mood of the depositors which make them transfer not pessimistic depositors. Eventually all the depositors will not withdraw in advance.

In short, fixed $\alpha - 1, 1.3, 1.6, 1.9$ and $v - 0.4, 0.3$, the attitude change rate of the depositor $v = 0.4$ and effect rate $\delta = 0.3$, when the average attitude of the depositors is pessimistic the depositors’ attitude will transfer to the not pessimistic through the positive speech of the financial authority. That is to say, the depositors believe that the bank is in well operate and the bank has sufficient liquidity funds. Finally all the depositors hold the positive attitude to the bank. Not only the pessimistic depositors are not affected by the not pessimistic depositors but also they are affected by the positive speech of the financial authority and transfer to the not pessimistic depositors. At this time, no depositors are pessimistic and the bank will not run into crisis. The whole bank system is stable. Actually, the number of the equilibrium points of the Eq. 2 varies with $\alpha$ and the bifurcation diagram of (2) is Fig. 4.

If the financial authority has some negative speech, the opposite is true. In a word, it’s important for banks to take corresponding measures to curb the contagion of pessimistic mood of the depositors.

**CONCLUSION**

In this study, employing the theory of dynamic differential equation, we build the pessimism contagion of depositors’ model. Through selecting the key variables that affects the attitude of the depositors, we take the interaction effect of depositors into consideration and also propose the dynamics differential equation models for depositors by dynamics modeling methods. Through the research of transmission rules and the dynamics behavior the causes of the bank runs will be revealed from the microscopic level of the pessimism contagion for depositors. It is significant to block and postpone the early withdrawal of depositors. At the same time, it can provide theoretical support for the bank management to prevent the banks from happening the banking crisis.

**ACKNOWLEDGMENTS**

This work was financially supported by China Postdoctoral Science Foundation funded project (2013M542183), Guangdong Province Natural Science Fund (S2013040014920) of China and Research Fund for the Doctoral Program of Guangdong University of

REFERENCES