Inventory Control Policy for E-tail Organizations Based on TOC

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Abstract: In the retail sector many traditional bricks-and-mortar companies have added online sales channels to their supply chains. Unfortunately, even though the combined e-tailer is becoming a common business model, there is very limited research addressing retail/e-tail operations. This study contributes to the literature by filling this gap and examining strategies that e-tailers can control the inventory dynamically by using the TOC methodology. The computational results indicate how specific problem characteristics influence the performance of whole system and demonstrate the efficiency of the proposed control policy.

Key words: Inventory control policy, e-tail organization, theory of constraints

INTRODUCTION

The advent of e-commerce has made retailing more complicated and more competitive. Today consumers can use the Internet to sidestep their corner store and patronize shops across the country or around the world. It is within this business context that many traditional bricks-and-mortar companies have attempted to increase sales and improve profitability by adding online retail channels for consumers. Unfortunately, even though the combined e-tailer appears to be emerging as the dominant business model for the retail sector, there is little research specifically addressing e-tail operations and how the inventory control policy should affect a firm’s supply chain network design.

In contrast to the substantial quantity of multi-channel research in the marketing and economics areas, research on multi-sales channel inventory systems is relatively sparse (Beamon, 1998).

We are not aware of any previous research that optimally specifies the subset of sites that is set up to handle online sales. The current study builds on the research in inventory control and online fulfillment assignment problems and extends the previously discussed literature by identifying where and how much online inventory should be held for a company that satisfies both in-store and online demand when potential synergies exist for pooling inventories across the sales channels. This study contributes to the literature by filling this gap and examining strategies that e-tailers can control the inventory dynamically by using the TOC methodology.

MATHEMATICAL MODEL OF THE METHOD

Prior to introducing the proposed policies and describe their detailed procedure and for a better understanding of their exposition, in what follows we first present the nomenclature that is used throughout this study and the basic model based on TOC.

Nomenclature:

\[ L_Q = \text{Regular replenish lead time} \]
\[ T = \text{Order review period which is the regular time interval between the reorder points. The buffer procedure regularly replaces the order at each order reorder point} \]
\[ L_E = \text{Emergency replenishment time which is the shortest time required to replenish the emergency order} \]
\[ R(t) = \text{Order arrived quantity at t period} \]
\[ Q(t) = \text{Regular order quantity at t period} \]
\[ Q_{E}(t) = \text{Emergency order quantity at t period} \]
\[ IT(t) = \text{On-road stock quantity at t period} \]
\[ IL(t) = \text{On-hand stock quantity at the beginning of t period} \]
\[ LP(t) = \text{Stock level at the beginning of t period which include on-road stock quantity} \]
\[ IL(t) = \text{On-hand stock quantity at the end of t period} \]
\[ D(t) = \text{Demand quantity at t period} \]
\[ BL(t) = \text{Back order quantity at t period} \]
\[ DS(t) = \text{Demand meeting quantity at t period} \]
\[ A_Q = \text{Regular order cost per time} \]

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We consider a discrete inventory control system in the model and sequences of events in each period are: (a) Order receipts, (b) Inventory verification, including on-hand stock, on-road stock and stock level as well, (c) Demand forecasting, (d) Place the order, (e) Meet the demand, the shortage quantity would become backorder and needs to be delivered next time and (f) Inventory verification again and calculate the throughput (net profit). As shown in Fig. 1.

Based on the Notation and assumptions above, as well as the event sequence in the inventory control system settings, after certain periods, we have:

$$R(t) = O(t-L_{l}) + O(t-L_{e})$$

The function of on-hand stock is:

$$IL_{l}(t) = IL_{l}(t-1) + R(t) - D(t-1)$$

As for the back order, we have:

$$BL_{l}(n) = (D(n) - IL_{l}(n))'$$

In which:

$$(x)' = \max \{x, 0\}, \quad (x)' = \max \{-x, 0\}$$

Thus, $BL_{l}(t)>0$ indicates that stock out happens in the period and the demand meeting quantity is:

$$DS(t) = \min \{D(t) + BL(t-1), IL(t)'\}$$

On-road stock is another important factor in the model, with its function:

$$IT(t) = \sum_{k=1}^{t}O(t-k) + \sum_{k=1}^{\infty}O(t-k)$$

Then we have the stock level at the beginning of a period:

$$IP_{l}(t) = IL_{l}(t) + IT(t)$$

While we calculate the throughput of the whole system, average throughput in $N$ periods will be considered:

$$TP(N) = TR(N) - OE(N) - TC(N)$$

In which $TR(N)$ stands for the average sales value, $OE(N)$ stands for the ordering cost which include regular order cost and emergency order cost, $TC(N)$ stands for the total holding cost which include both holding cost and back order cost:

$$TC(N) = \sum_{n=1}^{N} \left[ H \times IL(0)' + B \times IL(0)' \right]$$

Total cost obviously depends on the order frequency and quantity each period (Gawrysi and Kilpatrick, 2004). Sometimes throughput $TP(N)$ could be considered as the net profit of the system which will be calculated as:

$$TR(N) = \frac{1}{N} \sum_{n=1}^{N} pDS(t)$$

The basic control procedure is as same as the genetic buffer management framework proposed in Ref. 6. Monitoring Window (MW) mechanism also has been set in our model which is still a time interval that functions as a reference for tracking the buffer consumption status and deciding which action must be taken (Hulten et al., 2001). The monitoring window should be higher than the average replenishment time between shipments. The monitoring window is reset under the following conditions (1) During the monitoring window, no safety buffer penetration occurs; the green buffer is adjusted and a new monitoring window is reset at the end of the previous monitoring window and (2) Whenever either the safety buffer level or the target buffer level is adjusted during the monitoring window, the monitoring window is reset from the period which follows the one in which the buffer was adjusted. Whenever the monitoring window is reset, the buffer consumption status of the previous monitoring window is excluded from the decision regarding this new monitoring window.
The inventory control system operates with following rules:

- **Target buffer stock level adjusts rules.** Penetration of the safety buffer is first checked for, if no penetration has occurred, then if the monitoring window has ended, then the target buffer stock level is decreased, the monitoring window is reset and the next monitor is initiated. The green buffer level is reduced because the lack of safety buffer penetration implies that the target buffer stock level is too high. Meanwhile, if stock out does occur, an emergency order is placed and the target buffer level is increased because the target buffer stock level is too small. Again, the monitoring window will be reset and the next monitor initiated since the target buffer stock level has also been adjusted.

- **Safety buffer stock level adjusts rules.** If the safety buffer is penetrated but this does not represent the second penetration of the safety buffer within the monitoring window, we check if the stock out occurs. If stock out does not occur, only the emergency replenishment order is triggered, if stock out does occur, an emergency replenishment order of the same amount is triggered but the safety buffer level is increased because both the green and the safety buffer levels are set too low.

Adjusting the safety buffer level allows the monitoring window to be reset and the next monitor to be initiated. As stated above, the actual size of the buffer is not critical as long as the buffer status continues to be monitored. Therefore, the extent to which the safety or green buffer levels should be increased or decreased is also subjective. If the safety buffer is penetrated but it represents the second penetration of the safety buffer within the monitoring window, we check if the stock out occurs. If stock out has not occurred, then an emergency replenishment order is replaced and the safety buffer level is decreased, because in two cases of penetration, the safety buffer level may have been set too high. Again, the monitoring window will be reset and the next monitor initiated, since the safety buffer level has been adjusted.

- **Order rules.** Every $T$ periods, there would have a regular order, the order quantity set as:

$$O_a(t) = \frac{S_a(t) + S_a(t)}{2}$$

Differ from Ref. 6, we consider that the purpose of emergency order is to regain the on-hand stock level to safety stock level as soon as possible. Thus we set the quantity of each emergency order as:

$$O_b(t) = S_d(t) - IL(t)$$

It's obviously that the dynamic adjust procedure would great affect the performance of the system. Unfortunately, quantitative research in this area are still rare, many relative literatures are still use subjective adjust method in buffer management. Based on the framework above, we proposed a novel inventory control policy.

As demand forecast remains a big issue in reality, we proposed another inventory control policy based on demand forecasting. Even though there are many forecasting method being applied in reality, no matter which one has been chosen in procedure, we note $D(t)$ as the forecasting demand quantity for the next period and the adjust rules are set as:

- If no penetration into safety buffer has occurred, then the monitoring window has ended and the target buffer stock level is decreased as:

$$S_a(t) = \min \left\{ D(t) \times \min (T, L_a) + L_a D(t), S_a(t) \right\}$$

- If the safety buffer is penetrated, we check if the stock out occurs and if stock out does occur, an emergency replenishment order of the same amount is triggered but the safety buffer level is increased as:

$$S_a(t+1) = \max \left\{ D(t) \times L_a, S_a(t) \right\}$$

- If penetration of the safety buffer has occurred and stock out does occur, an emergency order is placed and the target buffer level is increased as:

$$S_a(t+1) = \max \left\{ D(t) \times \min (L_a, T) + D(t) \times L_a, S_a(t) \right\}$$

- If the safety buffer is penetrated and it represents the second penetration of the safety buffer within the monitoring window and stock out has not occurred, then the safety buffer level should be adjust as:

$$S_a(t) = \min \left\{ D(t) \times L_a, S_a(t) \right\}$$
SIMULATION AND ANALYSIS

As non-stationary demand character often appear as short time demand fluctuation in e-tail operation and ARIMA model has shown its efficiency in short period forecasting, thus, we implement ARIMA in our simulation for demand forecast module, then we have to use exponential smoothing method to eliminate the seasonal effect:

\[ D_t = D_{t-1} + (1 - \theta)(D_{t-1} - D_{t-1}) \]

In which θ stands for the prediction coefficients.

And we use following function to simulate the demand fluctuation:

\[
\begin{align*}
D_t(t) &= \mu + \alpha\varepsilon_t \\
D_t &= \mu + \alpha D_{t-1} - (1 - \gamma)\varepsilon_{t-1} + \varepsilon_t
\end{align*}
\]

In which:

\[ \alpha(t) = \frac{\sin(\pi t/12)}{12} \]

stand for seasonal coefficients, \( \mu \) and \( \gamma \) are objective parameters for different market environment, \( \{\varepsilon_t\} \) is the white noise and \( \text{mean}(\varepsilon(t)) = 0 \), \( \text{Var}(\varepsilon(t)) = \sigma^2 \) in our simulation, we set the relative parameters as: \( \sigma^2 = 100 \), \( \mu = 20, \theta = 0.6, \gamma = 0.6 \). The simulation is running with 120 sale periods, without loss of generality, demand fluctuation in the last 20 periods are steady and smooth.

The performance of backorders and demand meeting are shown in Fig. 2, the TOC policy has shown its stability and robust character and adaptive (s,S,T) policy has shown its great reliability on forecasting accuracy which may not so countable in reality as shown in Fig. 3.

As for the performance evaluation, we consider holding cost, order cost and throughput as our main index. Other important parameters are set as:

Fig. 2: Inventory adjustment process under TOC control policy

Fig. 3: Inventory adjustment process under adaptive (s,S,T) control policy
Table 1: Performance of TOC control policy

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<thead>
<tr>
<th>Ao</th>
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<th>H</th>
<th>B</th>
<th>P</th>
<th>Total holding cost</th>
<th>Total backorder cost</th>
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Table 2: Performance of adaptive (s,S,T) control policy

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\( \lambda [100,160], \pi = 200, H_c [2,5], p = 200, B = 40 \)

Still, for adaptive (s,S,T) policy, we set \( H_c [1,9] \).

Based on the parameters we input above, the simulation output based on proposed TOC control policies are shown in Table 1, from which we can conclude that TOC policy keeps a proper service level and inventory level and exhibit no sensitive to regular order cost \( A_0 \) and the holding cost \( H \), thus it guarantees both service level and the throughput of the system. Meanwhile, from Table 2 we can conclude that adaptive (s,S,T) policy are much more sensitive to holding cost \( H \) compared with the regular order cost \( A_0 \), in conclusion, TOC performed best among all the policies we proposed in this study.

**CONCLUSION**

We proposed an inventory control policies for non-stationary demand environment in this study which based on TOC generic replenishment method with the back ground of e-tail organization. With the monitoring window review of the inventory target level and red buffer level, the procedure rigorously defined a means of sizing the buffer, monitoring it and correcting it when necessary adaptively. And we use simulation method to reveal the feasibility of the procedure. Adopting the proposed procedure in a specific application environment is not difficult if the variables are defined according to the application environment. Furthermore, through the comparison, we’ve showed that the proposed buffer management procedures outperforms on prior generic buffer control management and adaptive (s,S,T) policy.

Though the forecasting accuracy is not a big issue in the discussion of our study, it’s an important aspect which may greatly impact the performance of the inventory system and we surely will concern it in our further research.

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