Identification of Nonlinear System Based on ANFIS with Hybrid Fuzzy Clustering

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Abstract: In this study, HFCM (Hybrid fuzzy clustering method) which is proposed by Niros and Tsekouras (2012) recently, is used to generate an initial TSK fuzzy model with the appropriate cluster centers number and performance index by adjusting the radius of a cluster center. To acquire a TSK fuzzy model with perfect performance, ANFIS (Adaptive neuro-fuzzy inference system) is combined to fine tune the premise parameters and consequent parameters by means of LM (Levenberg-Marquardt) Algorithm. A simulation to a dynamic nonlinear system demonstrates the effectiveness of this method.

Key words: Hybrid fuzzy clustering, ANFIS, dynamic nonlinear system

INTRODUCTION

Takagi, Sugeno and Kang have established what is called the Takagi-Sugeno-Kang (TSK) method (Takagi and Hayashi, 1991; Sugeno and Kang, 1988; Sugeno and Tanaka, 1994). This neural-network-based fuzzy reasoning scheme is capable of learning the membership function of the IF part and determining the amount of control in the THEN part of the inference rules. What’s more, it is nicely suited to mathematical analysis and usually works well with optimization and adaptive techniques. Subsequently, many improved algorithms and extensions were developed for the TSK model. In particular, the Adaptive Neuro-fuzzy Inference System (ANFIS) is an important approach to implement the TSK fuzzy system which has been put forward in 1993 (Jang, 1993).

But, as the number of input increasing, the complexity of ANFIS increases exponentially and the practicality of ANFIS decreases. So, Jang proposed a method to reduce number of rules and to decrease the complex of ANFIS (Kim and Lee, 2003), proposed a “pruning technique” according to impact factor of membership function on input variable, to reduce the number of membership function and then the number of rule. Actually, employing clustering algorithm to “prune” is also a good solution. On one hand, the combination of clustering algorithm and ANFIS could reduce the complexity of ANFIS effectively, on the other hand it makes each rule more “valuable”.

In a recent publication, Niros and Tsekouras (2012) have developed a Hybrid Fuzzy Clustering Method (HFCM) which combined the K-means algorithm and FCM. The purpose of this study is to validate the practicality of HFCM applied in ANFIS pruning using famous Box-Jenkins dataset. The rest of the study is organized as follows. Section 2 provides some necessary background information and details of the model. Then the simulation process and results are discussed in Section 3. Finally, Section 4 presents the summary of this study.

PROPOSED SYSTEM

In this section, we introduce the basic theory of ANFIS model and HFCM which has been used in this experiment, then describe the initialization mechanism of \( a_i \).

Adaptive neuro-fuzzy inference system (ANFIS): Both artificial neural network and fuzzy logic are used in ANFIS architecture. ANFIS consists of if-then rules and couples of input-output. For ANFIS training, learning algorithms of neural network are also used.

To simplify the explanations, the fuzzy inference system under consideration is assumed to have two inputs \((x, y)\) and one output \((f)\). For a regular ANFIS model, a typical rule set with basic fuzzy if-then rules can be expressed as if \( x \) is \( A_i \) and \( y \) is \( B_i \), then:

\[
f_i = p x + q y + r_i
\]

where, \( p, q, r \) is linear output parameters. The ANFIS architecture with two inputs and one output are as shown in Fig. 1. This architecture is formed by five layers and nine if-then rules.
**Layer 1:** Every node i in this layer is a square node with a node function:

\[ O_{i,j} = \mu_{A_i}(x) \tag{2} \]

where, \( x \) is the input to node \( I \) and \( A_i \) is linguistic label for inputs. In other words, \( \mu_{A_i} \) is the membership function of \( A_i \). Usually \( \mu_{A_i}(x) \) and \( \mu_{A_i}(y) \) are chosen to be Gaussian-shaped with maximum equaling to 1 and minimum equaling to 0, such as:

\[ \mu_{A_i}(x) = \exp(-(x-a_i)^2/c_i^2) \tag{3} \]

where \( a_i, c_i \) are the center and spread, respectively.

**Layer 2:** Every node in this layer is a circle node labeled \( \prod \) which multiplies the incoming signals and sends the product out. For instance:

\[ O_{i,j}(\prod) = \mu_{A_i}(x)\mu_{B_j}(y) \tag{4} \]

Each node output represents the firing strength of a rule.

**Layer 3:** Every node in this layer is a circle node labeled \( \Sigma \). The i-th node calculates the ratio of the i-th rules firing strength to the sum of all rules firing strengths:

\[ O_{\Sigma_i} = \frac{\bar{w}_i}{\sum w_i} \tag{5} \]

**Layer 4:** Every node i in this layer is a square node with a node function:

\[ O_{f_i} = w_i f_i \tag{6} \]

where, \( w_i \) is the output of layer 3.

**Layer 5:** The single node in this layer is a circle node labeled \( \Sigma \) that computes the overall output as the summation of all incoming signals:

\[ O_{\Sigma} = \sum w_i f_i = \frac{\sum w_i f_i}{\sum w_i} \tag{7} \]

**Hybrid fuzzy clustering method:** The K-Means algorithm is very sensitive to initialization but it is a fast procedure while the fuzzy K-Means is able to reduce the dependence on initialization but it remains a slow process (Pal and
In a recent publication, Niros and Tsekouras (2012) have developed a hybrid fuzzy clustering method which combined the K-means and the FCM. The basic idea of this study is originated on that learning algorithm and utilizes the following objective function:

$$ J_k = \sum_{k=1}^{K} \sum_{i=1}^{N} u_{ki} \| x_i - v_k \|^2 + (1 - \theta) \sum_{k=1}^{K} \sum_{i=1}^{N} u_{ki}^2 \| x_i - v_k \|^2 $$  \hspace{1cm} (8)

where, $K$ is the number of clusters, $\theta \in [0, 1]$ and $u_{ki} \in [0, 1]$ is the membership degree of the k-th training vector to the i-th cluster. Notice that when $\theta = 0$, the objective function is transformed to the fuzzy k-means with $m = 2$ and when $\theta = 1$ it becomes the k-means algorithm. Therefore, the function possesses a hybrid structure enabling the switch from fuzzy to crisp conditions depending on the value of $\theta$. Niros and Tsekouras (2012) defined the set $T_k$ as the aggregate of the cluster centers affected by $x_k$. Initially, the set $T_k$ includes all cluster centers and its cardinality is: $\#(T_k) = c$, where $c$ is the number of radial basis. The proposed hybrid clustering algorithm can be operated as follows: Select values for $c$ and $\theta$. Randomly initialize $v_1, v_2, v_3, \ldots$. Set $\gamma = 0, \forall k: \#(T_k) = c$ and $T_k(0) = \{v_1, v_2, \ldots, v_c\}$.

**Step 1:** Set $\gamma = \gamma + 1$

**Step 2:** Update the sets $T_k(\gamma)$ and their cardinalities $\#(T_k(\gamma)) (1 = k = n)$

**Step 3:** Calculate the membership degrees $u_{ki}$ $(1 = k = n, 1 = i = c)$

**Step 4:** If $u_{ki} < 0 (1 = k = n, 1 = i = c)$ then set $u_{ki} = 0$

**Step 5:** Calculate the normalized membership degrees

**Step 6:** Update the cluster centers

**Step 7:** If there are no noticeable changes for the cluster centers then stop, else turn the algorithm to step 1.

**Constructing ANIFS employing HFCM:** For $n$ m-dimensional data pairs $X = (x_1, x_2, \ldots, x_n)$, the i-th data pair $X_i$ could be described as $X_i = (x_i(1), x_i(2), \ldots, x_i(m))$, to construct a m-1 input and one output model. Assume $c$ clustering centers are needed, HFCM could offer $C = (c_1, c_2, \ldots, c_c)$ as clustering centers, where $c_i = (c_i(1), c_i(2), \ldots, c_i(m-1))$ and $p = 1, 2, \ldots, c$. The number of rules is as same as $c$. For the p-th rule, the IF part can be describe as:

$$ \mu_{pi} = \exp(-\| x_i - c_i \|^2 / c_i) $$  \hspace{1cm} (9)

where, $\mu_{pi}$ is the membership that the i-th data pair belongs to the p-th center.

The final output is:

$$ y_i = \frac{\sum_{p=1}^{c} u_{pi} f_{pi}}{\sum_{p=1}^{c} u_{pi}} $$  \hspace{1cm} (10)

where, $f_{pi} = k_1 x_i(1) + k_2 x_i(2) + \ldots + k_n x_i(m-1) + k_n$ is the Then part of the p-th rule.

**SIMULATION**

Famous Box-Jenkins dataset is the benchmark dataset to validate the performance of fitting method. The Box-Jenkins dataset represents the CO$_2$ concentration as output, $y(t)$, in terms of input gas flow rate, $u(t)$, from a combustion process of a methane air mixture. Lots of early work has been done on fitting Box-Jenkins dataset. Among them, input-type has been used widely:

\begin{align*}
A & u(k-4), y(k-1) \\
B & u(k-3), y(k-1) \\
C & u(k-3), u(k-4), y(k-1) \\
D & u(k), u(k-1), y(k-1), y(k-2) \\
E & u(k-1), u(k-2), y(k-1), y(k-2) \\
F & u(k), u(k-1), u(k-2), y(k-1), y(k-2), y(k-3) \\
G & u(k-1), u(k-2), u(k-3), y(k-1), y(k-2), y(k-3)
\end{align*}

MSE was used to measure the performance and its expression is given below:

$$ \text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (y(i) - \bar{y}(i))^2 $$  \hspace{1cm} (11)

In this simulation, GD-LSE Algorithm has not been used in training, for LM (Levenberg-Marquardt) algorithm would obtain more accuracy results and cost less time. All above have been achieved by modifying nntool toolbox in Matlab 7.8.0. Figure 2 depicts the MSE of each model while the number of clustering centers varies from 12 to 20. Some feature of HFCM based ANIFS could be drawn.
Fig. 2: Trend of MSE with varying number of clustering centers

![Graph showing MSE trend with varying number of clustering centers](image)

Fig. 3(a-b): Comparison between predicted value, actual value and error (input-type: G, number of clustering centers: 20, MSE = 0.0191)

![Graph comparing predicted value, actual value and error](image)

Table 1: Results of simulation

<table>
<thead>
<tr>
<th>Centers</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>MSE</th>
</tr>
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<tr>
<td>12</td>
<td>0.1110</td>
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<td>0.0370</td>
<td>0.0191</td>
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</tr>
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</table>

With the increasing of dimension of input, the errors decrease and the trend of decreasing will finally slow down.

Comparing between the curves of the same input dimension contributes to find the most suitable input. If the dimension of input is given, then it is easy to decide which to select.

With the number of clustering center increasing, the error becomes unstable but has a trend to decrease.

Figure 3 depicts the actual value, predicted value and the error between them when input-type is G and the number of clustering centers is 20.

Simulation result is performed in Table 1. To compare with other models, the errors of other models are listed in Table 2. After comparison, we can draw a conclusion that HFCM based ANFIS is superior to other models.

**CONCLUSION**

HFCM (Hybrid Fuzzy Clustering Method) is a fast and effective method to determine the clustering centers. When equipped for ANFIS, it could prune useless rule
and improve the performance. The simulation results of famous Box-Jenkins dataset present that selecting suitable input-type and corresponding number of clustering centers contributes to reduce the error. In addition, more clustering centers will not guarantee better performance.

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REFERENCE


