R&D Subsidy, Spillovers and R&D Cooperation in Duopoly

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Abstract: This study introduces the degree of R&D cooperation as an endogenous in a duopoly with R&D subsidy and spillovers. We examine the subsidy policy and find that the optimal degree of R&D subsidy is a constant in order to maximize social total welfare. And the further analysis shows, under the best R&D subsidy and positive spillovers, the best R&D strategy for the firms in duopoly is R&D cooperation to maximize their profit and social total welfare. Furthermore, the greater is the technological spillovers, the greater should be the firm’s profit and social total welfare which the duopoly can gain from R&D cooperation.

Key words: R&D subsidy, technological spillovers, R&D cooperation

INTRODUCTION

R&D cooperation in the presence of spillovers is generally seen as welfare enhancing in a lot of theoretical literature. D’Aspremont and Jacquemin (1988) is seen as the seminal study that provides a theoretical base for the favourable attitude towards cooperative R&D, they show that if spillovers are high enough, R&D cooperation (with subsequent competition at the output stage) leads to more output, innovation and welfare. In order to foster innovative activities, governments in numerous countries have introduced R&D support programs aimed at increasing R&D effort and motivating R&D cooperation, so R&D subsidies have become ubiquitous in practice. They are one of the largest and fastest growing forms of industrial aid in developed countries. In United States, European Union and Japan, where our data originate, R&D subsidies were drastically increased in the past decade and are now the most important tool of innovation policy (Takalo et al., 2013).

In theory, R&D subsidies should lower the marginal cost of R&D, increasing R&D investments and thereby firm profitability and social total welfare (Kamien et al., 1992; Vonortas, 1994; Poyago-Theotoky, 1995) and there is by now a huge body of literature about the effects of R&D subsidies. The main motivation for R&D subsidies is that investments in R&D are perceived to be below a social optimum. Almus and Czarnecki (2003), for example, analyze whether and how R&D subsidies influence private R&D activity and find that Eastern German firms that received public R&D subsidies increased their innovation activities. And some empirical researches also show that subsidies stimulate R&D spending within firms (Clausen, 2009; Carboni, 2011), increase R&D intensity in SMEs and increase the probability of patent application for small and medium sized firms (Reikowski et al., 2010).

Furthermore, economic theory shows that coordinating R&D policies appear to be desirable from a welfare point of view (Chu, 2009) because these allow some distortions to be internalized, especially those related to the presence of knowledge spillovers in R&D activities. For instance, Montmartin (2013) analyze the effects of a centralized R&D subsidy policy upon steady state and welfare using an agglomeration and growth model composed of two asymmetric countries and the result shows subsidy policy can eliminate some distortions and improve global welfare.

There are also some authors investigate the effects of both subsidies and spillovers on firms’ R&D efforts and collaboration (Petrakis and Poyago-Theotoky, 2002; Gil-Moltó et al., 2011). For instance, the latter study examine the use of subsidies to R&D in a mixed and a private duopoly market and find that the socially optimal R&D subsidy is increasing in the degree of spillovers, but it is lower in the private duopoly. However, all these papers focused on subsidies and spillovers and did not incorporate the degree of R&D cooperation into the frame of analysis.

In this article, we emphasize the degree of R&D cooperation as an endogenous in a duopoly market and examine the optimal R&D subsidy rate under different R&D cooperation conditions. And then we discuss the effects of both R&D cooperation level and technological spillovers on firms’ R&D investment, output, profit and social total welfare.

MODEL

Consider an industry with two firms facing an inverse demand function \( p(Q) = a-bQ \), where \( a>0 \) and \( Q = q_1+q_2 \) is the total quantity produced. And the firm i carry out cost-reducing R&D investment \( x_i \), at a cost \( y x_i^2/2 \), where \( y \) represents the relative effectiveness of R&D and is
assumed that $\gamma > 2$ so that the R&D cost function is convex enough to ensure that the second order conditions for R&D maximization problems hold (Banerjee and Lin, 2003). In addition, there are constant returns to scale in production, with unit costs of production $c_i$, a firm can lower its unit cost by engaging in cost-reducing R&D and it can also benefit from other firms' R&D through spillover, implying a marginal production cost $c_i = c - x_i - \beta x_i$, where $(i \neq j)$ and $\beta \in (0, 1)$ captures the degree of spillovers and $c$ is the initial unit production cost with $a > c$ (D'Aspremont and Jacquemin, 1988).

In order to encourage R&D cooperation, the regulator sets a subsidy policy which takes the form of a subsidy toward the costs of cooperative R&D. Thus, the R&D cost function of the firm $i$ becomes 
$$(1 - \lambda \delta) x_i / 2,$$
where $\delta$ is the subsidy parameter with $0 < \delta < 1$ and $x_i$ captures the degree of R&D cooperation. Clearly, the extreme values $\lambda = 0$ and $\lambda = 1$ represent, respectively, non-cooperative or full cooperative R&D, intermediate values for yields imperfect cooperation. Therefore, the profit function of firm $i$ is given by:

$$\pi_i = (p(Q) - c_i) q_i - (1 - \lambda \delta) x_i / 2$$

The timing in the model is as follows. In the first stage, the regulator sets a subsidy policy, firms choose their cooperative level and the cost-reducing R&D simultaneously while at the same time receiving a subsidy on their R&D cost. Finally, firms compete in the product market by choosing output.

The game is solved proceeding backwards. In the output stage firms simultaneously choose quantities to maximize profit, taking the R&D investments and subsidy as given. The first-order conditions of the maximization program are given by:

$$a - b(q_i + q_j) - c = q_i$$

Given that firms are identical ex ante, they take the same decisions ex post. The Nash-Cournot equilibrium can be computed to be:

$$q_i = a - c + (2 - \beta) x_i - (1 - 2\beta) x_i / 3b$$

(1)

At the preceding stage, in which firms choose R&D levels, profits can be written as:

$$\hat{\pi}_i = b q_i^2 - (1 - \lambda \delta) x_i^2 / 2$$

In the R&D cooperative stage, for a cooperative level given $\lambda$, firm $i$ sets its R&D levels so as to maximize the sum of its profit and a fraction $\lambda$ of the profit of the firm $j$. Therefore, owners of firms are assumed to maximize:

$$q_i = \hat{\pi}_i + \lambda \hat{\pi}_j (i \neq j)$$

The first-order condition is:

$$\frac{\partial q_i}{\partial x_i} = 2q_i \frac{\partial \hat{\pi}_i}{\partial x_i} - (1 - \lambda \delta) x_i + 2q_i \frac{\partial \hat{\pi}_j}{\partial x_i} = 0$$

which, from Eq. 1 becomes:

$$2(2 - \beta)[a - c + (1 + \beta)x_i] + 2\lambda(2\beta - 1)$$

$$[a - c + (1 + \beta)x_i] - 9b(1 - \lambda \delta) x_i = 0$$

In the symmetric equilibrium:

$$x_i^* = \frac{2(a - c)[(2 - \beta) + (2\beta - 1)\lambda]}{9b(1 - \lambda \delta) - 2[2(2 - \beta) + (2\beta - 1)\lambda](1 + \beta)}$$

(2)

where, $9b(1 - \lambda \delta) - 2[2(2 - \beta) + (2\beta - 1)\lambda](1 + \beta) > 0$ and $a > c$.

Using Eq. 2 we obtain output per firm:

$$q_i^* = \frac{3y(1 - \lambda \delta)(a - c)}{9b(1 - \lambda \delta) - 2[2(2 - \beta) + (2\beta - 1)\lambda](1 + \beta)}$$

(3)

and profit per firm is:

$$\bar{\pi}_i = \frac{y(1 - \lambda \delta)(a - c)^2}{9b(1 - \lambda \delta) - 2[2(2 - \beta) + (2\beta - 1)\lambda](1 + \beta)}$$

(4)

where, $9b(1 - \lambda \delta) - 2[2(2 - \beta) + (2\beta - 1)\lambda]^2 > 0$ from the second-order condition.

In the first stage, the regulator chooses the R&D subsidy parameter $\delta$ to maximize total welfare. Total welfare in the present setting consists of the sum of consumer surplus and firms' profit and then minus the R&D subsidy:

$$W(\delta) = \int_a^b (a - bx) dx - \int_a^b [a - (1 + \beta)x_i^*] k_i dx$$

$$-2(1 - \lambda \delta) x_i^* / 2 - \lambda \delta x_i^*$$

which is equivalent to:

$$W(\delta) = 2(a - c) + (a + \beta)x_i^* k_i - 2bq_i^2 - \lambda \delta x_i^*$$

(5)

Substituting output and R&D from Eq. 2-3, respectively and then solving the associated first-order condition:
\[ \frac{\delta W(x)}{\delta} = \frac{9b\gamma^2(a - c)^2(1 + \beta)(1 - 2\lambda \delta)(2 - \beta + (2\beta - 1)\lambda)}{4.5b\gamma(1 - \lambda \delta)(2 - \beta + (2\beta - 1)\lambda)(1 + \beta)} = 0 \]

Because only firms undertake R&D, they would get R&D subsidy. And according to Eq. 2, when \( x^* > 0 \), it is easy to see that \( (1 - 2\beta + (2\beta - 1)\lambda) < 0 \). So, we obtain the optimal R&D subsidy parameter \( \delta^* = 1/2\lambda \). So, the R&D subsidy in \( \delta \lambda = 1/2\lambda \).

**Proposition 1:** In a duopoly market, when firms act non-cooperatively in output and cooperatively in R&D, the best subsidy toward the costs of cooperative R&D is constant \( 1/2 \), whether the degree of R&D cooperation is low or high.

Substituting \( \delta^* = 1/2\lambda \) into Eq. 2-5, we obtain the equilibrium values for R&D, output, profit and total welfare:

\[ x^* = \frac{(a - c)(2 - \beta + (2\beta - 1)\lambda)}{2.25b\gamma - (2 - \beta + (2\beta - 1)\lambda)(1 + \beta)} \]

\[ q^* = \frac{0.75(a - c)}{2.25b\gamma - (2 - \beta + (2\beta - 1)\lambda)(1 + \beta)} \]

**SIMULATION ANALYSIS**

The question addressed in this section is: What is the effect of changes in technological spillovers and R&D cooperation level on R&D investment, output, profit and total welfare. However, because the resulting problem would be too difficult to solve, we use computer simulations to explore the impact of \( \beta \) and \( \lambda \) on the equilibrium values. And in the basic simulation model, four parameters \( a, b, c \) and \( \gamma \) were set to 100, 20, 0.6 and 7.

**Effects of cooperative R&D level:** In order to explain the effects of technological spillovers, we select five specific values, 0.1, 0.3, 0.5, 0.7, 0.9 of \( \beta \) during a simulation run and Fig. 1a-d are generated.

In Fig. 1a-b the values of R&D and output are shown as functions of \( \lambda \). As the two figures make clear both \( x^* \)

![Graphs showing effects of \( \lambda \) on various parameters](image)

**Fig. 1(a-d):** (a) Effects of \( \lambda \) on \( x \), (b) Effects of \( \lambda \) on \( q \), (c) Effects of \( \lambda \) on \( \pi \) and (d) Effects of \( \lambda \) on \( W \)
and \(q^*\) decrease with \(\lambda\) when \(\beta = 0.1, 0.3\) and increase with \(\lambda\) when \(\beta = 0.7, 0.9\). In particular, both \(x^*\) and \(q^*\) are constants when \(\beta = 0.5\).

**Proposition 2:** In a duopoly market: (1) If \(0 < \beta < 0.5\), then both the firm’s R&D and output decrease with cooperative R&D level \(\lambda\). (2) If \(0.5 < \beta < 1\), then both the firm’s R&D and output increase with cooperative R&D level \(\lambda\) and (3) If \(\beta = 0.5\), then both the firm’s R&D and output are constants.

In Fig. 1c-d the values of a firm’s profit and total welfare are also shown as functions of \(\lambda\). As the two figures make clear both \(\pi^*\) and \(W^*\) increase with \(\lambda\) when \(\beta = 0.5\). In particular, both \(\pi^*\) and \(W^*\) are constants when \(\beta = 0.5\). The two figures also show that when \(\beta = 0.1, 0.3, 0.5, 0.7, 0.9\), respectively, both the firm’s profit and social total welfare satisfy \(\pi_{\lambda = 0.1} > \pi_{\lambda = 0.3} > \pi_{\lambda = 0.5} = \pi_{\lambda = 0.7} < \pi_{\lambda = 0.9}\) and \(W_{\lambda = 0.1} > W_{\lambda = 0.3} > W_{\lambda = 0.5} = W_{\lambda = 0.7} < W_{\lambda = 0.9}\).

**Proposition 3:** In a duopoly market: (1) If \(\beta > 0.5\), then both the firm profit and social total welfare always increase with cooperative R&D level \(\lambda\) and (2) If \(\beta = 0.5\), then both the firm’s profit and social total welfare are all constants.

Combining the results in propositions 1, 2 and 3, the following corollary is immediate:

- **Corollary 1:** Under positive technological spillover with the best R&D subsidy, the duopoly should choose R&D cooperation to maximize their profit and the consequent enhance the social total welfare.

**Effects of technological spillovers:** In order to explain the effects of Technological spillovers, we select five specific values, 0.1, 0.3, 0.5, 0.7, 0.9 of cooperative R&D level \(\lambda\) during a simulation run and Fig. 2a-d are generated.

In Fig. 2a-b the values of R&D and output are shown as functions of \(\beta\). As figure 2a make clear \(x^*\) decreases with \(\beta\) when \(\lambda = 0.1, 0.3\) and increases with \(\beta\) when \(\lambda = 0.5, 0.7, 0.9\) and \(q^*\) always increases with \(\beta\). Further-more Fig. 2a-b also show that when \(\lambda = 0.1, 0.3, 0.5, 0.7\) and 0.9, respectively, the firm’s R&D and output

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**Fig. 2 (a-d):** (a) Effects of \(\beta\) on \(x\), (b) Effects of \(\beta\) on \(q\), (c) Effects of \(\beta\) on \(\pi\) and (d) Effects of \(\beta\) on \(W\).
satisfy $x^{*}_{81}>x^{*}_{82}>x^{*}_{83}>x^{*}_{87}>x^{*}_{89}$, $q^{*}_{91}>q^{*}_{92}>q^{*}_{93}>q^{*}_{94}$, with $0<\beta<0.5$ and $x^{*}_{01}<x^{*}_{02}<x^{*}_{03}<x^{*}_{07}<x^{*}_{09}$, $q^{*}_{01}<q^{*}_{02}<q^{*}_{03}<q^{*}_{07}<q^{*}_{09}$ with $0.5<\beta<1$.

Proposition 4: In a duopoly market (i) If $0<\lambda<0.5$, then the firm's R&D decrease with technological spillover $\beta$, (ii) If $0.5<\lambda<1$, then the firm's R&D increase with technological spillover $\beta$, (iii) When $0<\lambda<1$, the firm's output always increase with technological spillover $\beta$, (iv) If $0<\lambda<1$ and $0<\beta<0.5$, then the greater is cooperative R&D level $\lambda$, the smaller should be the firm's R&D and output and (v) If $0\lambda<1$ and $0.5<\beta<1$, then the greater is cooperative R&D level $\lambda$, the greater should be the firm's R&D and output.

In Fig. 3c-d the values of a firm's profit and total welfare are all shown as increasing functions of $\lambda$, in other words, both $\pi^*$ and $W^*$ increase with $\lambda$. And the two figures also make clear that when $\lambda = 0.1, 0.3, 0.5, 0.7, 0.9$, respectively, the firm's profit and social total welfare satisfy $\pi^{*}_{31} \leq \pi^{*}_{32} \leq \pi^{*}_{33} \leq \pi^{*}_{37} \leq \pi^{*}_{39}$ and $W^{*}_{31} \leq W^{*}_{32} \leq W^{*}_{33} \leq W^{*}_{37} \leq W^{*}_{39}$.

Proposition 5: In a duopoly market, if the two firms undertake R&D cooperation, then (1) Firm's profit and social total welfare all increase with technological spillover $\beta$. (ii) the greater is cooperative R&D level $\lambda$, the greater should be the firm's profit and social total welfare.

Combining the results in propositions 1, 4 and 5, the following corollary is immediate:

- **Corollary 2**: Under R&D cooperation with the best R&D subsidy, the greater is the technological spillovers $\beta$, the greater should be the firm's profit and social total welfare which the duopoly can gain from R&D cooperation.

CONCLUSION

This study focused on R&D cooperation in duopoly with spillowers and subsidy, in which case optimal subsidy policy should balance a trade-off between market power and efficiency. Furthermore, even if there is growing evidence that R&D cooperation facilitates firms' profit and social total welfare, it is unclear whether imperfect cooperation would yield an even better outcome in terms of firms' profit and total welfare with different technological spillovers. To explore these issues, we introduce a parameter that captures different degrees of R&D cooperation and examine the optimal degree of R&D subsidy and under which the effect of R&D cooperation level and technological spillowers on R&D, output, profit and total welfare.

The analysis shows that the optimal degree of R&D subsidy is a constant 1/2. And under the best R&D subsidy condition, considering positive technological spillowers, the duopoly should choose R&D cooperation to maximize their profit and the consequent enhance the social total welfare, furthermore, the greater is the technological spillowers, the greater should be the firm's profit and social total welfare which the duopoly can gain from R&D cooperation. Finally, numerical simulations confirm the results.

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REFERENCES


