Performance Analysis of Random Waypoint Mobile Model Based on Asymptotic Spatial Distribution Theory

Hongde Zhang, Weiren Shi, Hui Han and Wei Zhou
Chong Qing University, Chong Qing, 400015, China
Chong Qing Key Laboratory of Emergency Communication, Chong Qing, 400035, China
Chong Qing Communication Institute, Chong Qing, 40035, China

Abstract: This study analyses the performance of the random waypoint mobile model in mobile sensor network based on asymptotic spatial distribution theory, which is mainly aimed at analyzing the connectivity and coverage performances of the network, in order to obtain the maximized coverage with connectivity performance. Meanwhile, it also analyses the relationship between the length of the longest edge of the Minimum Spanning Tree (MST) and the minimum transmission distance of node in Geometric Random Graph and provides the minimum transmission distance of mobile sensor network for ensuring connectivity. Finally, the theoretical analysis results are verified based on the method of experimental simulation, which shows that the bigger the number of the node, the more consistent the results of theoretical analysis and experimental result, higher reliability.

Key words: Random waypoint, mobile model, asymptotic spatial distribution, geometric random graph

INTRODUCTION

Random waypoint model is one of the most common mobile models used in the mobile sensor network, which has been realized in NS2 and other large-scale simulation softwares. This mobile model belongs to the active movement and the expected network performance is the connectivity and the coverage performance of network. So, research on this content has a very important significance. Because if all transmission range of network nodes are set to be the minimum transmission range value in order to guarantee the connectivity coverage, it can obtain the maximum network capacity (Dong et al., 2013; Wang et al., 2010) at the same time minimizing the energy consumption of network. Meanwhile, this research content is also very important to the other research fields of mobile sensor network, including routing protocol (Chakraborty and Khan, 2013; Singh et al., 2010; Okdem et al., 2011; Karaboga et al., 2012) and target tracking algorithm (Ahmed et al., 2010; Li et al., 2011). The impact from the mobility to the routing protocol includes: (1) Frequent routing configuration; (2) Contingent network disconnection. Obviously, frequency of the network disconnection depends on the set of node transmission range. The bigger the transmission range is, the easier to be connected but with bigger energy consumption. Furthermore, the intercommunication interference between nodes will reduce network capacity. On the other hand, if the node transmission range is set to be too small, it will cause the network not to be connected frequently, so as to trigger low packet transmission rate and at this moment it will be wrong to think the routing protocol lacking of route maintenance function. Similarly, the condition of failing target tracking caused by the network disconnection also will be mistook for the performance deficiency of the tracking algorithm. Therefore, to the mobile sensor network, it is very important to set a reasonable node transmission distance to ensure the connectivity of the network.

Research in this study is on the premise that assuming each node is in homogeneous, namely transmission distances of all the nodes are the same, aiming at commonly used mobile model, namely the Random WayPoint Mobility Model (RWP) (Martinez et al., 2011). This study is aimed at what the biggest coverage performance is, on the premise of ensuring the network connected. That is to say, it should ensure the minimum transmission distance needed by ensuring the connectivity coverage under the circumstances number of the nodes have been known, or ensure the number of the minimum network node under the circumstances ensuring the network connected. At present, there are a few results which can be directly referred to the research on this question in the mobile sensor network, but part theoretical results of AdHoc self-organizing network has been grown now. So, this study mainly draws on these theoretical results.

Corresponding Author: Weiren Shi, Chong Qing University, Chong Qing, 400015, China
The question of network coverage based on the self-organizing network for ensuring its connectivity is mainly aimed at researching the minimum transmission distance of the nodes. At present, the current research is focused on static networks, assuming the node complying with uniform distribution. In Wan et al. (2010), the writer researches the transmission distance meeting the connectivity condition in unit circle area:

\[ r = \sqrt{\frac{\ln n + c(n)}{mn}} \]  

(1)

wherein, \( n \) is the node number, \( c \) is the function of the number of a certain node, of which the condition for establishing the above equation is that the function of \( n \) must satisfy \( c(n) \to \infty \). The minimum transmission distance which can satisfy the connectivity of high probability network is called critical transmission range. In Kim (2012), the writer researches the condition of the node distributed in a unit circle area \( R = (0, 1)^2 \) and gives a similar expression on the critical transmission range, namely to a certain constant:

\[ r = c\sqrt{\frac{\ln n}{n}} \]

(2)

All of the above researches are aimed at the static network, as well as it is very few to aim at the mobile network. Chen et al. (2012) is focused on researching RWP which is a most commonly used mobile model. However, the writer does not provide any explicit mathematical formulas to the detailed minimum transmission distance of mobile network, only conducting qualitative research in experiments. In addition, Sperling et al. (2012) studied its critical transmission range to the static network and mobile network by using a simulation tool. As the results show that, comparing with static network, mobile network can not obtain good network connectivity until its transmission range increased 25% as the cost.

THEORY OF GRG AND ASYMPTOTIC SPATIAL DISTRIBUTION

The traditional random graph theory can not be directly applied in the sensor network. Because in the random graph theory, the probability assuming two nodes directly connected by edges are independent. But for the wireless communication network, this assumption is not set up. GRG (Geometric Random Graph) (Thiedmann et al., 2011) generated from the random graph theory is suitable for the research of mobile sensor network. Because GRG does not simply consider it just an abstractions composed of point set connected by the edges, but a geometry composed of the point set in d-dimensional space, wherein connectivity between the nodes are determined based on proximity relation. Therefore, this study uses the geometric random graph model. In GRG model, the set composed of \( n \) nodes is distributed in a d-dimensional area with a certain probability density function, in which the connectivity probability between the nodes is determined by a rule. For example, whether it is connected or not is determined according to Euclidean distance between the nodes, as well as when the distance is larger than a certain value \( r \), the nodes are connected.

The longest edge theory of Minimum Spanning Tree (MST) (Penrose, 1999) of GRG is used in this study.

Theorem 1: Assuming that \( n \) nodes are distributed independently with a probability density function \( F \) in the region \( R \) randomly, \( \Omega \) in this region is connected with smooth boundary, \( F \) is also continuous in the boundary of the region, \( M \), represents the length of the longest edge of MST established based on the set of the nodes, then the following equation is generated when the \( n \) is big enough:

\[ \frac{mn(M)}{\log n} = \frac{1}{\min_{\Omega} F} \]

(3)

To the mobile network, probability density function of the nodes is relatively complex comparing with the static network, which is determined by the characteristic of the mobile model of nodes. This study analyzes these problems based on the asymptotic node spatial distribution of mobile network.

Definition 1: Asymptotic node spatial distribution:

Assuming that there are \( n \) nodes distributed in a particular area \( R \) according to a certain probability density function \( F \) in the beginning and then the nodes begin to move according to a certain mobile model \( M \), the asymptotic node spatial distribution of the mobile network \( M \) as \( F \) being its initial distribution is defined as:

\[ F_n = \lim_{t \to \infty} F_t \]

(4)

wherein, \( F \) is the probability density function of the node spatial distribution at the time period \( t \). If the formula on the right does not exist, it represents that the mobile model as \( F \) being its initial distribution is unstable.

At present, in the known asymptotic distribution models, RWP is the only model commonly used (Santi, 2005).
**Theorem 2:** Assuming that the node moves in the random waypoint model in $\mathbb{R}(0, 1)^2$, pause time is $t_p$ and speed is $v$, then its asymptotic spatial density function is:

$$F_{\text{asym}} = \begin{cases} P_{\text{pause}} & (x, y) \in [0, 1]^2 \\ 0 & \text{otherwise} \end{cases}$$

(5)

Where in:

$$P_{\text{pause}} = \frac{t_p}{t_p + \frac{0.521405}{v}}$$

(6)

$$F_x = \begin{cases} 0 & (x - 0) \text{or} (y - 0) \\ F_x(x, y) & \text{otherwise} \end{cases}$$

(7)

$F_x$ is determined by the values of $x$ and $y$ coordinates.

**NETWORK PERFORMANCE ANALYSIS OF RANDOM WAYPOINT MODEL**

In Eq. 3, definition of $M_n$ is that there is a determined relationship between the length of the longest edge of MST established based on the set of the nodes and the minimum transmission distance for ensuring the connectivity of mobile sensor network.

**Theorem 3:** If putting $n$ nodes in area $\mathbb{R}(0, 1)^2$, the minimum transmission distance $r_{\text{min}}$ composed of $n$ nodes for ensuring network connectivity is equal to the length of the longest edge of MST composed of the $n$ nodes in Euclidean distance.

This theorem is proved as follows:

- Making $e$ as the longest edge of $T$ of a network and $l(e)$ as its length.
- If distributing $l(e)$ to each node in homogeneity, it will produce a figure containing $T$ (namely the $T$ is a sub-figure), thus there exists a minimum transmission distance $r_{\text{min}} \leq l(e)$ to ensure the connectivity.

If deleting the edge $e$, it will produce two connected sub-figures, namely $T_1$ and $T_2$ as shown in Fig. 1. Assuming $P_1$ and $P_2$ are any two nodes of $T_1$ and $T_2$, that is $p_1 \in T_1$, $p_2 \in T_2$. According to the definition of MST we can get to know the edge $e$ is the shortest distance connected to $\{p_1, p_2\}$ and the distance between any one node in $T_1$ and any one in $T_2$ is at least $l(e)$, which means the figure will not be connected if transmission distance is less than $l(e)$. So, has the equation $r_{\text{min}} \geq l(e)$.

![Fig. 1: MST after the longest edge is deleted](image)

In summary we can obtain $r_{\text{min}} = l(e)$.

The theorem 3 has been proved.

Combing the above theorems we can see, the Theorem 1 indicates that the minimum transmission distance for ensuring network connectivity in any probability density function $F$ is only determined by the minimum value of $F$, therefore the Theorem 1 can be changed into:

$$\frac{\ln(r_{\text{min}})^2}{\log n} = \frac{1}{\min F}$$

(8)

The condition for Theorem 1 is that the boundary of the area in network is smooth, but in $\mathbb{R}(0, 1)^2$ the boundary may not be very smooth because of the existing of corner, but it can be solved by smoothing the corner. Thus, to the mobile sensor network composed of $n$ nodes, assuming just moving as a certain mobile model, the minimum transmission distance for ensuring the network connectivity can be obtained only by obtaining the minimum value of the asymptotic node spatial distribution $F$ of this mobile model.

If using $F_{\text{asym}}$ to represent the asymptotic distribution of RWP mobile model, it can be divided into two parts:

$$F_{\text{asym}} = F_r + F_u$$

(9)
In which \( F_n \) is heterogeneous component caused by the movement of its nodes; however, \( F_p \) is homogeneous component caused by the pause (namely the stop) of the nodes. The heterogeneity measurement of \( F_{RWP} \) is determined by relative strength of the homogeneous component \( F_n \) and heterogeneous component \( F_p \). The longer the pause time is, the bigger the \( F_p \) is, but the smaller the heterogeneity of \( F_{RWP} \) is, in extreme cases, if the pause time is 0, namely the node is moving all the time without any stop, the heterogeneous component \( F_m \) is the smallest. Therefore in Theorem 2:

\[
F_p = P_{\text{pause}} \tag{10}
\]

\[
F_n = (1 - P_{\text{pause}}) F_m (x, y) \tag{11}
\]

According to Theorem 3, we can obtain:

\[
\min_{RWP} = P_{\text{pause}} \tag{12}
\]

Hence:

\[
\frac{n \sigma(t_m)^2}{\log n} = \frac{1}{P_{\text{pause}}} \tag{13}
\]

Combing the above Eq. 13 and 6, we can obtain the following relation theorem between the minimum transmission distance for ensuring the network connectivity and the number of the nodes.

**Theorem 4**: Assuming that \( n \) nodes move in the random waypoint model in \( R(0, 1)^2 \), pause time is \( t_p \) and speed is \( v \), then the minimum transmission distance for ensuring the network connectivity is:

\[
t_{\text{min}} = \frac{1}{P_{\text{pause}}} \sqrt{\log n \over \pi n} = \sqrt{\log n \over \pi n} \tag{14}
\]

From the above equation we can see, the longer the pause time in RWP mobile model is, the even the distribution is, but the smaller the relevant minimum transmission distance is.

**SIMULATION EXPERIMENT**

In order to verify the above mobile sensor network connectivity coverage conclusion given, namely the relation between the minimum transmission distance for ensuring the network connectivity and the number of the nodes, this study does a detailed simulation experiment to the random waypoint model.

Environment for the simulation experiment is set up as follows: Number of the node of mobile sensor network is \( n \), coverage area is \( R(0, 1)^2 \), values for each experiment are from 10 to 5000, moving step number of the node is 10000 times with the purpose of ensuring the node distribution can be restrained into the asymptotic spatial distribution, each experiment is repeated for 10000 times, finally to obtain the averaging. Parameters in RWP models include \( t_p \) shorting for pause time, as well as the moving speed is a fixed value, namely \( v = 0.01 \).

Process of the experiment is that: First evenly distributing the \( n \) nodes into the area \( R(0, 1)^2 \) independently and randomly, then moving according to the parameter set in RWP, after moving with 10000 steps, calculating the length of the longest MST edge, this value is acted as the minimum transmission distance for ensuring network connectivity. Figure 2 and 3 are the experimental results with \( t_p = 100 \) and \( t_p = 200 \).

From Fig. 2 and 3 we can see, the experimental results and theoretical analysis results are basically consistent,
but the trend has certain rules, namely when the node number is very small, such as less than 200, the consistency between the theoretical and actual values is poor, on the contrary, when the node number increases, greater than 500, the theoretical and experimental values are substantially consistent, especially when the node number increases to 1500, the error is extremely small.

On the other hand, comparing the two figures we can see, to the two mobile sensor networks with the same node number, such as n = 500, when the pause time in random waypoint model is very small, the error between theoretical and actual values is much smaller, as well as both of the two values are bigger than the minimum transmission distance of the network with longer pause time. In this experimental result, under this circumstance n = 500, when t_p = 100, theoretical value of the minimum transmission distance of the network is 0.078, actual value is 0.112, error is 0.034; as well as when t_p = 200, the theoretical value of the network is 0.071, actual value is 0.134, error is 0.063.

In order to further research the relation between node number and minimum transmission distance in the mobile sensor network with high moving speed, in this study the mobile speed is set up to be 0.1 for further experiment, of which the experimental results are shown in Fig. 4 when pause time t_p = 100, as well as shown in Fig. 5 when pause time t_p = 200.

By comparison we can see, under the circumstance of network node moving in a high speed, the former experimental results still stand, namely the bigger the node number is, the more consistent the theoretical and actual values are, as well as the smaller the error between the theoretical and actual values when the pause time of the random waypoint model is very small, in which both of the two values are a bit bigger than the minimum transmission distance with longer pause time.

From Fig. 4 and 5 we can see, when the mobile sensor network moves just as the random waypoint model with much higher speed, both the theoretical and actual values of the minimum transmission distance become smaller. This is because following the speed of node increasing, the node is equal to become thick in unit time, the connectivity degree of the node is increased, so that the needed transmission distance also becomes smaller.

CONCLUSION

In this study, it analyzes the connectivity and coverage of the mobile sensor network performances in random waypoint model based on the asymptotic spatial distribution theory, which obtains the relation between node number and minimum transmission distance for ensuring the connectivity, as well as using the simulation experiment method to analyze its theoretical and experimental values, of which the experimental results show that, the two results trend to be consistent when node number is very big. However, this conclusion is not suitable for the particular case of the random waypoint model, namely t_p = 0, which requires further study.

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