FAHP Weight Method for Hydraulic Metal Structure Health Diagnosis

1,2 Guangming Yang and 1 Pande Jing
1 College of Energy and Electrical Engineering, Hohai University, Nanjing 210098, China
2 Research Center for Renewable Energy Generation Engineering, Ministry of Education, Nanjing 210098, People’s Republic of China

Abstract: Considering the characteristics of AHP and fuzzy comprehensive evaluation, the fuzzy consistent matrix is being drawn into AHP. In view of respective advantages and complementarity between the two methods, the FAHP weight method is put forward, which is applicable for hydraulic metal structure health diagnosis and effective in practice. The hierarchy model and corresponding judgment matrix for hydraulic metal structure healthy diagnosing are established, which improve the consistency inspection standard, the constitution and adjustment method of comparative judgment matrix. The results show that fuzzy analytic hierarchy process weight method make the weight results more scientific and reasonable, the analysis and diagnosis conclusion more credible. In addition, it provides the necessary theory foundation and new ideas for the development of hydraulic metal structure health diagnosis technology.

Key words: Hydraulic metal structure, weighting, FAHP method, fuzzy consistent matrix, weight coefficient

INTRODUCTION

Hydraulic metal structure health diagnosis is a multi-level, multi-standard and multi-factor comprehensive diagnosis system, which includes quantitative indexes and qualitative indexes. Diagnosis indexes are the basis and key elements to build the system and the determination of index weight is directly related to the rationality and reliability of the diagnosis results. At present, many ways of determining the weight are roughly divided into two groups (Hui, 2009; Ma et al., 1999) subjective weight method and objective weight method. The subjective weight method, which generally adopt qualitative method, reflects the experts and decision-makers of their subjective judgment or intuition to each factor weight, like the Delphi method, Analytic Hierarchy Process (AHP), sequential evaluation method and entropy weight method, etc. The weights are obtained by experts’ experience and subjective judgment on account of the method. The objective weight method is concluded by the correlation between indexes or each index variation coefficient, which mainly includes principal component analysis, variation coefficient method and fuzzy comprehensive evaluation method, etc. Based on the method, the weights are obtained by the statistics analysis with objective information. Each method has two sides and applicable scope. Due to the reduction of subjective factors, problems essence and the logic relations between system elements, the fuzzy analytic hierarchy process (FAHP) weight method is put forward.

In addition, fuzzy consistent matrix is being drawn into AHP, which is applicable for hydraulic metal structure health diagnosis and effective in practice.

Based on the hydraulic metal structures health diagnosis index system and the basic principle of fuzzy multi-level weight analytical method (Yang, 2012) the author had constructed, the hydraulic metal structures health diagnosis hierarchical structure model is established. By using Fuzzy Analytic Hierarchy Process (FAHP) method, the relative weight of indexes layer for the standard layer can be calculated. The aim of the research is to put forward a scientific health diagnosis method (FAHP), making it reasonably using in diagnosis case. Taking the gate as an example, the related case calculation and analysis is proceeded.

FAHP METHODOLOGY

Problem raised: Fuzzy comprehensive evaluation method (Chen et al., 1992) use the principle of fuzzy transformation and maximum membership degree, considering the diagnostic target related multi-objective and multi-level factors and making comprehensive evaluation for diagnostic target. Fuzzy comprehensive evaluation method, which apply fuzzy information theory to integrated diagnosis, overcome the defects of possible deviating from the objective reality in different degrees in traditional comprehensive diagnosis and solving the problem of the fuzziness and uncertainty judgment excellent. But the method also exist some disadvantages.

Corresponding Author: Guangming Yang, College of Energy and Electrical Engineering, Hohai University, Nanjing 210098, People’s Republic of China
at the same time. Fuzzy comprehensive evaluation method is a kind of indirect method. In order to find out membership function of each factor, it is necessary to put each index characteristic processing, which lead to a certain degree of error. And in the process of evaluation, it is hard for policy makers to accurately grasp the fuzzy boundaries, etc.

Compared with fuzzy analysis method, AHP method (Saaty, 1980), which combine quantitative and qualitative analysis, is a multi-objective and multi-criteria decision-making method. The complex problem is decomposed into each component factors, which form an orderly hierarchy structure according to the influence relationship between these factors. The relative importance of all factors relative to an index is determined by multiple comparison in the structure. With expert judgment comprehensive, the total order of each factor relative importance is determined. But it also has limitations, with subjective factors heavy, the constructed judgment matrix is often difficult to meet the requirement of consistency and the subjective consciousness influence the utility effect of diagnostic method.

For the proposition of comprehensive health diagnosis scheme, the mapping relationship is always some uncertainty and fuzzy. The result is often some fuzzy concepts in diagnosis and judgment, such as "excellent", "good", "medium", "bad" or "fine", "qualified", "unqualified", etc. It is also need to consider and determine the effect size of various factors in the diagnosis. In the view of respective advantages and the mature complementarity between the two methods, based on domestic and foreign scholars research achievements (Ma et al., 1999; Chen et al., 1992; Saaty, 1980; Zhang and Qiang, 2006; Wang, 2003; Du, 2000; Fan et al., 2002; Forman and Gass, 2001; Zhang, 2000), fuzzy consistent matrix being drawn into AHP and the FAHP weight method is put forward by author, which is applicable for hydraulic metal structure health diagnosing and effective in practice.

**Basic principle of FAHP method:** FAHP, also called fuzzy multi-level weight analytic method, improve the comparative judgment matrix construction and adjustment and consistency inspection standards on the basis of AHP, making consistency inspection and adjustment work more scientific, accurate and simple in AHP.

Firstly FAHP method need to construct the fuzzy consistent matrix. Fuzzy consistent matrix R shows that hierarchy elements, which are related to an upper hierarchy element, make relative importance comparison with the upper hierarchy element. Definition the fuzzy consistent matrix as follows:

Setting matrix $R = (r_{ij})$, if meet:

$$0 \leq r_{ij} \leq 1, \text{ (i,j = 1,2,...,n)}$$

where, R is fuzzy matrix.

If fuzzy matrix $R = (r_{ij})$ meet $\forall i, j, k$

$$r_{ij} = r_{ik} - r_{jk} + 0.5 \ (i,j,k = 1,2,...,n)$$

where, Fuzzy matrix R is called fuzzy consistent matrix. And the fuzzy consistent matrix is must be fuzzy complementary, i.e.

$$r_{i} + r_{j} = 1$$

where, In type (3): The fuzzy consistent matrix R has the following properties: $r_{i} = 0.5, i = 1,2,..,n, r_{ij} = 1-r_{ij}, i,j = 1,2,..,n; n$ is the elements sum of the i-th row and the j-th column in matrix R.

In fuzzy consistent matrix R, $r_{i}$ quantitatively describe the importance of element $c_{i}$ and $c_{j}$ relative to an upper criterion, which has a fuzzy relationship "the membership degree of who is more important than who" and adopting 0.1-0.9 scale in Table 1.

The consistency test of fuzzy consistent matrix is based on the following theorem (Zhang, 2000) when any specified two lines corresponding elements proceed subtraction and the result is constant, the fuzzy complementary matrix $R = (r_{ij})$ is a necessary and sufficient condition for fuzzy consistent matrix.

**Basic steps:** Based on FAHP method proceeding hydraulic mental structure comprehensive health diagnosis, the basic steps are as follows:

**Step 1: Construction of hydraulic metal structures health diagnosis hierarchy structure**

Using FAHP method on the hydraulic metal structure health diagnosis index for empowerment, the diagnosis target, influence factor and the relationship between

<table>
<thead>
<tr>
<th>Scale</th>
<th>Definition</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>Equal importance</td>
<td>Two elements are equally important</td>
</tr>
<tr>
<td>0.6</td>
<td>Moderate importance</td>
<td>Compared two elements, one slightly important than other element</td>
</tr>
<tr>
<td>0.7</td>
<td>Obvious importance</td>
<td>Compared two elements, one significantly important than other element</td>
</tr>
<tr>
<td>0.8</td>
<td>More important</td>
<td>Compared two elements, one very important than other element</td>
</tr>
<tr>
<td>0.9</td>
<td>Extreme importance</td>
<td>Compared two elements, one top important than other element</td>
</tr>
<tr>
<td>0.1-0.4</td>
<td>Contrary comparison</td>
<td>The judgment of $c_{i}$ and $c_{j}$ multiple comparison is $r_{ij} = 1-r_{ij}$</td>
</tr>
</tbody>
</table>

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various factors all need to be determined at first and then all the related factors is layered according to the diagnostic target. The elements of a same hierarchy, as a standard, plays a dominate role to lower hierarchy factors, which also received domination from upper hierarchy elements; in the way the dominant relationship from top to bottom form hierarchical structure. This article adopt three-layer architecture which proposed by Saaty (1980) the target layer, middle layer (rule layer) and solution layer. Combined with the gate and hoist comprehensive health diagnosis index system framework, the health diagnosis ladder hierarchical structure mode of hydraulic metal structure was established as Fig. 1.

**Step 2: Determining the relative weights of each factor**

- **Construction fuzzy judgment matrix:** From the second sub-goal target of hydraulic metal structure comprehensive health diagnosis hierarchy model (including security, flexibility, durability, etc.), all factors of a hierarchy make multiple comparison, which is subordinated to upper hierarchy. In addition, the relative important extent is determined according to the evaluation scale and finally the fuzzy judgment matrix is established. Provided that the upper layer element B relate to the next layer elements of C_1, C_2,...,C_m, the fuzzy judgment matrix can be expressed as Fig. 1:

\[
R_{B,C} = \begin{bmatrix}
  r_{11} & r_{12} & \cdots & r_{1n} \\
  r_{21} & r_{22} & \cdots & r_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{m1} & r_{m2} & \cdots & r_{mn}
\end{bmatrix}
\]  

(4)

In the fuzzy judgment matrix R_{B,C}, the meaning of r_{ij} is shown in Table 1.

**Transform fuzzy judgment matrix into fuzzy consistent matrix:** The fuzzy judgment matrix consistency reflects the consistency of people thinking and judgment, which is very important in the fuzzy judgment matrix construction. If the established fuzzy judgment matrix accord with the consistency check theorem, then the judgment matrix satisfies the consistency requirement, which need not consistency adjustment. Considering the complexity of comprehensive health diagnosis, the constructed judgment matrix is always not consistent, which need to be adjusted according to sufficient and necessary condition of fuzzy consistency matrix. Specific steps are as follows.

**Calculation the sum of row:** According to the established judgment matrix in type 4, calculation the sum of row shows as follow:

\[
p_j = \sum_{i=1}^{n} r_{ij} \quad (i=1,2,...,n)
\]

(5)

Establish a precedence relation matrix:

\[
P = \begin{bmatrix}
  p_{11} & p_{12} & \cdots & p_{1n} \\
  p_{21} & p_{22} & \cdots & p_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  p_{m1} & p_{m2} & \cdots & p_{mn}
\end{bmatrix}
\]

(6)

In type: \( p_{ij} = (p_i - p_j) / 2n + 0.5 \) (j = 1, 2, ..., n).

**Determine the relative weight:** Using 9 grades judgment scale, the eigenvector of fuzzy consistent judgment matrix can be determined; then normalization processing be carried out, finally the indexes relative weight about the criterion layer can be calculated.
Step 3: General ranking of weight: By means of comprehensive calculation of each layer factors relative important weights, the comprehensive weight of all levels importance ranking relative to the top level (total goal) is gotten through synthetic.

Step 4: Identify specific index membership degree:
Membership degree reflects the extent of all elements subordinate to index levels fuzzy set. If the membership degree of index is higher, then element subordinate to the index fuzzy set is higher and vice versa. This article adopts the semi trapezoidal distribution method to construct membership functions.

Step 5: Fuzzy comprehensive diagnosis: The grade theory domain of hydraulic mental structure comprehensive health diagnosis is set as \( V = (A, B, C, D) \)

The specific index membership degree matrix is gotten by means of membership degree function. Then using the received weight multiplied by the membership matrix, the membership degree of target layer relative to diagnosis grade theory domain A, B, C, D is received. In addition, according to the maximum membership degree principle, the target layer belonging to which level is obtained, finally gaining the comprehensive health diagnosis conclusion.

RESULTS AND DISCUSSION

The FAHP method is used to calculate, analysis and determine the weight for gate health diagnosis in the following.

In gate, for instance, in order to make the quantitative description of any two factors relative importance, 5 scaling from 0.1 to 0.9 are commonly used to establish quantitative index. According to senior industry experts and design engineers evaluation to each middle layer relative importance, the judgment matrix \( R \) is obtained by each level standard factors multiple comparisons. The matrix \( R \) of the first standard (sub-goal) shows as follow:

\[
R = \begin{bmatrix}
0.5 & 0.8 & 0.7 \\
0.2 & 0.5 & 0.4 \\
0.3 & 0.6 & 0.5 \\
\end{bmatrix}
\]  

Checking the fuzzy judgment matrix consistency: If multiple subtraction of arbitrary two rows corresponding elements is constant in the matrix \( R \), the fuzzy judgment matrix \( R \) meet consistency requirement. According to the single weight formula:

\[
w_i = \frac{1}{n} \frac{1}{\alpha} \frac{1}{\zeta} \sum_{k=1}^{n} \frac{1}{t} (i = 1,\ldots,n)
\]  

In type: \( n \) is the order number of \( R \):

\[
\alpha = \frac{n-1}{2}
\]

where, Calculated: \( w_i = (0.50,0.20,0,0.3) \).

The same method is used to construct the secondary standard factors judgment matrix \( B_1, B_2, B_3 \), obtaining each weight vector:

- By means of the next level of safety indexes pair-wise, the judgment matrix \( R_{1i} \) of safety standard factors \( B_1 \) showed as follow:

\[
R_{1i} = \begin{bmatrix}
0.5 & 0.6 \\
0.4 & 0.5 \\
\end{bmatrix}
\]

The normalization weight set of various factors safety standard can be gotten by calculating: \( w_{2i} = (0.6,0.4)^T \).

- The judgment matrix of applicability standard factors \( B_2 \) showed in Table 2.

Calculating fuzzy consistent judgment matrix:

\[
R_{22} = \begin{bmatrix}
0.50 & 0.55 & 0.55 & 0.60 & 0.52 & 0.58 \\
0.45 & 0.50 & 0.50 & 0.55 & 0.47 & 0.53 \\
0.45 & 0.50 & 0.50 & 0.55 & 0.47 & 0.53 \\
0.40 & 0.45 & 0.45 & 0.50 & 0.42 & 0.48 \\
0.48 & 0.53 & 0.53 & 0.58 & 0.50 & 0.56 \\
0.42 & 0.47 & 0.47 & 0.52 & 0.44 & 0.50 \\
\end{bmatrix}
\]

The normalization weight set of various factors applicability standard can be gotten by calculating: \( w_{2i} = (0.1867,0.1667,0.1667,0.1467,0.1796,0.1538)^T \).

- The judgment matrix of durability standard factors \( B_3 \) showed in Table 3.

Table 2: Result of applicability index pair-wise judgment matrix

<table>
<thead>
<tr>
<th>( B_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
<th>( C_5 )</th>
<th>( C_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_2 )</td>
<td>0.5</td>
<td>0.6</td>
<td>0.6</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>0.3</td>
<td>0.4</td>
<td>0.4</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>0.5</td>
<td>0.6</td>
<td>0.6</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>( C_6 )</td>
<td>0.3</td>
<td>0.4</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

\( B_i \) is applicability standard factors; \( C_j (j = 1,\ldots,6) \) is the \( j \)th sub index of \( B_i \).

Table 3: Result of durability index pair-wise judgment matrix

<table>
<thead>
<tr>
<th>( B_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_2 )</td>
<td>0.50</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>0.10</td>
<td>0.50</td>
<td>0.40</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>0.20</td>
<td>0.60</td>
<td>0.50</td>
</tr>
</tbody>
</table>

\( B_i \) is durability standard factors; \( C_j (j = 1,\ldots,3) \) is the \( j \)th sub index of \( B_i \).
Using the same method constructed the third level standard factors of judgment matrix $C_1$-$C_7$, their weight coefficient can be received by corresponding calculation. By using FAHP to all levels index weight calculation, the sub-goals weight coefficient of safety, applicability and durability respectively are 0.5 0.2 and 0.3. In addition, the second and third level corresponding index weight coefficients are obtained and shown in Table 4.

According to the above method, each level corresponding index weight coefficient of hoist comprehensive diagnosis is also obtained.

By using FAHP, the index weight for each layer of hydraulic metal structure comprehensive health diagnosis is proceeded analytical calculation, finally obtaining the gate and hoist corresponding index weight coefficients.

Compared with the index weight coefficient result of AHP method by Yang and Jia (2011), it can be found that the result of each health diagnosis index, obtained from research and analysis by using FAHP and AHP, is consistent with the actual situation. In addition, the index weight coefficients diagnosis result of the two methods is basically consistent. At the same time, compared with traditional safety assessment method, AHP method and FAHP in gate comprehensive health diagnosis model (Yang, 2012, Yang and Jia, 2011), the final diagnosis results are basically identical. The hydraulic metal structure comprehensive health diagnosis based on FAHP can avoid artificial factors, which combines the respective advantages of AHP and fuzzy comprehensive evaluation method and its complementary. With fuzzy comprehensive diagnosis expert scoring evaluation, its diagnosis is closer to the actual situation of equipment and more scientific, reasonable and reliable. Therefore, there would be further illustrate and validate the rationality and reliability of the method mentioned by the authors.

**CONCLUSION**

- Considering the characteristics of AHP and fuzzy comprehensive evaluation, the fuzzy consistent matrix is being drawn into AHP. In view of respective advantages and complementarity between the two methods, the FAHP weight method is put forward, which is applicable for hydraulic metal structure health diagnosis and effective in practice. As a result, the method makes the person's subjective factors restricted to a single small scope, which reduce the difference between objective and subjective. It improve subjective diagnosis and index weight coefficient more accurate, making analysis and diagnosis conclusion more believable.

- According to diagnosis target, influence factors and the relationship between various factors, the hierarchical structure model and the corresponding fuzzy judgment matrix for hydraulic metal structure
healthy diagnosing are established. By improving the comparative judgment matrix construction, consistency inspection standards and comparative judgment matrix adjustment method, in the way, the consistency of constructed judgment matrix is adjusted, making empowerment results more scientific and reasonable. Each level index weight coefficient of gate and hoist at is obtained by using the FAHP weight method.

- FAHP solved the problems of matrix consistency and the difference between matrix and the person's thinking, which simplified the calculation process and improved the calculation efficiency. FAHP combine the fuzziness of fuzzy mathematics and the systematic of AHP, providing necessary theoretical basis and new ideas for development of the hydraulic metal structure health diagnosis technology.

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