Development of a Mathematical Model of Dual Fuel Engine Speed Characteristics

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Abstract: To test and analyze engine speed characteristics, most previous studies have adopted plots of speed and power and plots of speed and torque as characteristic curves of engine speed. This method of qualitative analysis does not look at other useful information in the data. To perform an in-depth analysis on the information embedded in the engine speed characteristics curve and provide quantitative parameters for dual fuel engine design or refit, this study uses a method of regression analysis to establish a parabolic model to describe the relationship between the power and rotational speed of a dual fuel engine and an inverse-logistic model to describe the relationship between torque and rotational speed. Using case studies, the model coefficients of determinations (R²) all exceeded 0.99 and the results include the rates of change of the power and torque, the maximum power and its confidence interval and the torque inflection point and steepest descent interval, therefore achieving a quantitative analysis of the dual fuel engine speed characteristics.

Key words: Dual fuel engine, speed characteristics, regression analysis, mathematical model, dynamic characteristics

INTRODUCTION

The energy crisis and environmental pollution are two major issues facing humanity, marking the development of alternative fuels for vehicles increasingly urgent. The alternative fuels under current development include natural gas and Liquefied Petroleum Gas (LPG). Dual fuel engine combustion processes and dynamics have been well-researched issues in the alternative fuels field.

Using a Back Propagation (BP) neural network, Dong et al., (2006) constructed a model to predict the heat release rate from a small-bore direct injection diesel engine's fuel injection system. Han-Yu et al. (2007) performed a comparative study of the smoke emission of the dual fuel engine and the original machine, mainly in the aspect of speed, load and replace ratio. Dong et al. (2008) proposed a multi-zone exothermic model of the dual fuel engine combustion characteristics and constructed differential equations to solve for the dual fuel engine heat release rate using measured indicators. Using a BP neural network (Dong et al., 2010) constructed a model that can predict the heat release of a biogas-diesel dual fuel engine and studied the impact on heat release by the dual fuel engine's performance parameters. Zeng-Gang (2010) proposed construction principles for and conducted an experimental trial on a high-power 190 diesel-clean natural gas dual fuel engine and performed power, economy and reliability analyses on the dual fuel engine. Guang-Yao (2011) analyzed two different oil cutoff control strategies for various electronically controlled dual fuel engines and high-pressure common rail electronically controlled engines. Kai (2012) studied and developed a diesel-natural gas dual fuel engine and added a dual fuel electric control system to allow the engine to work in two modes: A pure diesel mode and a diesel/natural gas dual fuel mode. Vijayabal and Nagarajan (2009) introduced a glow plug into the combustion chamber to improve the performance at lower loads.

Papagianakis et al. (2010) showed that deterioration of the engine efficiency under diesel-NG dual fuel mode was evident at low and intermediate torques, while at high torque and high diesel supplementary ratios the engine efficiency improved. Yoon and Lee (2011) analyzed the combustion and emission characteristics for different fuel models and evaluated the combustion pressure and the rate of heat release. Rai et al. (2012) discussed the application of fuzzy logic for modeling performance and emission parameters in a LPG-Diesel dual fuel engine.

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Elnajjar et al. (2013) conducted an experiment to examine the effect of LPG composition on performance of a dual fuel engine running on diesel as a pilot fuel. Maghbouli et al. (2013) presented a coupled 3D-CFD/Chemistry model firstly and used the model to study the combustion and emission characteristics of the engine under different pilot fuel quantities.

Numerous studies have shown that a dual fuel engine fueled with natural gas or LPG has fewer pollutant emissions and better efficiency but reduced dynamic performance (Xianwen et al., 2002). In the experimental studies described previously, the dynamic performance is usually studied through a speed characteristic test. By collecting data about the engine power and torque at different rotational speeds, a chart of the speed characteristic curve can be drawn to intuitively describe the dynamic performance of the engine (Zhi-Fa and Jin-Yu, 2002). However, this type of analysis of the dynamic performance of the engine is too simple and therefore, it is difficult to achieve a quantitative analysis. The goal of this study was to apply a mathematical method of regression analysis using collected power, torque and speed data to construct a dual fuel engine speed characteristic model and to provide a new analysis method to test the performance of dual fuel engines.

**CONSTRUCTION OF THE MATHEMATICAL MODEL OF SPEED CHARACTERISTICS**

**Dual fuel engine experimental data:** In an experiment to collect engine speed characteristics, the power and torque data at different speeds are usually obtained by keeping the compression ratio and ignition timing constant. Table 1 lists the speed characteristic data for the engine adopted in this study (Xin-Shun and Tian-Qiang, 2002). The set of experimental data is representative, i.e., the engine speed characteristics are usually manifested in the following way. Initially, the effective power, \( P_e \), increases as the rotational \( n \) speed, increases; after \( n \) increases to a certain level, the effective power begins to decrease and the torque, \( M_e \), decreases monotonically in an "S" shape as \( n \) increases.

<table>
<thead>
<tr>
<th>Rotational Speed (( n \min^{-1} ))</th>
<th>Power (kW)</th>
<th>Torque (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gasoline</td>
<td>LPG</td>
</tr>
<tr>
<td>1200</td>
<td>26.5</td>
<td>22.9</td>
</tr>
<tr>
<td>1400</td>
<td>29.3</td>
<td>25.7</td>
</tr>
<tr>
<td>1600</td>
<td>31.8</td>
<td>27.9</td>
</tr>
<tr>
<td>1800</td>
<td>33.1</td>
<td>29.8</td>
</tr>
<tr>
<td>2000</td>
<td>33.4</td>
<td>30.5</td>
</tr>
<tr>
<td>2200</td>
<td>33.1</td>
<td>30.7</td>
</tr>
<tr>
<td>2400</td>
<td>31.8</td>
<td>30.4</td>
</tr>
</tbody>
</table>

**Construction of the mathematical model of the power:** A parabolic model was adopted to relate the power, \( P_e \) and the rotational speed, \( n \):

\[
P_e = an^2 + bn + c (a > 0)
\]  

Equation 1 can be derived using a polynomial regression with \( x_1 = n \) and \( x_2 = n^2 \). Because \( a > 0 \), the maximum value of \( P_e \) and the corresponding rotational speed are:

\[
Pe_{max} = \frac{4ac - b^2}{4a}, n_{max} = -\frac{b}{2a}
\]  

When:

\[
X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}, X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}
\]

And:

\[
X = \begin{bmatrix} n_1 \\ n_1^2 \end{bmatrix}
\]

and if \( L_{max} \) is the deviation array of \( X \), then the standard deviation of \( P_e \) at the observed value is:

\[
S_{pe} = \sqrt{\frac{1}{f-3} L_{max}^{-1} (X_n - X)^2 \sigma^2}
\]

The variance of \( P_e \) can be estimated as:

\[
\sigma^2 = \frac{1}{f-3} L_{max}(0 - R^2)
\]

where \( L_{max} \) is the sum of the squared deviations of the observed values of \( P_e \), \( R^2 \) is the coefficient of determination and \( f \) is the sample size. Corresponding to \( n_{max} \), the \((1-\alpha)\times100\%\) confidence interval for the observed values of \( P_e \) is:

\[
[P_{e_{max}}-t_{\alpha} \times S_{pe}, P_{e_{max}}+t_{\alpha} \times S_{pe}]
\]

The number of degrees of freedom for the t-distribution is given by \( f - 3 \). The range of the corresponding rotational speed can be derived using Eq. 1 through (5).
Construction of the mathematical model of the torque: An inverse-logistic model was adopted to relate the torque, \( M_e \) and the rotational speed, \( n \):

\[
M_e = \frac{b}{1 + Ae^{-n}}, (A,B,r > 0)
\]  

(6)

The differential form of Eq. 6 is:

\[
\frac{dM_e}{dn} = rMe \left( \frac{Me}{B} - 1 \right), (B > 0, r > 0)
\]  

(7)

where, \( B \) is the upper limit of the torque. Equation 7 shows that:

\[
\frac{dM_e}{dn}
\]

is consistent with the parabolic law (i.e., as \( n \) increases, \( M_e \) decreases). When \( M_e = 0 \) or \( B \):

\[
\frac{dM_e}{dn} = 0
\]

When:

\[
M_e = \frac{B}{2}, \frac{dM_e}{dn}
\]

has the minimum value of:

\[
\frac{B \times r}{4}
\]

When:

\[
\frac{d^2M_e}{dn^2} = 0
\]

the inflection point coordinates can be derived as:

\[
n_t = \frac{1}{r} \ln A M_e - \frac{B}{2}
\]  

(8)

When:

\[
\frac{d^2M_e}{dn^2} = 0
\]

the steepest descent interval of \( M_e \) can be derived as:

\[
\left[ \frac{3 - \sqrt{3}}{6} B \right] \times \left[ \frac{3 + \sqrt{3}}{6} B \right]
\]

(9)

and the corresponding speed range is:

\[
\left[ \frac{1}{r} \ln 2 - \sqrt{3} \right] \times \left[ \frac{1}{r} \ln 2 + \sqrt{3} \right]
\]

(10)

RESULTS AND DISCUSSION

Relationship between power and speed: Using the data in Table 1, the parabolic model was used to fit the data.

Relationship between power and rotational speed when fueled with gasoline: When the engine is fueled with gasoline, the established fitting model between the power and rotational speed is:

\[
P_e = -0.00001086n^{1.04358929n-10.24047619, R^2 = 0.9982 (p<0.01)}
\]

The maximum power, \( P_e_{max} = 3.5 \text{ kW} \) and the corresponding rotational speed, \( n_o = 2006.9 \text{ r min}^{-1} \). The variance of \( P_e \) is 0.0170 and the standard deviation at the corresponding \( n_{mo}, S_{n_{mo}} = 0.1480 \).

For \( n_{mo} \), the 99% confidence interval of \( P_e \) (kW) is [32.8, 33.88] and the corresponding rotational interval (r min\(^{-1}\)) is [1756.9, 2256.9].

Relationship between power and rotational speed when fueled with LPG: When the engine is fueled with LPG, the established model between the power and the rotational speed is:

\[
P_e = -0.000000830n^{1.03616071n-8.5857129, R^2 = 0.9987 (p<0.01)}
\]

The maximum power is 30.89 kW and the corresponding rotational speed is 2178.4 r min\(^{-1}\).

The variance of \( P_e \) is 0.0079 and the standard deviation of \( P_e \) at \( n_o \) is 0.1001.

For \( n_{mo} \) the 99% confidence interval of (r min\(^{-1}\)) (kW) is [30.3, 31.3] and the corresponding rotational speed interval (r min\(^{-1}\)) is [1993.0, 2423.8].

From the above analyses, the following are known. (1) Because the coefficient for the \( n \) term is small:

\[
\frac{dP_e}{dn}
\]

is approximately equal to the coefficient of the \( n \) term. Clearly, the rate of increase in \( P_e \) as \( n \) increases is larger.
when the engine is fueled with gasoline (0.0436) than, when the engine is fueled with LPG (0.0362), demonstrating that the increase in Pe as n increases occurs faster with gasoline than it does with LPG. (II) The $P_{e_{max}}$ of gasoline (33.5 kW) is larger than that of LPG (30.89 kW). Relative to gasoline, the power loss of LPG is 8.0%. At maximum power, the corresponding rotational speed when the engine is fueled with gasoline ($2006.9 \text{ r min}^{-1}$) is smaller than the corresponding rotational speed when the engine is fueled with LPG ($2178.4 \text{ r min}^{-1}$); the difference between these rotational speeds is $171.5 \text{ r min}^{-1}$. This result shows that at maximum power, the rotational speed of the engine when fueled by gasoline is greater than that when fueled by LPG by $171.5 \text{ r min}^{-1}$ and therefore, the engine stability is better when it is fueled with gasoline. (III) Based on the confidence interval of the maximum speed rate and the corresponding speed interval, the length of the intervals are similar for both fuels (approximately 1 kW and 500 r min$^{-1}$). Therefore, the lower limit of the rotational speed when the engine is fueled with gasoline is $1756.9 \text{ r min}^{-1}$ and when the engine is fueled with LPG, the lower limit is $1933.0 \text{ r min}^{-1}$. The rotational speed achieved using gasoline exceeds that achieved using LPG by $176.1 \text{ r min}^{-1}$.

**Relationship between torque and rotational speed**: The data in Table 1 were fitted using the inverse-logistic model.

**Relationship between torque and rotational speed of the engine fueled with gasoline**: The fitted model of the engine when fueled with gasoline is:

$$Me = \frac{217.7}{1 + 0.0326e^{0.004n}}, R^2 = 0.9998(p < 0.01)$$

The corresponding speed rate equation is:

$$\frac{dMe}{dn} = 0.00141Me \left[ \frac{Me}{247.7} - 1 \right]$$

When:

$$0 < Me < 247.7 \text{ N•m}, \frac{dMe}{dn} < 0,$$

indicating that $Me$ decreases as $n$ increases.

When $Me = 123.8 \text{ N•m}$, the minimum decreasing speed rate of $Me$ as $n$ increases is $-0.087 \text{ N•m min}^{-1}$.

The inflection point coordinates for $Me$ as $n$ decreases are $n_1 = 124.8 \text{ r min}^{-1}$ and $Me_1 = 123.85 \text{ N•m}$. The steepest descent interval for $Me$ as $n$ decreases (N•m) is [52.35, 195.4] and the corresponding rotational speed interval ($r \text{ min}^{-1}$) is [1494.0, 3362.0]. This result shows that when $Me$ decreases to 195.4 N•m, or when the rotational speed increases to 1494.0 r min$^{-1}$, enters the steepest descent interval and the power rises quickly.

**Relationship between torque and rotational speed of the engine fueled with LPG**: The relationship between the torque and rotational speed when the engine is fueled with LPG is:

$$Me = \frac{212.1}{1 + 0.03466e^{0.0026n}}, R^2 = 0.9996(p < 0.01)$$

The equation describing the speed rate is:

$$\frac{dMe}{dn} = -0.00129Me \left[ \frac{Me}{212.1} - 1 \right]$$

When $0 < Me < 212.1 \text{ N•m}$ and $\frac{dMe}{dn} < 0$, $Me$ decreases as $n$ increases. When $Me = 106.1 \text{ N•m}$, the minimum decreasing speed rate for $Me$ as $n$ increases is $-0.068 \text{ N•m min}^{-1}$.

The inflection point coordinates for decreasing $Me$ are $n_2 = 2608.1 \text{ r min}^{-1}$ and $Me_2 = 106.1 \text{ N•m}$. The steepest descent interval for $Me$ as $n$ decreases (N•m) is [44.8, 167.3] and the corresponding rotational speed interval ($r \text{ min}^{-1}$) is [1587.3, 3629.1].

This result demonstrates that when the engine is fueled by LPG, when $Me = 167.8 \text{ N•m}$ or $n = 1587.3 \text{ r min}^{-1}$, $Me$ enters the steepest descent interval and the power rises quickly.

These analyses show that when the engine is fueled by gasoline, the upper limit of the steepest descent interval for $Me$ (195.4 N•m) is larger than the upper limit when the engine is fueled by LPG (167.3 N•m). The lower limit of the speed rate interval when the engine is fueled by gasoline (1494.0 r min$^{-1}$) is smaller than the lower limit when the engine is fueled by LPG (1587.3 r min$^{-1}$), with a difference of 93 r min$^{-1}$. Therefore, under conditions of high torque and low speed rate, gasoline fuel results in a better power performance than does the LPG fuel.

**CONCLUSION**

To analyze the engine speed characteristics, most studies have used connected curve plots using rotational speed and power data and plots of rotational speed and torque data as the foundations for further analyses. This method of qualitative analysis does not uncover other useful information in the data. Using the mathematical model and other analysis methods established in this study, useful information such as the power change rate,
the maximum power confidence interval, the torque change rate and the steepest descent interval can be obtained to achieve quantitative analyses of the dual fuel engine speed characteristics, which can lead to an in-depth understanding of the dynamic engine characteristics and provide useful quantifiable parameters for dual fuel engine design and refit.

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