Research on Intelligent Test Paper Based on Hierarchical and Self-Adapting Genetic Algorithm

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Abstract: The generating test study is a research in constrained multi-object optimization. It is one of the key technologies in examination management systems. It relates directly to the efficiency and quality of the generating test paper. The test paper generation method suggested in this paper is based on hierarchical adaptive genetic algorithm. It is able to solve the problem of premature convergence or slow convergence in global optimization. On the one hand, the M subpopulations operate on adaptive genetic algorithm and save the intermediate result; then they operate with the top adaptive genetic algorithm until a satisfied paper is found. On the other hand, the minimum weighted mean square error model is used to establish the objective function and to inspect the error between the expectations and the actual value on types, knowledge topics, difficulty and the degree of differentiation of the test paper. It discusses also the error of the answer time, total score and luminosity. It improved the speed of generating test paper of the system. It avoided the problem of premature convergence which often appears in standard genetic algorithm. The high quality of paper generation and the good robustness generated in this algorithm can meet the practical needs of users.

Key words: Intelligent test paper, genetic algorithm, hierarchy, self-adapting

INTRODUCTION

The generating test paper is to optimize the multi-objects constraint. It is one of the key technologies in the examination management systems, thus it directly relates to the quality of test paper and the running time and space overhead. There are problems of time-consuming, low success rate and generated papers could not satisfy the need of actual test in traditional algorithms of generating test paper such as the priority strategy, random strategy, parallel strategy and backtracking strategy etc., (Zitzler and Thiele, 1999; Holland, 1975; Dorrónsoro and Alba, 2006; Leung and Wang, 2000; Jerald et al., 2006; Huang, 2011). However, the genetic algorithm can meet the requirements of intelligent-generating test paper with characteristics of adaptive global optimization, intelligent search and good convergence (Martinez-Gomez et al., 2010). The random algorithm is also popular for its simpleness (Nebro et al., 2009; Ombuki et al., 2006). The system proposed in this study combined those two algorithms to pumping test paper generation.

Test theory (Golino, 2005; Huang, 2011) pointed that the study quality is related to the following aspects:

- Whether the test points of the paper cover the most topics to be examined
- Whether the types and numbers of the questions are reasonable
- Whether the profundity and the difficulty of the paper meet the requirements
- Whether the distinction degree of the paper is suitable, that is whether the paper can test the real level of the students
- Whether the test is fair, that is whether the test questions are frequent in the study process

Intelligent-generating test paper is to optimize the multi-objects constraint (Galante, 1996). The constraints are so many that it is hard to satisfy all. What’s worse it can lead to generating test paper failure easily.

Genetic algorithm is a simulation of the natural biology evolution and the genetic mechanism of the biological evolution process. It is a method which
laearches the optimal solution by simulating the natural biological evolution process. It was first proposed by Professor J. Holland of Michigan University in 1975 (Holland, 1975).

In application, the secondary constraints are relaxed to make sure that the importantly constraints are satisfied. In this paper, the following aspects of generating test study are taken into account:

- Determine the proportion of questions by types and knowledgetopics in the paper with the syllabus.
- Determine the proportion of questions with different difficulty level in the study according to the test target.
- Determine the questions proportion of each knowledge level in the study.

MATERIALS AND METHODS

Genetic algorithm: The mathematical model of genetic algorithm can be expressed as:

\[ GA = (C, E, P, M, \Phi, \tau, \Psi, T) \]

In this Equation, C is the individual coding method, Fixed length encoding is generally used in GA, E is the individual fitness function, \( P_i \) is the initial total group, M is the number of total individuals in the group, \( \Phi \) is the selection operator, \( \Psi \) is the crossover operator, \( T \) is the algorithm termination condition. The program flow chart of the standard Genetic Algorithm is shown in Fig. 1.

Mathematical model of intelligent test paper: A hierarchical adaptive genetic algorithm is adopted for intelligent test paper in this model. The weighted least square error model is used to establish the mathematical model.

Paper constraint and attribute: It is supposed that A test paper consist of M questions, each question consist of N attribute values, thus the attribute values can constitute a matrix A of a \( m \times n \):

\[ A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \]  \hspace{1cm} (1)

where, \( A \) is the goal state matrix in solving the problem of intelligent-generating test paper, \( a_{ij} \) is the j attribute of the i question. The value range of l is (1, 2, \ldots, m), \( a_{il} \) is the

![Flow chart of the standard genetic algorithm](image-url)

Fig. 1: Flow chart of the standard genetic algorithm

I score, \( a_2 \) is the type of I question, \( a_3 \) is the knowledge topic of the I question, \( a_4 \) is the difficulty level of I question, \( a_5 \) is the distinguish degree of I question, \( a_6 \) is the answer time of I question, \( a_7 \) is the total score of I question, \( a_8 \) is the exposure degree of I question.

In this system, the proportion of each tests column and each attribute value in matrix A put forward by users is defined as pumping test paper constraint (target value). The difficulty, discrimination, out of the value and the answer time metering constraint are taken the arithmetic mean values. Counting constraints of the types, chapters and knowledge topics are directly set by the users. The task of pumping test paper is to make each of the generated attribute value in the papers as close as possible to the target value.

The difficulty of the test is the difficulty degree of questions in the test. P represents the difficulty of a question, \( \bar{S} \) represents the average score on this question of a student, S represents the full mark, the difficulty can be calculated as the following equation:

\[ P = 1 - \frac{S}{\bar{S}} \]  \hspace{1cm} (2)

The smaller value of P is, the easier the question is; the bigger value of P is, the harder the question is.
The distinction degree of a paper means discriminability of the real level of a student. Distinction degree D can be calculated according to the following equation:

\[ D = \frac{X_H - X_L}{N(H - L)} \]  

(3)

In the equation, \(X_H\) is the total score of the high score group, \(X_L\) is the total score of low score group, \(N\) is 25% of the total number of students, \(H\) is the highest score, \(L\) is the lowest score.

**Objective function:** The weighted minimum mean square error model is used in this paper to establish the objective function. The error between the expected value and the real value of types, knowledge topics, difficulty and degree of differentiation is calculated as the following:

\[ Q_k(j) = \sum_{i=1}^{\infty} a_{i,k} \delta_k(i,j) \]

where, \(Q_k(j)\) is the score of j-th k attribute (\(k = 1, 2, 3, 4\) represent the type, the knowledge topic, the difficulty and the degree of differentiation).

\[ \delta_k(i,j) = \begin{cases} 1 & a_{i,k} = j \\ 0 & a_{i,k} \neq j \end{cases} \quad k = 1, 2, 3, 4, j = 1, 2, \ldots t \]

so:

\[ e_k = \sum_{j=1}^{t} \left( \frac{(L_k(j) - Q_k(j))/L_k(j)}{L_k(j)} \right) \]  

(4)

where, \(L_k\) is the objective score of j-th k attribute.

The error of the answer time and the total score is calculated as follows:

\[ e_k = \left( L_k - \sum_{i=1}^{\infty} a_{i,k} \right) / L_k \]

(5)

In the equation, \(k = 5, 6\) represent the answer time and the total score. \(L_k\) represents the target score of the corresponding attribute.

The exposure degree is calculated by the following equation:

\[ e_k = \left( L_k - 1 / \left( \sum_{i=1}^{m} a_{i,k} + 1 \right) \right) / L_k, \quad k = 7 \]  

(6)

The objective function is W-MMSE:

\[ \min f = \sum_{i=1}^{7} a_i e_i^2 \]  

(7)

where, \(a_i\) is the weight of each indicator:

\[ \sum_{i=1}^{7} a_i = 1 \]

**Sampling strategy design**

Step 1: Users determine pumping problem constraints

Step 2: Question module selects samples in random according to the value of the types, chapters, knowledge topics in the constraints to generate the initial population--- chromosome (paper). The gene of each chromosome is expressed by question number

Step 3: Calculate and inspect the fitness degree of each gene in chromosome; quit pumping if the error of the result and the objective value is in the allowable range or reaching the maximum number of iterations

Step 4: Choose 2/3 chromosome of the high fitness degree as the male parent individuals in the population, then cross by cross over probability; mutate by mutation probability and generate new chromosome individuals

Step 5: Compare the fitness degree between new individual and the male parent individual and keep the better one

Step 6: Return to step 3 to the next generation evolution.

**Constraints and process of generating**

**Setting generating constraints:** Interact with the operator, complete the generating constraints setting of the course (test item). The program flow chart is shown in Fig. 2.

**Pumping generating paper:** Complete the pumping of the course (test item) according to the pumping constraints. The program flow chart is shown in Fig. 3.

**Hierarchically-adapted genetic algorithm:** GA intelligent-generating test paper operation is composed of chromosome coding, initial population, fitness function, genetic operator and controls parameter.

**Chromosome coding:** Chromosome coding converts the solution space of the problem into searching space which GA could deal with. Setting the number of questions in
Fig. 2: Flow chart of generating constraints

the question bank is $L$ which coded in a binary string; the form is $x_1, x_2, x_3, ..., x_L$.

Fig. 3: Flow diagram of pumping test paper

$$x_i = \begin{cases} 1 & \text{if number } i \text{ is selected} \\ 0 & \text{else} \end{cases}$$
**Initial population generation:** The initial population is composed of a number of chromosomes in a scale or the set of solutions. In this article, the initial population consists of different constrained chromosomes which are generated randomly.

**Fitness function:** The fitness function is the gist for the optimal solution based on genetic algorithm. The higher the fitness degree of an individual is, the higher the chosen probability is. The fitness function can be defined as the following:

\[ F = \frac{1}{1 + f} = \frac{1}{1 + \sum_{i=1}^{n} o_i e_i} \]  \hspace{1cm} (9)

**Genetic operator:** The genetic operator includes selection, cross over and mutation. In the searching process, it firstly calculates fitness degree, then choose corresponding individual to cross and mutate according to the choice mechanism, thus it generates new individuals and keep the variety of individuals.

**Select:** This algorithm combines the best individual conservation method and the roulette to realize the selection operation; calculate fitness value of each individual in the population. The highest degree of the individual is directly copied to the next generation of adaptation in each generation. For the rest of the M-I individuals, roulette wheel selection method is used. The best individuals are directly genetic to the next generation which can guarantee the emergence of the optimal individual in the next generation and improve the local search ability and the rate of convergence.

The roulette wheel selection probability equation is as follow:

\[ P_i = F_i / \sum F_i \]  \hspace{1cm} (10)

where, \( F_i \) represents the \( i \) individual fitness value.

**Crossover:** It is in favor of small genetic preservation of population to use a large cross probability and mutation probability. However, it will cause the difference of individual fitness among populations getting smaller. The small crossover probability and the big mutation probability can increase the diversity of individuals populations, enhance the global search ability of the algorithm and avoid the premature phenomenon of the algorithm. This study adopts the self-adaptive crossover mutation probability. The calculation equation is as follows:

\[
P_c = \begin{cases} 
P_{cl} \frac{(P_{cl} - P_{ct})(F_c - F)}{(F_{max} - F)} & \text{if } F_c \geq F \\ 
P_{ct} & \text{else} \end{cases}
\]

(11)

\[ P_{cl} = 0.9, P_{ct} = 0.6, P_{m1} = 0.01, P_{m2} = 0.001, F_{max} \text{ indicates the maximum fitness value in the group, } F \text{ represents the } F_c \text{ average colony adaptation degree of each generation, is the larger fitness value of the two individuals in the crossover.}

The cross probability was calculated by using the cross formula. If \( P_c > \text{random}(0.1) \), crossover operation is performed at the crossing position to get two new individuals, otherwise the cross operation is not performed.

**Mutation:** mutation equation is as follows:

\[
P_m = \begin{cases} 
P_{m1} \frac{(P_{m1} - P_{m2})(F_{max} - F_m)}{(F_{max} - F)} & \text{if } F_m \geq F \\ 
P_{m2} & \text{else} \end{cases}
\]

(12)

where, \( F_m \) is the fitness value of the variation individual.

Mutation probability can be calculated through the variation equation. If \( P_m > \text{random}(0.1) \), variability will generate in individual variation. It will increase chromosome types in a certain extent which will maintain the diversity of species and avoid the premature. In addition, the variation only change the value of a position in the chromosome which can avoid convergence problems arising due to generating too many new chromosomes.

**Genetic arithmetic termination condition:** The test paper generating algorithm adopts the following three termination conditions:

- Population meets fitness requirements
- Genetic evolution reaches to a specified maximum number of iterations
- Best individual in the group has not improved in continuous several generations

**Adaptive test paper algorithm based on hierarchical genetic algorithm:** In the proposed algorithm, firstly, the M populations operate on adaptive genetic algorithm and store the intermediate results, then operate the top adaptive genetic algorithm (mainly the selection, crossover and mutation operations). The calculation keeps circulating until the paper which meets the constraints is found (Huang, 2011).

The intelligent test paper based on hierarchical and self-adapting genetic algorithm is as following:
Step 1: Setting test constraints and control parameters (the constraint conditions: types, quantity, difficulty, discrimination, knowledge topics and the control parameters: the maximum number of iterations, the size of the initial population initialization and the crossover and mutation probability)

Step 2: Segment coding

Step 3: Determining the objective function and the fitness function

Step 4: Randomly generating the initial population according to the constraints and dividing them into M sub populations in which each sub population has the same number of paper individuals

Step 5: Encoding each individual sample in the sub populations and calculating the fitness value of each individual according to the fitness function with the first I test index and the error value between the weight of the individual sample index and the expected index

Step 6: Sub populations G(i) got from the initial M operate respectively self-adaptive genetic operation for a certain number of times. If there is a test paper meets the performance requirements, the results of the test paper output; the paper will exit, otherwise it goes to the fourth step

Step 7: Saving the above M genetic algorithm results to an array of G(M, m) and calculating the average fitness values of M populations; then recording the results in the array A(M)

Step 8: Selecting in the G(M, m) in M sub populations, the highest average fitness sub populations will be directly copied to the next generation. As for the remaining M-I sub populations, the biased Roulette Wheel Selection (RWS), method is adopted to select roulette wheel selection probability equation:

\[ \frac{E_i}{\sum E} \]

For each population individual inside, a similar choice method is also used, then the individuals crossover according to the crossover probability

Step 9: Mutation operation is done to G(M, m) in accordance with the mutation probability

Step 10: The newly produced sub populations again operate the self-adaptive genetic operation respectively for a certain number (such as the 10 generation, the 20 generation)

Step 11: If there is a individual sample which meets the quality requirements, the test paper output and exit the test paper generating system; otherwise it executes step 7. The operation goes repeatedly, until the satisfied test sample is found

RESULTS

Encoding operation: Ranking in layer and numbering the questions according to the constraint pumping level settled by users (for example, the first layer is the type, the second layer is the section or the knowledge topic, the third layer is the difficulty or the discrimination). The question number, as the genetic material of the chromosome, is unified by 4 decimal digit string Table 1.

Question extraction operation: Pumping test randomly within constraints according to the user specified layer rank of count constraint hierarchy (such as types, chapters, knowledge). The selected question numbers constitute chromosomal genes Table 1.

Genetic operation: In order to meet the user specified count constraints, genetic operations such as the selection, crossover, mutation (Table 2 and 3) and other genetic operations are operated in the same counting

<table>
<thead>
<tr>
<th>Table 1: Sketch map of chromosome gene coding</th>
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<tbody>
<tr>
<td><strong>Multiple choice</strong></td>
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<tr>
<td>---------------------</td>
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<tr>
<td>0009 0102 0102 0104 0105 0106 0107 0108 0109 0122 0222 0228</td>
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<th>Table 2: Sketch map of chromosome genetic crossover operation</th>
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<td><strong>Multiple choice</strong></td>
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<tr>
<td>(a) Before cross operation</td>
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<tr>
<td>Chromosome A</td>
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<td>0009 0102 0102 0104 0105 0106 0107 0108 0109 0122 0222 0228</td>
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<tr>
<td>Chromosome B</td>
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<td>0102 0104 0106 0202 0204 0208 0209 0209 0210 0211 0212</td>
</tr>
<tr>
<td>(b) After cross operation</td>
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<td>Chromosome A</td>
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constraint hierarchies in random. For example, when the user specifies the types of constraints, the crossover and mutation are proceeded in the same gene questions randomly.

**Recode elimination:** The adoption of gene encoding may cause chromosome gene duplication in the genetic operation. The duplicated mutation to the duplicated gene can eliminate the duplication.

**Control parameters:** The probability of crossover directly determines the speed of generation of new chromosomes. However, if the crossover probability is too big may undermine the excellent characteristics of the population. It is set as 0.8. Mutation probability also affects the generation speed of the individual. It will reduce the performance of the algorithm if the mutation probability is too big, thus it is 0.1. The maximum number of iterations is 200 generation; the average difficulty, discrimination and the allowable completion time error service is ±0.05.

**Transformation of difficulty:** “Easier, easy, medium, hard, difficult” 5 levels of difficulty are mapped to Table 4, in degree of difficulty involved in calculation of average difficulty.

**Distinction of conversion:** “Higher, high, moderate, poor, poorer” 5 grades respectively mapped to Table 5 and join the average discrimination calculation.

**DISCUSSION**

**Experiment comparison:** The comparison of the standard genetic algorithm with the hierarchical and self-adapting genetic algorithm (HSAGA) (Huang, 2011) is shown in Fig. 4.

In the experiment, we test the intelligent test paper algorithm by running the algorithm 200 times with the standard genetic algorithm and the HSAGA. The experimental results are shown in Fig. 4. The blue curve indicates the standard genetic algorithm and the red curve is the HSAGA. From Fig. 4, it can be seen that the standard genetic algorithm keeps the better solutions than the HSAGA in the first 60 generations. However, with the increase of the number of iterations, in the 90 generation or so, the HSAGA becomes gradually stabilized and the advantage of HSAGA is shown. The solution is obvious better than standard genetic algorithm.

**CONCLUSION**

Intelligent-generating test paper is to automatically select the test from the database by computer according to the requirements of the teacher and teaching to form a test paper which can meet the requirements of the knowledge distribution, the type distribution, the cognitive level distribution, the difficulty distribution, the distinction degree distribution, the time distribution and the fraction distribution. The prematurity and slow convergence problem are easy to appear in global optimization in the traditional genetic algorithms. This
study adopts the packet coding and establish objective function with the weighted minimum mean square error model which can avoid the individual decoding. The calculation speed and the quality of generating paper are improved. The robustness is good. The generated test papers can satisfy the actual needs of users. With the combination of the generating test paper algorithm and the test database, this system, generates papers by extracting questions satisfy the constraints. It realized the separation of teaching and examination and liberated the teachers from the tedious examinations. At the same time, this system can also ensure the fairness of the exam and reflect the true ability level of student.

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