Multiple Emergency Locations Communication Resource Scheduling Model Based on Non-cooperation Game in China

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Abstract: In recent years, occurrence of worldwide unconventional emergencies becomes more frequent with the rapid development of technology and the environment changes gradually. Dispatch emergency communication resources legitimately for the first time to restore the damaged communication network for the further rescue. The unconventional emergencies often create multiple emergency locations at the same time. So create emergency communication resource allocation model of multiple disaster locations based on the non-cooperative game theory is constructed firstly. And then, the Nash equilibrium solution of this model was gained by iterative algorithm. At last, the feasibility and effectiveness of this model was verified through the example analysis. Research shows that scheduling strategy based on iterative Nash equilibrium algorithm can achieve the purpose about cost minimization and utility maximization of emergency communication resource scheduling for multiple disaster locations.

Key words: Multiple disaster location, resources scheduling, game modeling, Nash equilibrium

INTRODUCTION

Whether to recover damaged communication network system to guarantee the communication smooth after emergency is the key for the whole rescue in response, command and schedule. The fairness, rationality and effectiveness of the emergency communication resource scheduling for multiple disaster location is the important premise and basis, will relate to the quality of emergency communication security work and the post-disaster emergency rescue action. For minimizing the loss of the disaster, the dynamic adjustment process of rescue plan according to the evolutionary trend and periodic salvation results of the event is similar to a dynamic game. Here, emergency decision makers tend to use decision analysis tools to construct simple decision model, so the game theory plays a role on the decision of emergency resource scheduling because it can satisfy the above requirement. There are many relevant studies all around the world. Shetty (2004) and Gupta and Ranganathan (2007) used game theory and a solution is developed based on the Nash equilibrium to optimize the allocation of resource allocation and management. Zhang et al. (2007) designed a improved utility function based on preference order, which describe effectiveness and timeliness of each disaster location in obtaining rescue and established the emergency resource scheduling game model of multiple disaster location. Based on the theory of complexity adaptive system, Yu and Xue (2008) study the emergency management of public health incidents through evolutionary game theory and simulate the evolutionary game through AWARM tool. Yibo et al. (2007) and Zhou (2009) proposed the non-cooperative game model of scheduling strategy in the stream media monitoring system and demonstrated the uniqueness and existence of the Nash equilibrium. The above research apply the game theory to the resource scheduling, however, there are few theoretical researches specifically on the multiple emergency locations communication resource scheduling and allocation. This study constructs the multiple emergency locations communication resource scheduling model based on non-cooperation game and get the optimal scheduling scheme based on the Nash equilibrium iterative algorithm.

MATERIALS AND METHODS

Mobile, fixed telephone and other communication interrupted by the emergency, it requires the rational schedule of emergency communication resource including power car, emergency communication vehicle, power generation, oil machine, cable repair vehicles, satellite phone and so on to restore the communication network. And the required resource of each disaster location is mutual conflict under the condition of limited resources.
after the emergency occurred. So, construct the non-cooperative game theory of emergency communication resource schedule of multiple rescue centers.

In this section, recall the basic concepts of the non-cooperative game, such as finite game, mixed strategy, payoff function and equilibrium point.

Non-cooperative games (Nash, 1951)

Finite game: Intuitively, a game consists of a set of players, rules according to which the players may act and probably some chance element. Moreover, every player assigns to each possible and position of the game a certain value and tries to optimize this value. There are many types of games depending on how much information the players get during play. For example, an n-person game is a set of n players, each with an associated finite set of pure strategies and corresponding to each player, i, a payoff function, p_i, which maps the set of all n-tuples of pure strategies into the real numbers. The n-tuples represents a set of n items, with each item associated with a different player.

Mixed strategy: The formulation of the strategy set is the most critical step in realizing a game, a strategy is not a single move in a game but a collection of moves for every possible position in the game. The game is non-cooperative in the sense that all players must choose their strategies without communication with the other players. For example, a mixed strategy of player i is a collection of non-negative numbers which have unit sum and are in one to one correspondence with his pure strategies:

\[ s_i = \sum \pi_n \]

with \( c_n \geq 0 \) and \( \sum c_n = 1 \) to mean such a mixed strategy, where the \( \pi_n \)'s are the pure strategies of player i. The \( s_i \)'s is regarded as points in a simplex whose vertices are the \( \pi_n \)'s. This simplex may be regarded as a convex subset of a real vector space and a natural process of linear combination for the mixed strategies.

Payoff function: For each strategy chosen by a player, there is an expected utility associated with it which the player would receive as a function of its selected strategy and the combination of other players' strategies. This utility is known as the payoff paid/received by the player, depending upon whether the payoff is modeled as a gain or a loss function. Here, the payoff is modeled as a function of loss incurred to the player playing the strategy and gains for the other players when they play their respective strategies, which every player tries to minimize. For example, the payoff function, p_i, used in the definition of a finite game above, has a unique extension to the n-tuples of mixed strategies which is linear in the mixed strategy of each player, this extension is denote by p_i.

Equilibrium point: The central notion of Nash’s theory are equilibrium points, they are positions in the game which are locally optimal for each player. The payoff function for all of the players are used to generate an optimal solution by implementing the Nash equilibrium algorithm. An n-tuple is an equilibrium point if and only if all players play optimal, namely, maximize their payoff when the strategies of the other players are kept fixed. Hence, equilibrium points are stable in the sense that no single player can gain by choosing another strategy.

Model assumption: The resource requirement between the multiple disaster locations is non-cooperative and the aim of game is to reduce the cost. Then, the assumption about the model of the emergency communication resource schedule as follows:

- A single often can't satisfy the needs of multiple disaster locations, so it needs other rescue centers to give assistance to complete the whole rescue task. Considering the rescue cost, the rescue center can receive the assistance from the other nearest rescue centre when the emergency communication resource cannot match its requirement in its own area (Yang et al., 2008)
- According the different extent of severity of each disaster location, classifying the emergency condition into four levels, 1 denotes the highest level and others is 2, 3 and 4 in order with the same principle
- The first resource assignment for each disaster location depends on the rescue cost minimum principle without considering the amount of rescue centre. As a multivariate composite function, the cost function has many influence factors including the severity of the disaster, response time, average speed and distance from rescue center to disaster location. So, sort the rescue cost of each disaster location pay to each rescue center in order firstly

RESULTS

According to the above problem description and assumption, combined with the non-cooperative game...
theory, the non-cooperative game model of emergency communication resource schedule of multiple disaster location is constructed as follows:

$$G = \left\{ \left( t, N, (S_i(t))_{p \in N}, (P(t))_{p \in N} \right) \right\}$$  \hspace{1cm} (1)

The $t$ denotes the rescue time period, $N = \{1, 2, \ldots, n\}$ is the set of the $n$ disaster location and $S_i(t)$ denotes the set of the whole optical strategy disaster. $P_i(t) = P_i(s_1, s_2, \ldots, s_n)$ is the utility function of disaster location $p$ in $t$ stage, which mapped by the reciprocal of schedule cost. $S = (s_1, s_2, \ldots, s_n)$ is the combined strategy of $n$ disaster locations. $s = (s_p, s_q)$ is the action strategy of the $p$ disaster location, $S_p$ denotes the combined strategy of the whole disaster location except disaster location $p$:

$$S = \prod_{p=1}^{n} S_p$$

is the set of whole action strategy, which compose a game strategy space.

The following is the relevant factors and mathematical expression of the game model.

**Speed of emergency communication resource scheduling** ($v^k(t)$): The emergency communication resource schedule speed of disaster location $p$ is closely related to its road condition in a specific period, which generated indirectly by the average speed $\bar{v}_p(t)$ from rescue center $k$ to disaster location $p$. The specific equation is as follows:

$$v^k_p(t) = \mu(t) \cdot \bar{v}_p(t)$$  \hspace{1cm} (2)

The $\mu(t)$ is the road condition coefficient, which reflect the quality of the road condition from rescue center $k$ to disaster location $p$. Here $\mu(t) \in [0, 1]$ the better the road condition is, the larger the $\mu(t)$ is.

**Total cost function of emergency communication resource scheduling** ($C_p$): The $q$th scheduling strategy of disaster location $p$ is:

$$s_{p,q} = (u^q_1, u^q_2, \ldots, u^q_n)$$

whose total schedule cost function can be denoted as follows:

$$C_{p,q} = \sum_{k \in M} c^k_p \cdot u^q_k \hspace{0.5cm} \forall k \in M, \hspace{0.5cm} \forall q \in h(p)$$  \hspace{1cm} (3)

The $M$ is the set of all rescue centers and $h$ is the set of all the disaster location. The $u^q_k$ denotes the resource amount scheduled by the disaster location $p$ from the rescue center $k$ when adopt the $q$th strategy. $C^k_p$ is the unit rescue cost when the disaster location $p$ schedule the emergency communication resource from rescue center $k$, the function of $C^k_p$ can defined as follows:

$$C^k_p = \left[ L_p \left( \frac{D^k_p}{v^k_p(t)} \right) \right]_0^\infty$$  \hspace{1cm} (4)

The $D^k_p$ denotes the distance between rescue center $k$ and disaster location $p$ and the $L_p$ denotes the level of disaster location $p$, $8$ means the unit cost is infinite when schedule emergency communication resource from the rescue center outside the game scope (Yang et al., 2008).

**Revenue function**: The revenue function means the expected utility level of each disaster in a specific strategy, the utility when the disaster location $p$ schedule resource from rescue center $k$ can be defined as the reciprocal of the scheduling unit cost:

$$P_p(k) = \frac{1}{c^k_p + \Delta_c} \hspace{0.5cm} \forall k \in M$$  \hspace{1cm} (5)

The $\Delta_c$ is the increased cost when disaster location $p$ schedule the additional resource that not satisfied in the rescue center $k$.

The utility of emergency communication resource which the disaster location $p$ scheduled from all rescue center $M$ satisfy the superposition theorem, so, the total revenue function of disaster location $p$ from the all rescue center $M$ can be denoted as follows:

$$P_p(M) = \sum_{k \in M} P_p(k) \hspace{0.5cm} \forall p \in N$$  \hspace{1cm} (6)

**Objective function**: The goal of emergency communication resource scheduling is to maximize the total utility function under the circumstance of fairness schedule. So, the objective function can be defined as follows:

$$F(P_p) = \max \sum_{k \in M} \frac{P_p(M)}{P_p(M)}$$  \hspace{1cm} (7)
ALGORITHM SOLUTION OF MODEL

Solving steps of the emergency communication resource schedule based on the Nash equilibrium algorithm: The process of the emergency communication resource schedule based on the Nash equilibrium algorithm is as follows:

- Firstly, distribute the initial resource and make sure the cost each disaster location should pay when it schedule the emergency communication resource from each rescue center.
- Distribute the emergency communication resource each disaster location need according to the principle of minimum cost without considering the largest supply of each rescue center (Yang et al., 2008).
- Distribute the resource based on the requirement of each disaster location directly if the amount of resource of the rescue center k can satisfy all disaster location at the same level. While each disaster location at the same level will compete the resource of k if the largest resource supply of k are less than the total requirement of disaster location at the same level, which is a game relationship.
- Construct the multiple emergency locations communication resource schedule game model and use the Nash equilibrium algorithm to solve the model, then distribute the resource of the rescue center by the equilibrium solution.
- The unmet resource of each disaster location at the same level according to solution of the Nash equilibrium algorithm is scheduled from the next rescue center by using the principle of next cost minimum until the requirement has been satisfied.
- Repeat the step of 3 to the end.

Existence and iteration of the nash equilibrium of emergency communication resource schedule strategy: Any rational participants will not have the urge to change strategy alone on the Nash equilibrium of non-cooperative game. Each player separate change their own strategy will not increase its utility while the others at constant strategies. That can not only guarantee the maximum profit of disaster location, but also reflects the social fairness of emergency communication resources schedule on the whole. Therefore, the solution of the Nash equilibrium algorithm is the optimal feasible strategies to the disaster location for emergency communication resources schedule.

Get something as follows by the non-cooperative game Nash equilibrium existence theorem and the corresponding iterative algorithm.

Theorem 1: If the optimum reaction function \( O_q(S-p) \) of each disaster location satisfy the following conditions in the non-cooperative game of the disaster location \( n \):

- \( s_q \) is the each schedule strategy of disaster location \( q \), here \( \forall q \neq p, O_q(S-p) \) is the differentiable function of \( s_q \):

\[
\sum_{p \neq q} \frac{2O_q(S_{p,q})}{\partial s_q} \leq \lambda_q < 1
\]

Then, there exist the nash Equilibrium point in this non-cooperative game of disaster location \( n \).

Proof: Firstly, define the distance in \( S = R^n \) and select two points, which are:

\[
s^{(1)} = \{ s_1^{(1)}, s_2^{(1)}, \ldots, s_n^{(1)} \}
\]

and:

\[
s^{(2)} = \{ s_1^{(2)}, s_2^{(2)}, \ldots, s_n^{(2)} \}
\]

Then, get distance between two points, which is:

\[
d(s^{(1)}, s^{(2)}) = \sum_{p=1}^{n} |s^{(1)}_p - s^{(2)}_p|
\]

In addition, \( S \) is the strategy space, here:

\[
\forall s = (s_1, s_2, \ldots, s_n) \in S
\]

Define:

\[
Xs = (O_1(s_1), O_2(s_2), \ldots, O_n(s_n))
\]

Obviously, \( X \) is the operator of \( S \times S \). So get something as follows:

\[
d(Xs^{(1)}, Xs^{(2)}) = \sum_{p=1}^{n} |O_p^{(1)} - O_p^{(2)}|
\]

According to the mean value theorem the study would obtain something as follows:

\[
O_p^{(1)} - O_p^{(2)} = \sum_{p \neq q} \frac{\partial O_q}{\partial s_q} \bigg|_{s_q=q_p} (s_q^{(1)} - s_q^{(2)})
\]

The \( q_p \) is the point on the line which connect \( s_p^{(1)} \) and \( s_p^{(2)} \), so get it as follows:

\[ d(Xs^{(0)}, Xs^{(0)}) = \sum_{p=1}^{n} \sum_{q=1}^{n} \frac{\partial Q_s}{\partial s_q} |s_q - s_q^{(0)}| = \sum_{p=1}^{n} \sum_{q=1}^{n} \frac{\partial Q_s}{\partial s_q} |s_q^{(0)} - s_q^{(0)}| \]

\[ -\sum_{p=1}^{n} \sum_{q=p}^{n} \frac{\partial Q_s}{\partial s_q} |s_q^{(0)} - s_q^{(0)}| \leq \frac{\lambda_0}{2} \sum_{p=1}^{n} |s_p^{(0)} - s_p^{(0)}| - \lambda_0 \sum_{p=1}^{n} |s_p^{(0)} - s_p^{(0)}| - \lambda_0 d(s^{(0)}, s^{(0)}) \]

Obtaining \( 0 \leq \lambda_0 \leq 1 \) according to conditions of the theorem, so \( X \) is a contractive factor from \( S \) into itself. There exist fixed point:

\[ s^* = \{ s_{1}^*, s_{2}^*, \ldots, s_{n}^* \} \]

Which can satisfy the requirement of \( Xs^* = s^* \) based on contractive mappings theorems, namely:

\[ Oj(s_{p}^*) = s_{p}^* (p = 1, 2, \ldots, n) \]

Thus it can be seen, \( s^* \) is the Nash equilibrium point of non-cooperative game model.

The above demonstration shows that the actual iterative computations can start at the arbitrary point, here \( s^{(0)} \in S \), let \( d(0) \) be the distance between initial point \( s(0) \) and the Nash equilibrium point \( s^* \), it can get:

\[ d(s^{(0)}, s^*) \leq \lambda_0 d(s^{(0)}, s^{(0)}) \]

After \( m \) times of iterative computation. Here \( 0 \leq \lambda_0 \leq 1 \), so use the iterative formula \( s^{(m+1)} = Xs^{(m)} \) to approximate \( s^* \) which is the Nash equilibrium point of non-cooperative game (Yang, 2009).

Theorem above shows that the Nash equilibrium of non-cooperative game of multiple disaster locations strategy is not only exits but also is unique when strategy each disaster location choose has little effect on others', namely:

\[ \sum_{x=1}^{n} \frac{\partial Q_s}{\partial s_q} \leq \lambda_0 < 1 \]

and \( \lambda_0 \) denotes convergence factor.

**ANALYSIS OF EXAMPLE**

The 7.1 earthquake occurred in yushu Tibetan autonomous prefecture of qinghai province in April 14, 2010, which attacked many areas. Select yushu county, zaduo county and qunmalai county as three disaster location which are numbered by 1, 2 and 3. And set their classification as 1, 2 and 3. There are 2 rescue center denoted as A and B in yushu state to support the emergency communication resource to each disaster location. And there are 1 rescue center named C in gansu province. The supply and demand relationship of emergency communication car between each disaster location and each rescue center are presented as Table 1 and 2.

**Hypothesis:** The disaster location at the same classification has the same unit cost when they schedule the emergency communication car from each rescue center:

\[ c_a^1 - c_a^2 - c_b^2 = 2 \]

\[ c_a^3 = c_a^2 = c_b^3 = 3 \]

\[ c_b^3 = c_b^2 = c_b^3 = 5 \]

As Table 1 presented, \( S_{A, supply} = 18 < Q_{demand} = 28 \), the rescue center A and B can not satisfy demand of the emergency communication car of yushu county, zaduo county and qunmalai county at the same time. So rescue C have to join the rescue action to provide the required emergency communication car to disaster location.

The optimal scheduling scheme of each disaster location is to schedule the needed emergency communication resource from the rescue center which have the lowest schedule cost based on the model assumption of 1.2. There exist competition in the example, such as the disaster location 3 have the same lowest schedule cost from rescue A and B, so there are three party resources competition without considering the classification.

The Nash equilibrium solution is calculated as follows based on the iterative algorithm in 3.1:

\[ s_1^* = (8, 5, 0), s_2^* = (0, 5, 3), s_3^* = (0, 0, 7) \]

So, the final scheduling strategy of emergency communication car as Table 3.
Table 3: Emergency communication car allocation from the rescue centers to the disaster locations

<table>
<thead>
<tr>
<th>Disaster location No.</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
</tbody>
</table>

According to the Eq. 3, the total cost of three disaster based on the Nash equilibrium solution as follows:

\[ C_{1q} = 2 \times 8 + 3 \times 5 + 5 \times 0 = 31; \]
\[ C_{2q} = 2 \times 0 + 3 \times 5 + 5 \times 3 = 30 \]
\[ C_{1q} = 2 \times 0 + 3 \times 0 + 5 \times 7 = 35 \]

It can get the best scheduling scheme of emergency communication car through the above analysis. Disaster location 1 is the most serious areas, so it should be considered at first. Disaster location 1 schedule 8 emergency communication cars from rescue center A. then, the other 5 needed emergency communication cars are scheduled from rescue center B based on the Cost minimum principle to achieve maximum utility. The classification of disaster location 2 is inferior to location 1, so the disaster location 2 schedule 5 emergency communication cars from rescue center B and the other needed 3 cars are scheduled from rescue center C. The disaster location 3, which is the lowest classification, is considered at last. Emergency communication cars in rescue A and B have been scheduled to the disaster location 1 and 2. So the disaster location 3 has to schedule the whole 7 emergency communication cars from rescue center C. The total cost of yushu county, zaduo county and qumalai county is \( C_p, q = C_1, q+C_2, q+C_3, q = 31+30+35 = 96 \). And the total cost is the best combined strategy of scheduling emergency communication car.

The revenue of each disaster locations, calculated based on the Eq. 5 and 6, is as follows:

\[ P_1 = \frac{1}{2+0} \times 8 + \frac{1}{2+(3-2)} \times 5 = \frac{17}{3} \]
\[ P_2 = \frac{1}{3+0} \times 5 + \frac{1}{3+(5-3)} \times 3 = \frac{34}{15} \]
\[ P_3 = \frac{1}{5+0} \times 7 = \frac{7}{5} \]

Then, the total utility is calculated as follows based on the Eq. 8:

\[ F = P_1 + P_2 + P_3 = \frac{28}{3} \] (8)

The result shows that the best total revenue of the emergency communication car schedule for yushu county, zaduo county and qumalai county is 28/3 based on the Nash equilibrium solution.

In conclusion, use the model based on the game theory applied in the emergency communication car schedule in the yushu earthquake successfully. And it also adapt to the other emergency resource schedule as well as the resource scheduling in business.

**DISCUSSION**

There are many provinces in China and each province has several emergency rescues. When the emergency occurred in a region, the emergency resource can’t meet the requirement of the disaster locations’ demand. So, the nearest rescue center out of the region will be join the resource scheduling based on the principle of minimum cost. Like the rescue center C in this example, the assumption of the principle of minimum cost is conforms to China’s national conditions and it is rational in china. So, this model is useful and rational when the emergency occurred in China.

In the above assumption, the resource scheduling is based on the different level of classification, so the emergency resource classification is the further research interest.

**CONCLUSION**

Construct the non-cooperative game model of multiple emergency locations communication resource schedule based on the unconventional emergencies, the cooperative game theory and Nash equilibrium theory. At the same time, present the Nash equilibrium iteration algorithm and analyze the accuracy and uniqueness of the Equilibrium Point. Finally analyze and calculate the Equilibrium Point combined with the actual situation of emergency communication resource schedule. There are many disaster locations after the yushu earthquake. The competition behavior of each disaster location is non-cooperative to get the needed resource. Select three disaster locations in the above example. According to the hypothetical unit cost and resource quantity to solve the Nash equilibrium based on iterative algorithm, the solution realize the cost minimization and utility maximization in the emergency communication resource scheduling.

This best scheduling scheme will offer a Strong decision support to help emergency manager to make the emergency decision, which making contribution to the
recovery of the communication in the disaster location fast and fully and laying a good foundation for the follow-up rescue. This model and algorithm, which have certain theoretical and practical significance, offer new solution and realization way for the resource scheduling in business.

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