Study on A New Hybrid Random Space Vector Pulse Width Modulation Strategy

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Abstract: The random space vector pulse modulation (SVPWM) strategy can weaken the harmonic spectrum peaks on or around the switching frequency and its multiple ones. The randomization makes the hardware and software system complex, so it is important to develop new random SVPWM strategy that can be realized easily. Aiming at the complexity and characteristic of hybrid random space vector pulse modulation (SVPWM), the study proposes a new hybrid random SVPWM strategy. The strategy is controlled by two random variables, one of which is to control the duration time partitioning method to two zero-vectors, the other of which is to control the pulse positioning method. The new hybrid random SVPWM strategy can be realized in many digital micro control units. Three-phase currents can be measured in the middle of a switching period because the ripple currents are zero or almost zero in the middle of a switching period for inductive load. The macro Harmonic Distortion Factor (HDF) formula of the hybrid random SVPWM strategy is deduced. Finally, using Monte Carlo method the macro HDF characteristic is analyzed and using flux/current/voltage trajectory triangles the phase ripple current and switching pulse are discussed by computer simulation. The theoretical analysis and simulation results show the excellent performance and advantages.

Keywords: Harmonic distortion factor, hybrid random space vector pulse width modulation, voltage vector, current ripple, harmonic spectrum

INTRODUCTION

The space vector pulse width modulation (SVPWM) technique has been used in many kinds of fields, such as electric vehicle, industrial control and everyday life (Holmes, 1996; Bowes and Lai, 1997; Hava et al., 1999). Because of the basic principle of volt-second balance of pulse width modulation (PWM), the harmonic is inevitable besides the required fundamental (Holmes and Lipo, 2003; Zheng et al., 2011). The harmonic brings out many problems, such as mechanical vibration, motor temperature increase, acoustic noise and electromagnetic radiation (Boudjerdia et al., 2011; Na et al., 2002; Kaboli et al., 2007; Reddy et al., 2011). Especially the deterministic SVPWM has maximum amplitude harmonics on or around the switching frequency and its multiple ones because of fixed switching frequency, deterministic partitioning method to duration time of two zero-vectors and deterministic pulse positioning method, which may aggravate the problem (Holmes and Lipo, 2003; Boudjerdia et al., 2011; Na et al., 2002; Kaboli et al., 2007; Reddy et al., 2011). Many random SVPWM (RSVPWM) strategies have been proposed and discussed to weaken the harmonic spectrum peaks on or around the switching frequency and its multiple ones. The basic RSVPWM strategy includes random switching frequency PWM (RSFPWM), random zero-vector distribution PWM (RZDPWM) and random pulse position PWM (RPPPWM) (Na et al., 2002; Ma et al., 2007a, b, 2008; Wang and Wang, 2007; Oh et al., 2009; Kirlin et al., 1994; Wu et al., 2011; Chen et al., 2012b, c). The control software for traditional SVPWM can be utilized to RZDPWM with minor modification and without modification on hardware. The asymmetric pulse waveform in a period of RPPPWM makes it difficult to generate six switching signals in many micro control units and current measurement can only be done at the beginning and end of a period. In order to make advantages of the three basic RSVPWM strategies adequately, the hybrid random SVPWM strategy (HRPWM) that is combination and modification of RZDPWM, RSFPWM and RPPPWM is complex but very effective and has been discussed in plenty of literature (Wang et al., 2005, 2009; Chen et al., 2012a). Two key issues are focusing on harmonic spectrum peak weakening effectiveness and complexity on hardware and software. Simple combination of two or three of the basic RSVPWM strategies cannot solve the above two issues. So, the study proposes a new HRPWM strategy that has excellent performance and can be realized by reusing the mature hardware and software system for traditional SVPWM with minor modification.

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PRINCIPLE OF THE NEW HYBRID RSVPWM

Principle of SVPWM: The classic two-level three-phase inverter has eight switching states and the corresponding space voltage vectors are shown in Fig. 1 (Holmes and Lipo, 2003). The six active voltage vectors are:

\[
\tilde{U}_k = \frac{2}{3} (u_{vk} + \alpha u_{vk} + \alpha^2 u_{vk}) = \frac{2}{3} U_{dc} e^{\alpha \omega \tau} \quad (k = 1, 2, \ldots, 6)
\]

(1)

where, \(U_{dc}\) is the DC bus voltage, \(u_{vk}\) are phase voltages relative to the float neutral point of load and \(\alpha = e^{j\pi/6}\).

An arbitrary target voltage vector inside the hexagon region shown in Fig. 1 can be expressed as:

\[
\tilde{U}_t = U_e e^{j\theta}
\]

(2)

where, \(U_e\) is the vector amplitude and \(\theta\) is the phase angle.

The target voltage vector \(\tilde{U}_t\) can be generated by two active vectors-starting and ending boundaries of the corresponding sector and zero-vectors. The duration time of \(\tilde{U}_t\) and zero vectors (\(\tilde{U}_0\) and/or \(\tilde{U}_1\)) \(T_{sv1}, T_{sv2}, T_{sv0}\) can be computed based on voltage-second balance principle and expressed by Eq. 3:

\[
T_{sv} \tilde{U}_t = T_{sv1} \tilde{U}_1 + T_{sv2} \tilde{U}_2
\]

(3)

where, \(T_s\) is the PWM period.

So,

\[
T_{sv1} = \frac{\sqrt{3}}{2} M T_s \sin \left(\frac{p}{3} - \theta\right)
\]

(4)

where, \(M\) is the modulation index:

\[
T_{sv2} = \frac{\sqrt{3}}{2} M T_s \sin \theta
\]

(5)

\[
T_{sv1} + T_{sv2} = \frac{\sqrt{3}}{2} \frac{U_{dc}}{U_{dc}} \sin \left(\frac{p}{3} + \theta\right)
\]

(6)

The total duration time of two zero vectors (\(T_{sv0}\) for \(\tilde{U}_0\) and \(T_{sv0}\) for \(\tilde{U}_1\)) is:

\[
T_{sv0} = T_s - T_{sv1} - T_{sv2} = T_{sv0} + T_{sv0}
\]

(7)

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Fig. 2: Working sequence of basic vectors in the first sector and duration time of every vector

To get excellence harmonic performance, less switching number and implementation facility in the digital system, the conventional duration assignment strategy in Fig. 2 is utilized widely. To traditional SVPWM (TSVPWM), the switching signal is symmetrical in a switching period, that is to say \( T_1 = T_7, T_2 = T_4, T_3 = T_5, T_4 = T_3, T_5 = T_4, T_6 = T_6 \).

**Method of the new HRPWM:** In the new HRPWM discussed in the study, the relation between \( T_1 \) and \( T_7 \) is controlled by one random variable \( R_1 \) on interval \([0, 1]\). And the duration time partitioning method of two zero-vectors is controlled by the random variable \( R_2 \) on interval \([0, 1]\), so:

\[
\begin{align*}
T_{zv0} &= R_1 T_0 \\
T_{zv10} &= (1 - R_1) T_{zv0} \\
(0 \leq R_1 \leq 1)
\end{align*}
\]  

\( (8) \)

The duration time \( T_1, T_2, T_3, T_4, T_5, T_6, T_7 \) of every segment can be expressed as:

\[
\begin{align*}
T_1 &= R_2 T_{zv0} \\
T_2 &= T_{zv1}/2 \\
T_3 &= T_{zv1}/2 \\
T_4 &= T_{zv0} \\
T_5 &= T_{zv2}/2 \\
T_6 &= T_{zv2}/2 \\
T_7 &= (1 - R_2) T_{zv10}
\end{align*}
\]  

\( (9) \)

The random variable \( R_2 \) should satisfy the following restrictive condition:

\[
\begin{align*}
T_1 \leq \frac{1}{2} T_{zv0} \\
T_7 + T_2 \geq \frac{1}{2} T_{zv0}
\end{align*}
\]  

or:

\[
\begin{align*}
T_1 \leq \frac{1}{2} T_{zv1} \\
T_7 + T_2 \geq \frac{1}{2} T_{zv1}
\end{align*}
\]  

\( (10) \)

Equation 10 can be expressed as:

\[
\begin{align*}
R_2 R_1 T_{zv1} \geq \frac{1}{2} T_{zv0} \\
R_2 R_1 T_{zv1} + (1 - R_1) T_{zv0} \geq \frac{1}{2} T_{zv1}
\end{align*}
\]  

\( (11) \)

So, the following equation can be gotten:

\[
\begin{align*}
R_2 \leq \frac{1}{2R_1} \\
R_2 \geq 1 - \frac{1}{2R_1}
\end{align*}
\]  

\( (12) \)

Equation 11 can be expressed as:

\[
\begin{align*}
(1 - R_2) R_1 T_{zv0} \leq \frac{1}{2} T_{zv0} \\
(1 - R_1) T_{zv0} + (1 - R_2) R_1 T_{zv1} \geq \frac{1}{2} T_{zv1}
\end{align*}
\]  

\( (13) \)
\[ \begin{align*}
(1-R_z)R_i & \leq \frac{1}{2} \\
1-R_z & \geq 1 - \frac{1}{2R_i} \\
(1-R_z)R_i & \leq \frac{1}{2R_i}
\end{align*} \quad (15) \]

And Eq. 15 can be expressed as:
\[ \begin{align*}
R_z & \leq 1 - \frac{1}{2R_i} \\
R_z & \leq \frac{1}{2R_i}
\end{align*} \quad (16) \]

Equation 15 and 16 can be rewritten as:
\[ \frac{1}{2R_i} \leq R_z \leq \frac{1}{2R_i} \quad (17) \]

So, the random variable \( R_z \) should be on interval \((K_1, K_2)\) and:
\[ K_1 = \begin{cases} 0 & \text{if } R_z \leq \frac{1}{2} \\
1 - \frac{1}{2R_i} & \text{if } \frac{1}{2} < R_z \leq 1 \\
1 & \text{if } R_z \geq 1 \end{cases} \quad (18) \]

\[ K_2 = \begin{cases} 1 & \text{if } R_z < \frac{1}{2} \\
\frac{1}{2R_i} & \text{if } \frac{1}{2} \leq R_z \leq 1 \\
1 & \text{if } R_z \geq 1 \end{cases} \quad (19) \]

In motor close-loop control system, three phase currents have to be measured. The phase current can be measured in the middle of a switching period because three phase currents are zero or almost zero for inductive load. To reduce electromagnetic interference from the switch of IGBT, it is suggested that some delay time should be set. If the delay time is \( T_{delay} \), Eq. 10 and Eq. 11 are expressed as:
\[ \begin{align*}
T_i & \leq \frac{1}{2} T_{zero} \\
T_i + T_e & \geq \frac{1}{2} T_{zero} + T_{delay}
\end{align*} \]

Or:
\[ \begin{align*}
T_i & \leq \frac{1}{2} T_{zero} \\
T_i + T_e & \geq \frac{1}{2} T_{zero} + T_{delay}
\end{align*} \quad (20) \]

If let \( \lambda = T_{delay}/T_{zero} \), then the value range of \( R_z \) can be deduced and expressed as:
\[ \frac{1}{2} \frac{1}{2R_i} + \frac{\lambda}{R_i} \leq R_z \leq \frac{1}{2} \frac{1}{2R_i} + \frac{\lambda}{R_i} \quad (21) \]

**Harmonic distortion factor:** Based on the research results in the literature (Holmes and Lipo, 2003; Wu et al., 2011; Chen et al., 2012c; Wang et al., 2009), the total harmonic mean square of three-phase ripple currents can be expressed as:
\[ \Delta I_{rms} = \langle \Delta I_{ab}^2 \rangle + \langle \Delta I_{ac}^2 \rangle + \langle \Delta I_{bc}^2 \rangle \]
\[ = \frac{1}{T_e} \int_{0}^{T_e} \left( \Delta I_{ab}^2 + \Delta I_{ac}^2 + \Delta I_{bc}^2 \right) dt \]  
\[ = \left( \frac{U_{dd}}{2L_e} \right)^2 \int_{0}^{T_e} 3f(M, \theta) \]  

where, \( \Delta I_{ab}, \Delta I_{ac}, \Delta I_{bc} \) are phase ripple currents, \( \langle \Delta I_{ab} \rangle, \langle \Delta I_{ac} \rangle, \langle \Delta I_{bc} \rangle \) are the MS (Mean Square) value of three-phase ripple currents, \( I_{eq} \) is the equivalent line-line inductance and \( f(M, \theta) \) is defined as the single phase micro HDF.

Through complex deduction the micro HDF can be expressed as:
\[ f_{HDF}(M, \theta) = aM + bM^2 + cM^3 \quad (23) \]

where, the coefficients \( a, b, c \) are complicated functions of \( R_z \) and \( R_i \).

Through integral in the first sector, the macro HDF can be gotten and expressed as:
\[ f_{HDF}(M, \theta) = \frac{-1}{T_e} \int_{0}^{T_e} \left[ \frac{64R_i^2 R_z R_{eq}^2 + 96R_i^3 R_z}{27} + \frac{128R_i^2 R_z}{12} + \frac{132R_i^3 R_{eq}^2}{12} \right] \]
\[ + \frac{15}{12} \left[ \frac{72R_i^2 R_z}{72} + \frac{144R_i^2 R_{eq}^2}{72} \right] \]
\[ + \frac{15}{12} \left[ \frac{72R_i^2 R_z}{144} + \frac{144R_i^2 R_{eq}^2}{144} \right] \]
\[ + \frac{15}{12} \left[ \frac{12R_i^2 R_z}{72} + \frac{144R_i^2 R_{eq}^2}{72} \right] \]
\[ + \frac{15}{12} \left[ \frac{72R_i^2 R_z}{144} + \frac{144R_i^2 R_{eq}^2}{144} \right] \]
\[ + \frac{15}{12} \left[ \frac{12R_i^2 R_z}{72} + \frac{144R_i^2 R_{eq}^2}{72} \right] \]
\[ + \frac{15}{12} \left[ \frac{72R_i^2 R_z}{144} + \frac{144R_i^2 R_{eq}^2}{144} \right] \]

**SIMULATION ANALYSIS AND DISCUSSION**

The macro HDF is a random function. That is to say, the macro HDF is a random variable for every discrete modulation index \( M \) and follows a certain distribution to the specified distributions of \( R_z \) and \( R_i \). Based on Monte
Fig. 3: Maximum, minimum and mean value for macro HDF (harmonic distortion factor) for the new hybrid random space vector pulse width modulation strategy

Carlo method, characteristics of the new HRPWM can be analyzed. The maximum, minimum and mean values for the macro HDF are shown in Fig. 3, if \( \lambda \) is set to 0 and \( R \), \( R_s \) follow uniform distribution. From the figure, some phenomena and conclusions can be found and drawn. The value ranges of the macro HDF differ greatly for different modulation index. The range becomes bigger and bigger with the modulation index increasing, reaches maximum value when the modulation index is about 0.6 and becomes smaller and smaller with the modulation index decreasing when the modulation index is more than about 0.6. The range is very small when the modulation index is \( 2\sqrt{3} \). The randomizing performance of the pulse is largely determined by total duration time of two zero-vectors and the random number \( R \). The duration time of two zero-vectors becomes shorter and shorter with the modulation index increasing and reaches the shortest when the modulation index is \( 2\sqrt{3} \) on the whole. The above characteristics can also be found in RZDPWM and RPPPWM (Chen et al., 2012b, c). The duration time of the zero-vector \( \bar{u} \) has heavy effects on the randomization level of the widest, for example the phase A pulse in the first sector.

The simulation platform can be built in engineering software. The study built a unified SVPWM platform for deterministic SVPWM and RSV PWM strategies including traditional SVPWM, DPWM0, DPWM1, DPWM2, DPWM3, DPWMMAX, DPWMMIN, RSF PWM, RPPPWM, RZDPWM and several HRPWM strategies.

The simulation parameters are set as follows. The modulation index is 0.6, the switching frequency is 2000 Hz, the distributions of two random variables are uniform distribution and the delay time is none. The simulation results are shown in Fig. 4 and 5. The phase voltage is sampled in the whole fundamental period and spectrum analysis is done using FFT (fast Fourier transform). The amplitude spectrum of phase voltage is shown in Fig. 4, which shows that the proposed HRPWM can significantly weaken harmonic frequency peaks on or around the switching frequency and its multiple ones, especially twice the switching frequency. This characteristic is similar to that of RZDPWM because the duration time partitioning ratio of two zero-vectors is random variable in the two RSV PWM strategies (Chen et al., 2012b, c).

In Fig. 5, three subgraphs demonstrate the ripple current of phase A, upper arm switching pulses of phase A, B and C from top to bottom. The phase ripple current is almost zero at the beginning and in the middle of a switching period shown in the top subgraph, so phase currents can be measured in the middle of a switching period, which can also be explained by flux/current/voltage trajectory triangles. Based on volt-second balance, a target output voltage vector is generated by two active vectors-starting and ending boundaries of the sector and zero-vectors. The errors exist between the target voltage vector and basic vectors. As pointed out by Holmes and Lipi (2003), if the load model is an inductance, the trajectories of harmonic current and harmonic flux are only different in scale. The harmonic current/flux trajectories of traditional SVPWM and HRPWM strategies in the first sector are shown in Fig. 6, where, \( \bar{e}, \bar{s}, \bar{h}, \bar{e}_s \) and \( \bar{e}_h \) are errors between the target voltage vector and \( \bar{u}, \bar{u}_2, \bar{u}_3, \bar{u}_4 \), respectively and D is the endpoint of the target voltage vector. D corresponds to the beginning and middle of the switching period.

The two triangles of traditional SVPWM, determined by modulation index and phase angle only, are centrosymmetric; that is to say, \( JD = ED - HD - GD \), \( IH = FG \) and \( JJ = FE \). To the new HRPWM, the two triangles are not centrosymmetric; that is to say, \( JD \neq ED \), \( HD \neq GD \), \( IH \neq FG \) and \( JJ \neq FE \). In order to measure the phase current in duration time of \( \bar{u}_i \), the endpoint of the target vector has to be on the sides of GE and HH, that is to say:

\[
\frac{ED - (EG + HJ)}{2} = EG + HJ
\]

\[
ED + GH = (EG + HJ)/2
\]

or:

\[
\frac{JD - (EG + HJ)}{2} = EG + HJ
\]

\[
JD + GH = (EG + HJ)/2
\]
**Fig. 4(a-c):** Amplitude spectrum of the phase voltage to the virtual neutral point of DC voltage for (a) HRPWM (hybrid random SVPWM) in one switching period, (b) HRPWM (hybrid random SVPWM) in another switching period and (c) Traditional SVPWM (space vector pulse modulation) with equal duration time of two zero-vectors.

**Fig. 5(a-d):** Simulation results of (a) Ripple current and three pulses of (b) Phase A, (c) Phase B and (d) Phase C.
Fig. 6(a-b): Harmonic current/flux trajectories in the first sector for (a) SVPWM (space vector pulse modulation) and (b) New HRPWM (hybrid random SVPWM)

Equation 25 is consistent with Eq. 10 and 11. The points E, F, G, H, I and J are switching instants, so the delay time $T_{delay}$ is set to avoid measuring phase currents at these points.

CONCLUSION

In this study a new HRPWM strategy is proposed. The strategy is controlled by two random variables, one of which is to control the duration time partitioning method to two zero-vectors, the other of which is to control the pulse positioning method. The new HRPWM strategy can be realized in many digital micro control units and three-phase currents can be measured in the middle of a switching period because the ripple currents are zero or almost zero in the middle of a switching period for inductive load. The HDF is a key index to assess a PWM strategy, so the micro and macro HDF formulas are deduced based on ripple currents. And the macro is a random function including two random variables and modulation index. In order to analyze the numerical character of HDF, Monte Carlo simulation is utilized and the maximum, minimum and mean values for the macro HDF are computed. The simulation result verifies significant harmonic spectrum peak weakened performance and presents ripple current characteristics. The analysis based on harmonic current/flux trajectories shows the reason that the phase ripple current is almost zero at the beginning and in the middle of a switching period for the proposed SVPWM strategy.

The control system for traditional SVPWM can be reused with minor modification and without modification on hardware, so the proposed SVPWM strategy can be easily used in many kinds of electric drive systems with close-loop control. For example, electric vehicles are excellent application objects because they require critical electromagnetic compatibility.

It is randomization that weakens harmonic spectrum peaks. However, the random probability distributions of the two random variables have heavy effect on performance. From the theory of stochastic process, the harmonic spectrum/probability density function is determined by random variables. In the future, the study will focus on how to accurately compute the probability distributions for random variables to get required harmonic spectrum. In other words, the harmonic spectrum may be customized.

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REFERENCES


