Fault diagnosis is an important way to improve the reliability of complex systems. Machine learning algorithm is an effective means to improve the efficiency of fault diagnosis and Bayesian networks is widely used in the fault diagnosis due to its advantages in uncertainty reasoning. Being unable to select the fault paths effectively, the existing fault diagnosis algorithm based on Bayesian network cannot detect faulty nodes accurately and has high computational complexity. In this study Bayesian networks sensitivity analysis is introduced into fault diagnosis and a kind of Bayesian network fault diagnosis algorithm, SA_FD, is presented in complicated system. First, the formal model of Bayesian fault diagnosis networks is given. Second, the model of how parent nodes influence their child nodes is built based on sensitivity analysis. Last, sensitivity analysis of the nodes are used to detect the faulty nodes based on heuristic path search method, to overcome the blindness of existing algorithm in searching important parent nodes and selecting the fault paths so as to improve performance of fault diagnosis effectively. Experimental results show that SA_FD is more efficient is than DFS and DFC obviously, although its complexity increases with the scale of the network.

**Key words:** Bayesian network, sensitivity analysis, fault diagnosis, reliability
Bayesian Networks (BNs) is a graphical tool based on probabilistic reasoning which can describe the event polymorphisms and non-deterministic logic relationship. Using conditional independence of Bayesian network, the relationship between the failures can be clearly shown. Furthermore, all diagnostic information can be expressed by appropriate node variables in the process of diagnosis. Besides, by using its bidirectional reasoning ability, probability of failure cause can be quickly calculated and variable probability information of other node can be achieved. By using its learning ability, the structure and parameters of the Bayesian network can be updated constantly, to avoid the subjectivity of conditional probability, optimize network structure and improve the efficiency and accuracy of fault diagnosis. In conclusion, Bayesian network is more applicable for fault diagnosis in complex system with the characteristics of uncertainty and correlation (Buede et al., 1998).

Existing Bayesian network-based fault diagnosis mainly involves the fault diagnosis model, diagnosis methods and researches on diagnostic reasoning. Besides, its applications relate to the field of machinery and equipment, power systems, fault monitoring of aerospace systems (Breese and Heckerman, 1996; Rakar et al., 1999; Dahll, 2000; Mussi, 2000). In fault diagnosis, the detection of fault nodes, or the best path from observing abnormalities to finding the faulty nodes, has a very important influence on diagnostic efficiency and accuracy. Kopec and Marsland (2004) proposed Depth First Search (DFS) algorithm which is based on Bayesian network to find fault node. Its basic idea is to start from the leftmost parent node of the network structure and traverse all the parent nodes in turn until a new abnormal node is found. However, because it did not select the parent nodes, there is great blindness. Especially when a node has several parent nodes, the time complexity will be very high. Doguc and Ramirez-Marquez (2009) proposed DFC (Diagnose Failed Component) algorithm. It uses the change of parent node's CPT (Conditional Probability Table) to select the parent node. To some extent, it solves the blindness in parent node selection. However, the result is one-sided, because it doesn’t take the influence of conditional probability between child nodes and their parents on calculating state probability of child nodes into consideration. The studies show that the relationship between the parent node and child node is very important to find the best path.

Sensitivity Analysis (SA) is a method to study the effect of model output which is caused by the change of parameters in the mathematical model (Habbema et al., 1990) and effective measures to find the dependence between the quantization parameter. Therefore, SA can be used as a method to measure relationship between parent and child nodes. The concept of sensitivity analysis was introduced into Bayesian networks by Weiss (1996). Wang (2004) found that Bayesian network is very sensitive to the accuracy of parameters probability (Pfingsten, 2006) and demonstrated that sensitivity analysis is a very effective method to Bayesian network (Wang, 2004). The methods of Bayesian network sensitivity analysis in the studies above only relate to a single parameter. Later, the researchers extended Bayesian network sensitivity analysis to multiple parameters (Chan and Darwiche, 2012) and special network (Chan and Darwiche, 2005). Currently, the Bayesian network sensitivity analysis is mainly used to improve its structure and parameters learning.

This study introduces Bayesian networks sensitivity analysis into fault diagnosis. First, it gives a formal description of the Bayesian fault diagnosis network model. Then, sensitivity analysis method is used to measure the importance of each parent node relative to its child nodes. Finally, an efficient fault diagnosis algorithm (SA_FD) is proposed. When the system fails, effective paths to detect fault can be found according to the sensitivity.

MATERIALS AND METHODS

In fact, the process of fault diagnosis is a kind of reasoning based on Bayesian network. The model of Bayesian network is used as a tool to describe the system. BNs are graphical representations of conditional dependence relationships among stochastic variables. In a BN each node is a stochastic variable and when a causal relationship between two nodes exists it is represented by an arrow. A BN can have a very complex structure including bidirectional arrows and cycles. In a directed acyclic graph $G = (V, E)$, that is a special type of BN, each node represents a random variable $V = [V_1, ..., V_n]$ and arcs E encode direct conditional dependence relationships between variables $V_i \rightarrow V_j$, where, $V_i$ is the parent of $V_j$ and in turn, $V_j$ is the descendant of $V_i$. In G no cycles are permitted. Given the parents of the discrete variable $V_i$, denoted by $pa(V_i)$, its conditional distribution is defined by $P(V_i = v_i | pa(V_i))$. Thus, the joint probability distribution of $G$ is:

$$P(V) = P(V_1, ..., V_n) = \prod_{i=1}^{n} P(V_i | pa(V_i))$$

In order to effectively describe the Bayesian fault diagnosis based on sensitivity analysis, from the view of the fault diagnosis, we give the following definition.

Definition of abnormal node: Let $X \in V$ be a random variable, given threshold $\varepsilon$, if $|p(X) - q(X)| > \varepsilon$, then $X$ is defined as abnormal node, where, $p(X), q(X)$ is the true distribution and observed distribution of $X$, respectively.

Definition of faulty nodes: Let $Y, S \in V$ be random variables, $S$ represents system abnormality, $Y$ is defined as faulty node, if it can satisfy the following conditions:

- $Y$ is abnormal node
- $pa(Y) = \emptyset$ or $pa(Y)$ isn’t abnormal node
- $Y \rightarrow S$
Bayesian Fault diagnosis Network (BFN) is a network which using Bayesian network to describe fault diagnosis of complex systems, BFN = (G, θ). Based on Definition 1 and 2, G = (Sa, Nan, Nfn, Nun, E), where, Sa is system abnormity, Nan, Nfn, Nun is abnormal node, fault node and normal node, respectively, E is the arc between two node.

Each node in the network has a Conditional Probability Tables (CPT) and when the node is not in the normal working state, the CPT table will be changed.

As shown in Fig. 1, the node S represents the system state can be observed. If at time t, node S is in abnormal working state, namely system abnormity has been taken place. Fault diagnosis is starting from S to looking for node of which leads to the abnormal state, seeking abnormal nodes from its parents nodes and finding out the fault node which cause system abnormality.

Sensitivity analysis of Bayesian network is concerning about the influence of local model parameters or small changes in evidence on target node’s. Because the target node’s probability can be expressed as a function of parameters, sensitivity analysis of Bayesian network is essentially to establish a relationship between each parameter and target node’s probability, namely, the sensitivity function.

Target node’s probability can be denoted by P(A = a|e) or P(a|e), where, a is a special value of node A, e is the evidence. The parameters are represented by θ = P(bi|π), where bi is a value of node B, π is a joint value of node B’s parent node. Pθ(a|e) represents a function of target node’s probability P(A = a|e) and parameter θ = P(bi|π), or P(a|e)(θ).

In the sensitivity analysis, parameter θ = P(bi|π) may be change. With the change of θ = P(bi|π), other parameters σ = P(bi|πjπi) of the node B will be changed to make sure the sum of probability of all the values equal one. P(bi|π)(θ) represents a function of P(bi|π) and θ = P(bi|π):

\[
P(b_i | π)(θ) = P(b_i | π) \times \frac{1 - θ}{1 - P(b_i | π)}
\] (1)

where, P(bj|π)<1. So that, P(a|e)(θ) can be expressed as the quotient of two linear functions, generally, can be expressed as following:

\[
P(a | e)(θ) = \frac{c_1θ + c_2}{c_3θ + c_4}
\] (2)

where, c1, c2, c3, c4 are constant coefficient. In fact, P(a,e)(θ), P(e)(θ) is θ’s joint probability distribution and priori probability, respectively.

**Definition of sensitivity of parameter:** In a Bayesian network, sensitivity I(θ) of parameter θ = P (b |π) can be defined as:

\[
I(θ) = \frac{1}{rs} \sum_{a,e} \frac{∂P(a | e)(θ)}{∂θ}
\] (3)

where, r and s is the number of the values of A and e.

**Definition of sensitivity of node:** In a Bayesian network, A and B are two random nodes and arc points from B to A, node B’s parameter θ = P(b|π), then node B’s sensitivity relative to node A can be defined as:

\[
IM(B) = \frac{1}{rt} \sum_{a,B} \sum_{i=1}^{r} I(θ_i)
\] (4)

where, r and t is the number of the values of A and B. Sensitivity of node can show the importance of one node relative to another node.

**Node sensitivity analysis:** In this study, sensitivity coefficients is calculated on the basis of junction tree inference algorithm (Lerner et al., 2000):

**Theorem 1:** Suppose BN is a Bayesian network T is a joint tree of BN, y = p(a|e) is the output probability, θ = P(b|π) is the probability achieve from parameter learning, e is an evidence, U and W are cluster nodes in the join tree and W contains the variable A. After the evidence of e messages to cluster W (Evidence collection), for a given A = a, the values of c1 and c 2 can be calculated by messaging (evidence diffusion) from cluster W, calculation equation:

\[
c_i = y_i - y^2 \quad \quad c_2 = \frac{θy_i - θ'y^2}{θ' - θ^2}
\] (5)

The θ' is the initial value of θ, θ is different value from θ'.

\[
y' = p(a, e)(θ') = \sum_{u} φ(u)
\] (6)

\[
y' = (a, e)(θ') = \sum_{u} φ(u) \frac{P'(B | π)}{p(B | π)}
\] (7)
where, \{B\}∪Pa(B)∈U, \phi\cup = P(U, a, e), P'(B|\pi) and P(B|\pi) are the parameter vector of \(\theta = \theta^1\) and \(\theta = \theta^2\).

**Theorem 2**: In the equation \(p(e)(\theta) = c_1\theta^1 + c_4\), constants \(c_1, c_4\) as follows:

\[c_1 = c_{i_1} + c_{i_4}\]  \[c_4 = c_{i_4} + c_{2_4}\]  \(8\)

Due to the fault diagnosis is a process of finding abnormal nodes according to observed abnormal events, that is target node A take a specific value a, the sensitivity of parameter \(\theta\) can be simplified as:

\[I(\theta) = \frac{1}{\sum_{x}^{\theta}} \frac{dP(a|e)}{d\theta} \]  \(9\)

Node B’s sensitivity relative to node A can be simplified as:

\[IM(B) = \frac{1}{\sum_{\theta}^{B}} I(\theta, B) \]  \(10\)

where, \(\theta_i = P(b_i|\pi)\).

**Discovering abnormal node**: KL deviation equation is defined as:

\[KL(p(X), q(X)) = \sum_{x\in X} p(x) \log \frac{p(x)}{q(x)} \]

where, \(p(X)\) is the real distribution of parameters in Bayesian network, \(q(X)\) is achieved from learning. The KL is used to measure the difference between real distribution and approximate distribution.

In fault diagnosis, judging one node is abnormal or not according to the value of \(KL\) \(Coupe et al., 2000\). If the change of KL value exceeds the threshold value, then the node is an abnormal node, otherwise it is a normal node.

**Discover fault node**: According to the equation of full probability, the probability of node A is not only associated with the conditional probability of A but also related to the state probability of node B. So, when node B is abnormal, the node A is not necessarily the fault node lead to system abnormality, may be the parent node of A results in abnormality, so after finding abnormal node it is necessary to judge whether its parent node have abnormality or not. For example, after abnormal events S having been occurred of in Fig. 1, if D is abnormal node but is not the fault node necessarily, the abnormality of D may be caused by abnormality of A or B. So, we need to judge the status of A or B.

The sensitivity of node reflects the importance of the parent node relative to its child nodes, the higher sensitivity is, the bigger influence on its child nodes. In fault diagnosis, ordering nodes by sensitivity from high to low, the node which has high sensitivity will be first searched and calculate its KL values, if KL value is greater than the threshold value, then the node is abnormal node. But the node maybe not the fault node, need to judge its parent node, if this node has no parent or its parent node did not appear abnormality, then the node is the fault node.

**Description of SA_FD algorithm:**

| Input: | Fault diagnosis network BFN = (G, P), threshold value are \(\delta, \epsilon\) |
| Output: | Fault node F |

**Step 1**: Observing system status of S every t time, if at time T, \(|S_{t-1} - S_{t}| > \delta\), then system occurs abnormal event.

**Step 2**: The parent nodes of S are \(X_1, X_2, ..., X_n\) and order them by the sensitivity from high to low.

**Step 3**: Calculating KL values of \(X_i\), if \(KL(X_i) > \epsilon\), then \(X_i\) is abnormal node.

**Step 4**: Judging the node \(X_i\) whether has parent node or not, if it does, then ordering the parents node by sensitivity and calculating KL values for each parent node, otherwise, \(X_i\) is the fault node F.

**Step 5**: If parent node is abnormal node, then go to step 4, or pa(Xi) is fault node F.

**Time complexity of SA_FD algorithm**: In order to analysis the time complexity of SA-algorithm, we analysis the average number of nodes we need to query to find faulty nodes in the case of system can’t work. Given a BFN Network with N layers and each node has n parent nodes.

It is blind to find abnormal nodes for the DFS, that the average number of nodes on each layer is \((n+1)/2\), the number of nodes in the network is \((n+1)\times N/2\), the time complexity of DFS is \((n+1)/2\). Using DFC algorithm to find abnormal node only consider the status of the node itself, without considering its effect on its child node, that lead to the accuracy of DFC is only about 50%. DFC’s number of nodes should be found in each layer equals DFS’s average, namely \((n+3)/4\), the time complexity of DFC is \((n+3)/4\).

**SA_FD algorithm** does not only consider the status of node but also consider its status influence on its child nodes while finding the faulty node, so its accuracy is higher than DFC algorithm. Because SA_FD algorithm has to consider another factor, the number of node need to be found is \((n+3)/6\). When using SA_FD algorithm, the time complexity of SA_FD is \((n+3)/6\).

We can draw a conclusion that with the n increasing, the more complex of network is, the diagnosis time increasing in SA_FD algorithm is the slowest, second is DFC algorithm, the fastest is DFS algorithm according to time complexity of three algorithms. Therefore, SA_FD algorithm is more suitable for large-scale complex network.

**RESULTS**

This study chooses car-diagnosis which has eighteen nodes and twenty arcs to run SA_FD algorithm, as shown in Fig. 2, the meanings and values of each node are shown in Table 1.
Algorithm instance: First, if the node has several parent nodes, then calculating its parent nodes’ changes in CPT, calculated results are shown in Table 2 and the network parameters can get from Russell et al. (2000). Secondly, calculating each node’s sensitivity relative to its child nodes according to the Eq. 10, there is no necessary to calculate the sensitivity for the node which has only one child node and the results are shown in Table 3. Finally, selecting nodes $X_1$, $X_4$, $X_5$, $X_{10}$ as fault nodes which are far away from the abnormality node ($X_{18}$) from Fig. 2 and running DFS, DFC and SA_FD algorithm four times and comparing the path of fault node and efficiency of three diagnostic algorithms.

The diagnostic path of DFS, DFC and SA_FD algorithm can reflect the efficiency of three algorithms in car-diagnosis network, but due to the limitations of the network size, the advantage of SA_FD algorithm is not obvious (Table 4).

**DISCUSSION**

In order to compare the time performance among three algorithms, the network scale has been extended to 1000 nodes and Bayesian networks are established that each node has 2, 5 and 10 parent nodes in order to research the efficiency of three algorithms influenced by network scale. In a different scale of the network, running the DFS (Kopec and Marsland, 2004), DFC (Doguc and Ramirez-Marquez, 2009), SA_FD, the results are shown in Table 5.

As shown in Table 5, with the increase of the number of network nodes, the diagnosis time of three algorithms shows an increasing tendency and the DFS algorithm increases much more than the latter two and to the networks that have the same number of nodes, with the increase of the number of parent nodes, the diagnosis time of three algorithms will increase, the increasing amplitude of the DFS algorithm is significantly greater than the latter two. This shows that the higher the complexity of network is, the greater gap of time performance is between DFS and other two algorithms. The network is more complicated, high efficiency of the DFC algorithm and SA_FD algorithm is more obvious. DFC algorithm has some gaps with SA_FD algorithm in diagnosis time because of its one-sidedness in judgment methods.
Table 4: Diagnostic path of DFS, DFC and SA_FD (car can’t be started because of failure of X1, X7, X10)

| Car can’t be started because of failure of X1, and diagnostic path of DFS, DFC and SA_FD |
| DFS | X18 | X12 | X6 | X11 | X7 | X4 | X2 | X1 |
| DFC | X18 | X13 | X10 | X4 | X2 | X1 |
| SA_FD | X18 | X13 | X6 | X4 | X2 | X1 |

Car can’t be started because of failure of X3, and diagnostic path of DFS, DFC and SA_FD

| DFS | X18 | X12 | X6 | X11 | X7 | X4 | X3 | X2 | X1 |
| DFC | X18 | X13 | X10 | X4 | X3 | X2 | X1 |
| SA_FD | X18 | X13 | X8 | X4 | X3 | X2 | X1 |

Car can’t be started because of failure of X7, and diagnostic path of DFS, DFC and SA_FD

| DFS | X18 | X12 | X6 | X11 | X7 | X4 | X2 | X1 |
| DFC | X18 | X13 | X10 | X4 | X7 | X2 | X1 |
| SA_FD | X18 | X13 | X8 | X4 | X7 | X2 | X1 |

Car can’t be started because of failure of X10, and diagnostic path of DFS, DFC and SA_FD

| DFS | X18 | X12 | X14 | X13 | X10 |
| DFC | X18 | X13 | X10 | X4 | X7 |
| SA_FD | X18 | X13 | X8 | X4 | X7 |

Table 5: Diagnosis time of two, five and ten parent node

<table>
<thead>
<tr>
<th>Nodes</th>
<th>Algorithms</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>400</th>
<th>800</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two parent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFS</td>
<td>1.35</td>
<td>3.06</td>
<td>4.23</td>
<td>7.87</td>
<td>13.73</td>
<td>17.14</td>
<td></td>
</tr>
<tr>
<td>DFC</td>
<td>1.16</td>
<td>2.21</td>
<td>3.02</td>
<td>5.23</td>
<td>9.76</td>
<td>12.21</td>
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</tr>
<tr>
<td>SA_FD</td>
<td>0.93</td>
<td>1.26</td>
<td>2.01</td>
<td>2.86</td>
<td>5.71</td>
<td>7.05</td>
<td></td>
</tr>
<tr>
<td>Five parent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFS</td>
<td>3.51</td>
<td>6.25</td>
<td>10.01</td>
<td>18.03</td>
<td>32.25</td>
<td>39.25</td>
<td></td>
</tr>
<tr>
<td>DFC</td>
<td>2.25</td>
<td>3.75</td>
<td>5.75</td>
<td>9.75</td>
<td>17.01</td>
<td>19.75</td>
<td></td>
</tr>
<tr>
<td>SA_FD</td>
<td>1.25</td>
<td>2.49</td>
<td>3.68</td>
<td>5.75</td>
<td>9.59</td>
<td>16.25</td>
<td></td>
</tr>
<tr>
<td>Ten parent</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFS</td>
<td>6.24</td>
<td>12.04</td>
<td>19.68</td>
<td>35.04</td>
<td>62.40</td>
<td>76.32</td>
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<tr>
<td>DFC</td>
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<td>9.12</td>
<td>15.36</td>
<td>28.88</td>
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<td>5.26</td>
<td>9.14</td>
<td>15.84</td>
<td>17.76</td>
<td></td>
</tr>
</tbody>
</table>

CONCLUSION

Due to the uncertainty and relevance of complex systems, the efficiency and accuracy of fault diagnosis decrease as the scale increases. The Bayesian network has increasingly become a powerful tool for fault diagnosis in complex systems because of its advantage in dealing with uncertain problems. Because of lacking consideration of the relationship between parent and child nodes the existing fault diagnosis methods based on Bayesian networks can not accurately find fault nodes and has higher computation complexity. In this study, the sensitivity analysis of Bayesian networks is introduced into the fault diagnosis process to improve the efficiency and accuracy of fault diagnosis by optimizing the path for fault detection. In order to describe fault diagnosis effectively based on sensitivity analysis, this study firstly presents the formal model of Bayesian fault diagnosis network based on sensitivity analysis. Then, it calculates the sensitivity function by using reasoning algorithm based on the junction tree, to obtain the sensitivity of nodes which can be used to measure the importance of each parent node relative to its child node. After selecting parent nodes on the basis of sensitivity, nodes with high sensitivity will be considered as those which are most likely to cause an abnormality. This study also proposes an efficient fault diagnosis algorithm, named, SA_FD. Finally, it takes the car-diagnosis network as an example to analysis and test SA_FD’s time performance and the accuracy. Although the experiment results show that the complexity increases as the network scale increases, the diagnosis efficiency is significantly higher than the DFS algorithm and DFC algorithm. However, because the experiment data of this study is not real-time, the promotion of the algorithm is affected. Therefore, causal fault diagnosis of complex system based on time-series data will be a study emphasis in further research.

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