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Mathematical Management of Simple Inventory System

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Abstract: The article discusses optimal inventory policy in a simple business organisation that has a known or predictable factors. Optimal inventory policy implies cost savings in inventory control, and achievement of maximum profits or/and to minimum costs. A mathematical model is used for this purpose and a case study adjusts mathematics (maths) results to practicality of a business situation. Theoretical optimisation (maths optimisation) and practical optimisation (adjustment of maths results) are compared.

Keywords: optimisation, inventory control, maximum benefits/profits, minimum costs/risks

Introduction

A common problem in inventory control when making decisions in physical storage is determining an efficient policy Gavirneni and Tayur, (2001). In establishing such a policy, it is vital to determine the order quantity, defined as the number of items to be stocked periodically, and the order point, which is the time when replenishment should occur Wu *et al*, (2000). A relevant controllable factor that includes inventory policy is inventory cost Boehle *et al*, (2000). In a typical business situation, involved costs for evaluation of inventory systems are ordering costs, procurement costs and holding costs Dekker & Frenk, (2000).

Mathematical model for economic order quantity (EOQ):According to Wu *et al* (2000), the EOQ model is considered a simple inventory model that involves a type of item (commodity) that has a known and constant demand, that is resupplied instantaneously, and where backordering is not permissible. The objective is to select an efficient inventory policy that minimises the annual inventory costs. Axsater (2001) points out that the policy should choose the order quantity and establish the optimal order point. To facilitate the discussion, the next paragraph defines the quantities used in the EOQ model.

Parameters of the appropriate mathematical model given by Lapin (1995) for the described problem are:

c_0 = fixed cost per order,
N = annual number of items demanded,
c = cost of procuring a single item,
h = annual holding cost (per currency unit) of items in inventory,
t = time between orders, and
q = order quantity.

The simple mathematical model for the EOQ case, implied by Wu *et al.* (2000) is:

total annual cost = ordering cost + holding cost + procurement cost (1)

The objective is to choose the number of items q to order in such a way that (1) is minimised.

Quantifying costs: New inventory begins when an old one ends, and the q items are replenished periodically. The order quantity q determines the cycles duration t (in years) that equals the proportion of the annual demand consumed in an inventory cycle. That is:

$$t = \frac{q}{N} \qquad (2)$$

Note: Items are used up to N units per year. The sequence is inventory depletions followed by replenishment of successive inventory cycles, and vice versa. Restocking occurs only when inventory has reached zero, because it is costly to hold items in inventory.

The mathematical expression for the problem objective start with the annual ordering cost (AOC), where AOC is based on the number of orders placed each year and the number of orders depends on the annual number of items demanded (N) and the order quantity (q) Wu *et al*, (2000). Therefore,

$$\text{number of annual orders} = \frac{N}{q}$$

which, after multiplying by the cost per order c_0,

$$\text{leads to annual ordering cost} = \left(\frac{N}{q}\right)c_0 \qquad (3)$$

Annual holding cost is based on the number of items placed in inventory and the length of time they are stored in inventory. Some items are sold immediately while others are held until the inventory is stocked Yee and Veatch, (2000). By considering an average value, it is noted that the inventory level in any cycle ranges from q to 0 with a uniform rate of depletion. This leads to:

$$\text{average inventory} = \frac{q}{2}.$$

The holding cost and other costs such as insurance and property taxes, are based on the value of the items held, and the value of an item is based on its procurement cost Collier and Johnson, (1998). Hence, the annual holding cost of an item in inventory is the annual holding cost per currency unit (h) multiplied by the unit procurement cost (c). This means that: annual holding cost of one item $= hc$. Therefore,

$$\text{annual holding cost} = hc\left(\frac{q}{2}\right) \qquad (4)$$

Moreover, for each whole year, N items will be demanded and then:

annual procurement cost $= Nc$ (5)

The function given by equation (1) is obtainable in quantified form from (3), (4) and (5) as the total annual cost (F) given by:

$$F(q) = \left(\frac{N}{q}\right)c_0 + hc\left(\frac{q}{2}\right) + Nc \qquad (6)$$

Optimal solution: The function F is called the objective function Liang and Wang, (2000). The value of F depends on the order quantity q, and the cost of each order is c_0, regardless of the value of N. To optimise F implies minimisation because it is a cost function.

The objective function is minimised with respect to F. Calculus is employed to derive the solution. The derivation, based on optimisation using calculus of stationary points, Davey, (1999), is:

$$\frac{dF(y)}{dq} = -\frac{N}{q^2} + \frac{hc}{2} = 0$$

which implies that:

$$q_{opt} = \sqrt{\frac{2Nc_0}{hc}} \qquad (7)$$

where q_{opt} is the order quantity that minimises the inventory cost for any given year.

Controllability of inventory costs: The total annual relevant cost given by (6) could be optimised by $TC = \left(\frac{N}{q}\right)c_0 + hc\left(\frac{q}{2}\right)$, which depends on the order quantity q.

The reader is reminded that the total annual relevant cost has two components, the annual ordering cost and the annual holding cost, which are expected to be equal when the optimal order quantity has been achieved. TC is the sum of two costs, hence, it is always larger than their values at any level q. The ordering cost is a hyperbola $\left(\frac{N}{q}\right)c_0$, c_0 constant. The function TC is optimised at its minimum point. The optimal order quantity $\left(q_{opt}\right)$ is the minimum cost point on the TC curve at slope zero.

A case study: Motswedi bakery was established in the year 1995 in the village of Masimong, north-east of the small town of Brits, South Africa. A consultant was requested to assist Motswedi bakery to find an optimal inventory policy for flour A. In 1998 the bakery appointed Sibongile, a new MBA graduate as the Marketing Executive in its Logistics and Inventory Division. Sibongile adjusted fairly well, and realised a problem with the manner in which the ordering of flour A was done. In particular, she did not fancy the monthly ordering of bulk flour A which, according to her, did not have justification for large quantity orders. Her argument was that even though it costs the bakery nothing to order and store too much flour, flour orders are delivered timeously when needed. Further, each unit order of flour made costs the same price to a buyer regardless of how it is dispatched to the buyer. Another argument that Sibongile presented is that fresh flour A could be bought when needed instead of using stale flour.

Motswedi bakery needs 156000 kg of flour A each year and the cost of each kg of flour A to the bakery is 75 cents. Each delivery to the bakery from the supplier is R12, regardless of the quantity ordered. Further, the orders are delivered on a same day delivery policy for orders made in the morning, and the following day for orders made in the afternoon. The monetary cost of flour held in inventory is estimated at 14 cents.

The objective is to establish an optimal inventory policy by minimising the annual cost. As a result of this objective, Sibongile wishes to assess the current policy of ordering flour A in the bakery. The following discussion explains how the consultant undertook the analysis for the bakery.

As-Is situation: The following constants required for the mathematical model have been deduced.

Co = R12 per order of flour A

N = 156000 kg, amount of flour A demanded annually

c = R0.75, cost of procuring a kg of flour A

h = 14c × 12 = R0.48, annual cost per rand value of flour A held in inventory.

For any policy used, Nc = R117000.00 is a fixed compulsory cost. Currently, the policy of ordering per month implies that the quantity of flour A bought every month is:

$$q = \frac{156000kg}{12} = 13000kg$$

The total annual relevant cost of the monthly policy for optimisation is calculated next.

$$TC = \left(\frac{N}{q}\right)c_0 + hc\left(\frac{q}{2}\right)$$

The annual ordering cost of R144 is much less than the annual R2340 holding cost. This huge difference shows that the situation is far from optimal because for optimal policy, these costs should be the same.

This means that the total annual cost for flour A is:

TC + Nc = R2484.00 + R117000.00 = R119484.00.

Optimal policy: The optimal inventory policy explained earlier at the EOQ model is given by (7) as:

$$q_{opt} = \sqrt{\frac{2Nc_0}{hc}} = \sqrt{\frac{2(156000)\times 12}{(0.48)(0.75)}} = 3225 \text{ kg.}$$

The optimal time between the orders is:

$$t_{opt} = \frac{q_{opt}}{N}$$

$\frac{3225kg}{156000kg}$ = 0.021 years X 365 days/year = 7.5 days.

Theoretically, this means that the optimal inventory policy is to order 3225 kg of flour A every 7.5 days, with a total annual relevant cost for optimisation of: $TC = \left(\frac{156000}{3225}\right) \times 12 + (0.48)(0.75)\left(\frac{3225}{2}\right)$

= 580.47 + 580.50 = R1160.97.

Total annual cost for flour A is $TC + Nc$ = R1160.97 + R117000.00 = R118160.97.

The optimal policy could save the bakery R1319.03 every year.

Practicality of the situation: One notes that for the mathematical optimal policy, the annual R580.47 ordering cost and the annual R580.50 holding cost are (almost) equal. However, this could occur for every 7.5 days of ordering, which practically could mean 7 or 8 days. In terms of a periodical pattern, 7 days is practically appealing because it means the weekly pattern, while 8 days could disrupt or defy normal weekly bookkeeping procedure. The weekly pattern is now compared with the optimal situation.

The policy of ordering per week implies that the quantity bought every week is:

$$q = \frac{156000kg}{52} = 3000kg$$

The total annual relevant cost of the monthly policy is:

$$TC = \left(\frac{N}{q}\right)c_o + hc\left(\frac{q}{2}\right)$$

$$= \frac{156000}{3000} \times 12 + (0.48)(0.75)\left(\frac{3000}{2}\right) = 625 + 540$$

= R1165.00.

The total annual cost for flour A in this case will be $TC + Nc =$ R118165.00.

Discussion

The annual ordering cost of R625 is a little more than the annual R540 holding cost. This small difference shows that the situation is not very far from optimal because for optimal policy, these costs should be the same. Compared with the monthly policy, the weekly approach could save the bakery about R1325 every year. This is not very different from the annual R1319.03 that the optimal policy could save the bakery every year. In effect, the optimal policy edges the weekly policy by implying that it could save the bakery only R4.03 every year relative to the weekly policy. This amount is negligible and we may consider the weekly policy to be practically optimal.

On using the fact that the weekly policy proposes that 3000 kg of flour A be purchased compared to 3225 kg after every 7.5 days (which is not practical, as it could be interpreted as 8 or 7 days). The difference between the weekly policy and the optimal case is not a problem because in the end the flour is finished when new stock is ordered. The weekly procedure becomes more appealing because it is both practical, convenient and implies more freshness in the flour A used.

The weekly policy is practically optimal while 7.5 days policy is theoretically optimal. Further, the weekly policy does not disgrace itself against the theoretical optimal policy as these two policies compare well. There is also a benefit that storage space of capacity equivalent to 100000 kg of flour could be used for other purposes.

Therefore, evident benefits in the efficient policy brought by the weekly orders of flour A are that:

- It is practicable when compared with the mathematically optimal one.
- It saves the bakery over a thousand rands
- Fresh flour could be used more often in the bakery, and
- There is storage space that could be used for other purposes.

Conclusion

There could be cases where the optimal solution could be implemented as derived directly from mathematical optimisation. In the case study used, it was a learning curve to understand theory in relation to actual industrial practice. From the case study it becomes evident that optimality from a mathematical approach serves as a guide to efficiency in real life, and it does not necessarily lead to a conclusion that could be implemented. It is concluded, however, that theory be pursued as it enhances better practice, while practice encourages improvement and advancement of existing theory. Practice could also trigger research that could lead to new theory.

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