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A Proportional-Integral-Derivative (PID) Controller Realization by Using Current Feedback Amplifiers (CFAs) and Calculating Optimum Parameter Tolerances

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Abstract: In this work the synthesis procedure of a proportional-integral-derivative (PID) controller implemented with a commercially active component, AD 844, which is equivalent to the combination of a second generation current conveyor having a gain of +1 (CCII+) and a unity gain voltage buffer discussed. These current-mode (CM) circuits are not used in analog controller design so far. Furthermore the optimum parameter tolerances for the proposed PID circuit by the use of parameter sensitivities are determined. These tolerances keep the relative error at the output of the controller due to parameter variations in tolerance region.

Key words: PID controller, Current Feedback Amplifier, Optimum Parameter Tolerances

Introduction

The proportional-integral-derivative (PID) controllers are one of the most important control elements used in process control industry Kuo, (1997). In practice operational amplifiers are generally used in analog controllers. On the other hand, current feedback amplifier(CFA) is an active component providing an excellent combination of AC and DC performance. It combines high bandwidth and very fast large signal response with excellent DC performance. It is also free from the slew rate limitations inherent traditionally in operational amplifiers and other current-feedback operational amplifiers. It can be used instead of traditional operational amplifiers, however its current feedback architecture results in much better AC performance and high linearity Roberts and Sedra (1989) and Wilson (1990). CFA is equivalent to the combination of a second generation current-conveyor having a gain of +1 (CCII+) and a unity gain voltage buffer Svoboda *et al.* (1991).

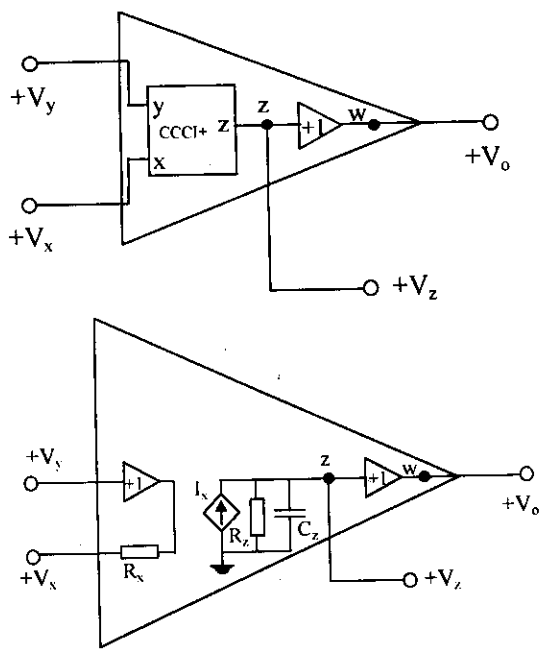
In spite of the above mentioned features, no work has been carried out for the generation controllers using CFAs except for the one realized by using current conveyors by Erdal *et al*, (2000).

The main purpose of this paper is to present a new circuit for the realization of PID controller using only four CFAs and passive components. This procedure is based on the signal-flow graph, which is a powerful tool in active circuit design.

The simulation of the proposed circuit is performed and the results are discussed. Furthermore, the optimum parameter tolerances by the use of parameter sensitivities are determined. These tolerances keep the relative error at the output of the controller due to parameter variations in tolerance region and they can also be used to improve and to control the sensitivity performance of the proposed PID controller.

Current-Feedback Amplifier (CFA): The circuit symbol of a current feedback amplifier (CFA) is shown in Fig. 1. An ideal CFA can be described, in s-domain, by the following equations Analog Devices, Linear Products Data Book, (1990):

(a



(b)

Fig. 1: (a). Circuit symbol of CFA (b). Equivalent circuit of CFA

$$V_x = V_y \quad V_o = V_y \quad I_y = 0 \quad I_z = I_x \tag{1}$$

where V_y, V_x, I_y, and I_x are positive and negative input terminal voltages and currents, respectively V_z and V_o are output terminal voltages. I_z is the z-terminal current.

An equivalent circuit of CFA is also shown in Fig. 1. where R_x is the input resistance of the negative input terminal. R_z and C_z are input resistance and capacitance respectively of the z-terminal. $R_x = 50\ \Omega$, $R_z = 3\ M\Omega$ and $C_z = 4.5$ pF are the typical values of a commercially available CFA, namely AD844/AD from Analog Devices (1990).

Note that both plus and minus signs or the letters y and x are used in literature to denote the inputs of CFA. For example see the references Liu (1995) and Svoboda *et al.* (1991). In this study y and x are preferred for the inputs of the commercially available current feedback amplifier.

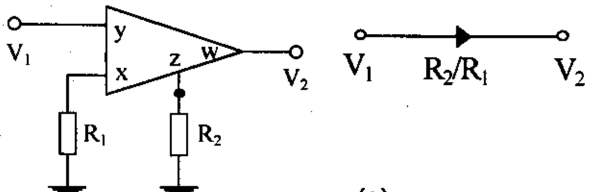
Taking the non-idealities into account the terminal equations of CFA can be written as follows:

$$I_y(t)=0,\quad V_x(t)=\beta V_y(t),\quad I_z(t)=\alpha I_x(t),\quad V_o=\gamma V_z \tag{2}$$

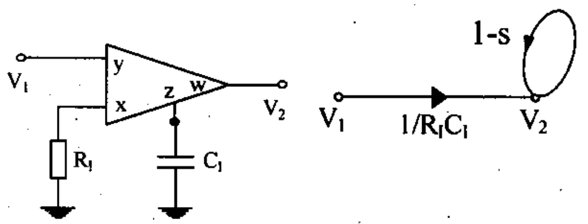
Here $\alpha=1-\varepsilon_i$ denotes the current gain, $\beta=1-\varepsilon_v$ denotes the voltage gain of the current conveyor, and $\gamma=1-\varepsilon_o$ denotes voltage gain of the voltage buffer. ε_i, ($|\varepsilon_i|<<1$) is the current tracking error; ε_v, ($|\varepsilon_v|<<1$) is the voltage tracking error of the input buffer, and ε_o, ($|\varepsilon_o|<<1$) is the voltage tracking error of the output buffer.

The low output impedance of the buffer enables easy cascading in voltage-mode operation.

Synthesis Procedure: Consider the current feedback amplifier circuits and their corresponding signal-flow graphs shown in Fig. 2.



(a)



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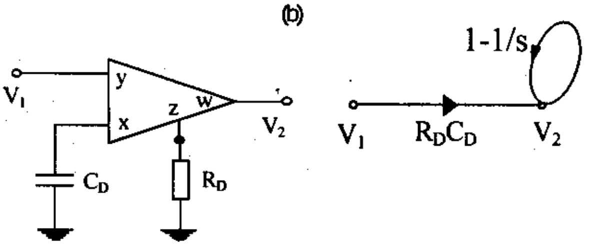
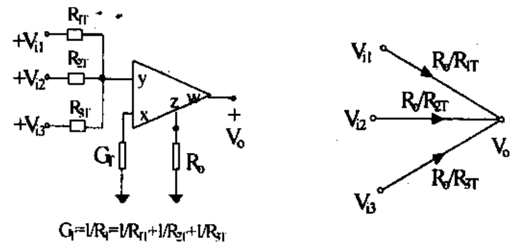


(c)



(d)

Fig. 2: Basic building blocks using CFAs togeher with corresponding signal-flow graphs (a) Amplifier circuit, (b) Integrator circuit, (c) Derivative circuit, (d) Summing circuit.

In Fig. 2(a), an amplifier circuit and its signal-flow graph are shown. The gain is R_2/R_1. In Fig. 2(b), an integrator and its signal-flow graph are shown. The integration time constant of this circuit is $1/R_IC_I$. In Fig. 2(c), a CFA based derivative circuit and its signal-flow graph are illustrated. The derivation time constant of this circuit is R_DC_D. To obtain a PID controller the three basic operations shown in Figs. 2 (a), (b) and (c) are transmitted to the output by CFA based summing circuit illustrated in Fig. 2(d). If a given transfer function is represented by a signal flow-graph, it can be easily observed from Fig. 2 that the corresponding circuit to the given transfer function can be realized by interconnecting these building blocks. Note also that, in non-ideal case all the transfer functions shown in Fig. 2 should be multiplied by the factor of $\alpha\beta\gamma$.

The transfer function of a general analog, proportional-integral-derivative (PID) controller can be written as follows Kuo (1997):

$$T(s)=\frac{V_o(s)}{V_i(s)}=K_P+\frac{K_I}{s}+sK_D \tag{3}$$

The signal-flow graph of the transfer function of an analog PID controller can be drawn such as in Fig. 3. Erdal and Toker (1998).

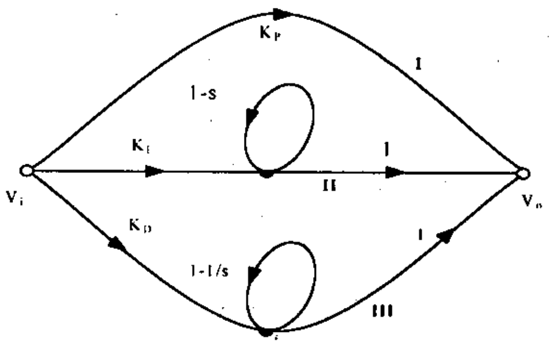


Fig. 3: A signal flow graph corresponding to the transfer function of the general proportional- integral-derivative (PID) controller

Using this signal-flow graph the controller transfer function T(s) can be realized using the active-RC circuits involving CFAs Acar (1996). The realization of the analog

CFA-based, PID controller circuit corresponding to the signal-flow graph in Fig. 3, which is realized by using the sub-circuits given in Fig. 2 is illustrated in Fig. 4. Note that in Fig. 4, a PI controller circuit and a PD controller circuit can be obtained by removing the path III and path II respectively.

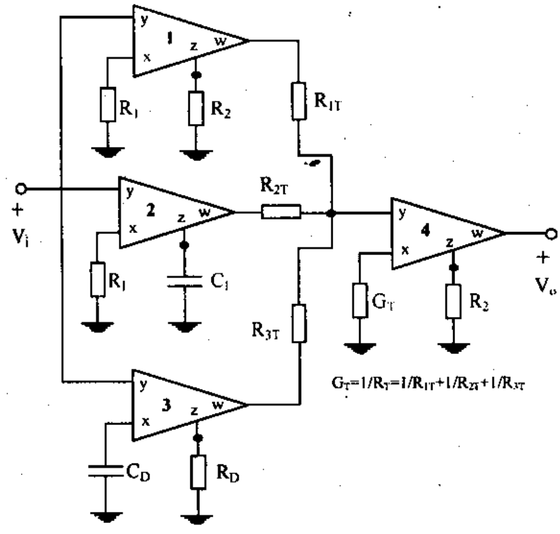


Fig. 4:A CFA-based PID controller realization corresponding to the signal-flow graph in Fig. 3

If the circuit in Fig. 4 is analyzed with taking the non-idealities of CFA into account the control coefficients K_P, K_I, K_D will be obtained as follows:

$$K_P = \alpha_1\beta_1\gamma_1\alpha_4\beta_4\gamma_4 \frac{R_2R_O}{R_1R_{1T}} \tag{4a}$$

$$K_I = \alpha_2\beta_2\gamma_2\alpha_4\beta_4\gamma_4 \frac{R_O}{R_1C_IR_{2T}} \tag{4b}$$

$$K_D = \alpha_3\beta_3\gamma_3\alpha_4\beta_4\gamma_4 \frac{R_DC_DR_O}{R_{3T}} \tag{4c}$$

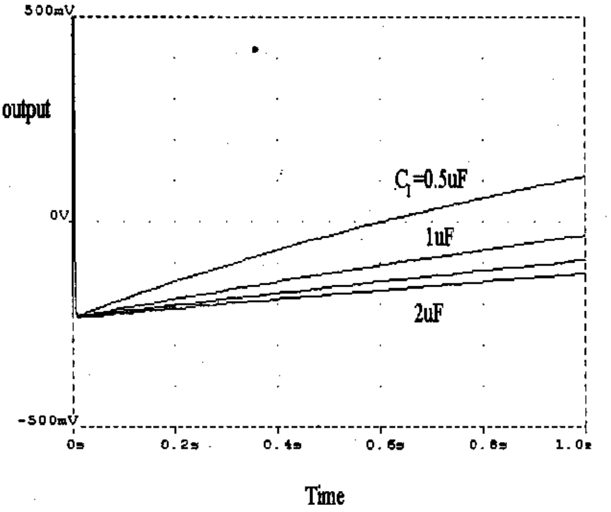
These control coefficients K_P, K_I, and K_D will be used in calculating the optimum parameter tolerances in section 5.

Simulation Results: SPICE simulations are performed for the PID circuit shown in Fig. 4 by using the macro-model of AD844/AD from Analog Devices (1990), to verify the theoretical explanations given above. In this circuit supply voltages of ± 12 V are used. The values of capacitors, C_I in the circuit are varied in the range from 0,5 μF to 2 μF with the 0,5 μF increments and for C_D from 2.5 μF to 10 μF with the 2.5 μF increments.

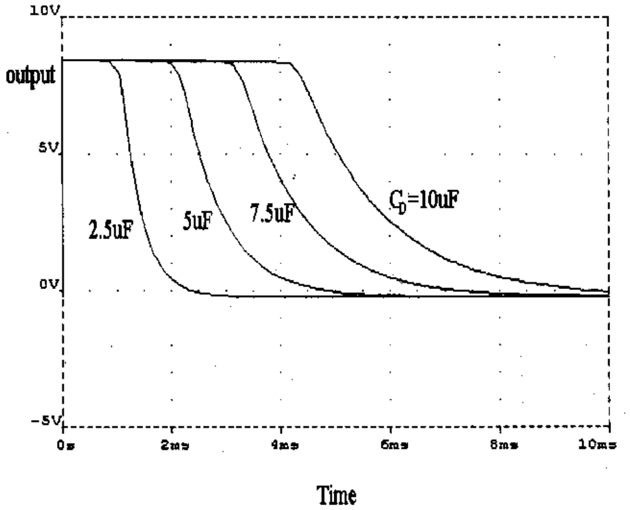
The simulation results of the output of the CFA based PID controller are given in Fig. 5 (a) and (b) respectively. In both situation, the proportional coefficient is taken zero otherwise the curves will shift in vertical direction corresponding to the value of K_P.

The values of the capacitances in simulation procedure are given in figure caption, the resistor values are the same as given in section 5. The capacitance values are selected such as to have a better illustration. From Figs. 5 (a) and (b) it is easily remarkable that the results are in good agreement with the theoretical expectations.

Calculating Optimum Parameter Tolerances: The optimum parameter tolerances are defined as the



(a)



(b)

Fig. 5: Simulation results of the PID controller in Fig. 4 for (C_I=0.5 μF, 1 μF, 1.5 μF, 2 μF), for (C_D=2.5 μF, 5 μF, 7.5 μF, 10 μF).

tolerances contribute equally to the upper bound of the relative error of the output voltage of the controller ($|\Delta V_o/V_o|$) given in Fig. 4. In general, it is not known in advance how much each parameter contributes to the output error. That is why this definition is quite reasonable, since the designer expects the contribution of each parameter variation on output deviation to be equal to each other. The formulation of these tolerances was given by Erdal *et. al.* (2001). As a result, we can define the optimum parameter tolerances as

$$t_{x_i} = t_0 / n\left|S^T_{x_i}(\omega_i)\right|_{max}, \quad i = 1,\ldots,22$$

where t_{x_i} is the ith parameter tolerance, t_o is the output tolerance of the controller, n is the parameter number, i.e. n=22 for the given configuration, and w_i is the angular frequency at which $\left|S^T_{x_i}(\omega)\right|$ takes its maximum value, i.e. $\omega \hat{I}[\omega_1,\omega_2]$ describes designer's specified frequency band. Hence $\left|S^T_{x_i}(\omega)\right| \le \left|S^T_{x_i}(\omega_i)\right|_{max}$, $\omega\hat{I}[\omega_1,\omega_2]$. It should be noted that ω_i belong to the interval $\omega\hat{I}[\omega_1,\omega_2]$, and $\left|S^T_{x_i}(\omega)\right|$ has its maximum value at this frequency. The designer can easily determine w_i by plotting $\left|S^T_{x_i}(\omega)\right|$ at this interval or by using already existing mathematical programs like Matlab.

For example, assuming that the proportional gain, $K_P = 10$, the integral gain, $K_I = 2\ s^{-1}$, and the derivative gain, $K_D = 5$ s, are given. Then the parameter values can be selected in Fig. 4 as follows:

$$R_1 = 2\ K\Omega,\ R_0 = R_2 = 10\ K\Omega,\ R_I = R_D = 50\ K\Omega, \tag{7a}$$

$$R_{1T} = 5\ K\Omega,\ R_{2T} = 40\ K\Omega,\ R_{3T} = 1\ K\Omega,\ R_T = 1.25\ m\Omega$$
$$C_I = 1\ \mu F,\ C_D = 10\ \mu F \tag{7b}$$

$$\alpha_i = 1, \quad i = 1,\ldots4; \quad \beta_j = 1, j = 1,\ldots4; \quad \gamma_k = 1,\ k = 1,\ldots4. \tag{7c}$$

For this example, the maximum values of the parameter sensitivities are calculated as follows:

$$\left|S^T_{x_i}(\omega_i)\right|_{max} = 1, \ i = 1,\ldots22 \tag{8}$$

If it is required that $|\Delta V_o/V_o| \le 0.01$, the parameter tolerances are to be chosen as follows:

$$t_{R_1} = t_{R_2} = t_{R_{1T}} = t_{R_{2T}} = t_{R_{3T}} = t_{R_o} = t_{C_I} = t_{C_D} = 0.45\%, \tag{9a}$$

$$t_{\alpha_i} = t_{\beta_i} = t_{\gamma_i} = 0.45\%, \quad i = 1,\ldots4. \tag{9b}$$

For this particular example, the optimum tolerances are found to be equal to each other, however they are usually different in general case. Choosing the parameter tolerances such as above, the designer can guarantee that the maximum deviation of the output voltage of the controller caused by the parameter variations due to the environmental effects will be less than or equal to 0.1. If $|\Delta V_o/V_o| \le 0.01$ is required the parameter tolerances must be chosen ten times smaller than the ones in Eq. (9) and so forth.

Conclusions

In this study, a CFA based PID design procedure is given and a PID circuit is proposed. The proposed circuit consisted only of four CFAs, two grounded capacitors and resistors. This circuit is also very suitable to control the rapidly changing signals and in the situations when the stable control is required since CFAs have suitable properties than operational amplifiers. Besides, the controller coefficients K_P, K_I and K_D depend on the time constants and resistor ratios. This property simplifies the use of the commercially available active component in implementation. The effects of parasitic input impedance of the CFA on controller performance can be reduced by selecting the impedance scaling factor correctly as stated by Svoboda (1994). Furthermore, the optimum parameter tolerances are determined. These tolerances keep the relative error at the output of the CFAs based PID controller due to parameter variations in its tolerance region.

References

Acar, C. 1996. Nth-order voltage transfer function synthesis using commercially available active component AD844: Signal-flow graph approach, Electronics Letters, 33:1933-1934.

Analog Devices, Linear Products Data Book.1990. Norwood, Massachusetts, USA.

Erdal, C., A. Toker and C. Acar. 2000. Current conveyor based proportional-integral-derivative (PID) controller and calculating optimum parameter tolerances, Proc. of 7th International Conference and Exhibition on Optimization of Electrical and Electronic Equipment (OPTIM), III: 575-578.

Erdal, C. and A. Toker. 1998. A method to design an OTA-C based proportional-integral-derivative (PID) controller, (In Turkish), Proc. of The National Automatic Control Symposium, TOK' I: 155-158.

Erdal, C., A. Toker, and C. Acar, 2001. OTA-C based proportional-integral-derivative (PID) controller and calculating optimum parameter tolerances, Turkish J. Electrical Engg. & Computer Sci., Elektrik, 9, No. 2, (in press).

Kuo, B.C.. 1997. Automatic Control Synthesis. Prentice-Hall Inc., N.J.,USA.

Liu, S. I. 1995. High input impedance filters with low component spread using current-feedback amplifiers, Electronics Letters, 3: 1042-1043.

Roberts, G. W.and A. S. Sedra. 1989. All current-mode frequency selective circuits, Electronics Letters, 25: 759-761.

Svoboda J. A., L. Mcgory, and S. Webb. 1991. Applications of a commercially available current conveyor, Int. J. Electronics , 70: 159-164.

Svoboda, J. A. 1994. Transfer function synthesis using current conveyors, Int. J. Electronics, 76: 611-614.

Wilson, B.. 1990. Recent developments in current conveyor and current-mode circuits, Proc of. IEE pt. G, 137: 63-77.