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Optimal Inventory Control Under Conditions of Permissible Delay in Payments Derived Without Derivatives

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Abstract: The present study was carried out to investigate Goyal's model (1985) and Teng's model (2002) using the algebraic method to determine the optimal cycle time under delay payments. This paper provides algebraic approach that must be considered as a pedagogical advantage for explaining the inventory concept to students that lack knowledge of derivatives. This algebraic approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus.

Key words: EOQ, inventory, permissible delay in payments, derivatives

INTRODUCTION

In most business transactions, the supplier will allow a specified credit period to the retailer for payment without penalty to stimulate the demand of his/her products. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of trade credit period. Recently, several papers have appeared in the literature that treat inventory problems with varying conditions under the consideration of permissible delay in payments. Some of the prominent papers are discussed below.

Goyal[1] established a single-item inventory model under permissible delay in payments. Chung[2] developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Aggarwal and Jaggi[3] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Chang *et al.*[4] extended this issue to the varying rate of deterioration. Liao *et al.*[5] and Sarker *et al.*[6] investigated this topic with inflation. Jamal *et al.*[7] and Chang and Dye[8] extended this issue with allowable shortage. Chang *et al.*[9] extended this issue with linear trend demand. Chen and Chuang[10] investigated light buyer's inventory policy under trade credit by the concept of discounted cash flow. Hwang and Shinn[11] modeled an inventory system for retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payment. Jamal *et al.*[12] and Sarker *et al.*[13] addressed the optimal payment time under permissible delay in payment with deterioration. Teng[14] assumed that the selling price not equal to the purchasing price to modify Goyal's model[1]. Chung *et al.*[15] discussed this issue under the selling price not equal to the purchasing price and different payment rule. Khouja and Mehrez[16] investigated the effect of four different supplier credit policies on the optimal order quantity within the EOQ framework. Shinn and Hwang[17] determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size and also the demand rate is a function of the selling price. Chung and Huang[18] extended this problem within the EPQ framework and developed an efficient procedure to determine the retailer's optimal ordering policy. Huang[19] extended this issue under two levels of trade credit and developed an efficient solution procedure to determine the optimal lot-sizing policy of the retailer. Huang and Chung[20] extended Goyal's model[1] to cash discount policy for early payment.

In previous all published papers which have been derived using differential calculus to find the optimal solution and the need to prove optimality condition with second-order derivatives. The mathematical methodology is difficult to many younger students who lack the knowledge of calculus. In recent papers, Grubbström and Erdem[21] and Cárdenas-Barrón[22] showed that the formulae for the EOQ and EPQ with backlogging derived without differential calculus. They mentioned that this approach must be considered as a pedagogical advantage for explaining the EOQ and EPQ concepts to students that lack knowledge of derivatives, simultaneous equations

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and the procedure to construct and examine the Hessian matrix. This algebraic approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus.

Goyal[1] is frequently cited when the inventory systems under conditions of permissible delay in payments are discussed. Then, Teng[14] assumed that the selling price not equal to the purchasing price to modify Goyal's model[1]. Therefore, this paper tries to investigate Goyal's model[1] and Teng's model[14] using the algebraic method to determine the optimal cycle time. In addition, we summarize some previously published results of other researchers to determine the optimal cycle time.

Goyal's model: For convenience, most notation and assumptions similar to Goyal[1] will be used in the present study.

Notation

- D = annual demand
- S = cost of placing one order
- c = unit purchasing price per item
- h = unit stock holding cost per item per year excluding interest charges
- I_d = interest which can be earned per \$ per year
- I_c = interest charges per \$ investment in inventory per year
- t = permissible delay period in years
- T = the cycle time in years
- Z(T) = the annual total relevant cost
- T* = the optimal cycle time of Z(T).

Assumptions

1. Demand rate is known and constant.
2. Shortages are not allowed.
3. Time period is infinite.
4. $I_c \geq I_d$.
5. During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When $T \geq t$, the account is settled at T=t and we start paying for the interest charges on the items in stock. When $T \leq t$, the account is settled at T=t and we do not need to pay any interest charge.

Algebraic modeling: The annual total relevant cost for the retailer can be expressed as Z(T)=ordering cost+stock-holding cost+interest payable-interest earned. Based on the above notation and assumptions, Goyal[1] showed that:

$$Z(T) = \begin{cases} Z_1(T) & \text{if} \quad T \geq t & (1a) \\ Z_2(T) & \text{if} \quad 0 < T \leq t & (1b) \end{cases}$$

where

$$Z_1(T) = \frac{2S + Dct^2(I_c - I_d)}{2T} + \frac{DT(h + cI_c)}{2} - DctI_c \quad (2)$$

and

$$Z_2(T) = \frac{S}{T} + \frac{DT(h + cI_d)}{2} - DctI_d \quad (3)$$

Since $Z_1(t)=Z_2(t)$, Z(T) is continuous for T>0.

Then, we can rewrite

$$Z_1(T) = \frac{2S + Dct^2(I_c - I_d)}{2T} + \frac{DT(h + cI_c)}{2} - DctI_c$$

$$= \frac{D(h + cI_c)}{2T}\left\{T^2 + \frac{2S + Dct^2(I_c - I_d)}{D(h + cI_c)}\right\} - DctI_c$$

$$= \frac{D(h + cI_c)}{2T}$$

$$\left\{T^2 - 2T\sqrt{\frac{2S + Dct^2(I_c - I_d)}{D(h + cI_c)}} + \left[\frac{\sqrt{2S + Dct^2(I_c - I_d)}}{D(h + cI_c)}\right]^2\right\}$$

$$+ \left\{\sqrt{D(h + cI_c)[2S + Dct^2(I_c - I_d)]} - DctI_c\right\}$$

$$= \frac{D(h + cI_c)}{2T}\left[T - \sqrt{\frac{2S + Dct^2(I_c - I_d)}{D(h + cI_c)}}\right]^2$$

$$+ \left\{\sqrt{D(h + cI_c)[2S + Dct^2(I_c - I_d)]} - DctI_c\right\} \quad (4)$$

Equation (4) represents that the minimum of $Z_1(T)$ is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value T_1^* is:

$$T_1^* = \sqrt{\frac{2S + Dct^2(I_c - I_d)}{D(h + cI_c)}} \quad (5)$$

Therefore, equation (4) has a minimum value for the optimal value of T_1^* reducing $Z_1(T)$ to

$$Z_1(T_1^*) = \left\{\sqrt{D(h + cI_c)[2S + Dct^2(I_c - I_d)]} - DctI_c\right\} \quad (6)$$

Similarly, we can derive $Z_2(T)$ without derivatives as follows:

$$Z_2(T)=\frac{S}{T}+\frac{DT(h+cI_d)}{2}-DctI_d$$
$$=\frac{D(h+cI_d)}{2T}\left\{T^2+\frac{2S}{D(h+cI_d)}\right\}-DctI_d$$
$$=\frac{D(h+cI_d)}{2T}\left\{T^2-2T\sqrt{\frac{2S}{D(h+cI_d)}}+\left[\sqrt{\frac{2S}{D(h+cI_d)}}\right]^2\right\}$$
$$+\left\{\sqrt{2SD(h+cI_d)}-DctI_d\right\}$$
$$=\frac{D(h+cI_d)}{2T}\left[T-\sqrt{\frac{2S}{D(h+cI_d)}}\right]^2$$
$$+\left\{\sqrt{2SD(h+cI_d)}-DctI_d\right\} \tag{7}$$

Equation (7) represents that the minimum of $Z_2(T)$ is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value T_2* is

$$T_2*=\sqrt{\frac{2S}{D(h+cI_d)}} \tag{8}$$

Therefore, equation (7) has a minimum value for the optimal value of T_2* reducing $Z_2(T)$ to

$$Z_2(T_2*)=\left\{\sqrt{2SD(h+cI_d)}-DctI_d\right\} \tag{9}$$

Decision rule of the optimal cycle time T*: From above equation (5) implies that the optimal value of T for the case of $T\geq t$, that is $T_1*>t$. We substitute equation (5) into $T_1*>t$, then we can obtain that

$$\text{if and only if } 2S-Dt^2(h+cI_d)>0 \tag{10}$$

Similarly, equation (8) implies that the optimal value of T for the case of $T\leq t$, that is $T_2*<t$. We substitute equation (8) into $T_2*<t$, then we can obtain that

$$\text{if and only if } 2S-Dt^2(h+cI_d)<0 \tag{11}$$

Of course, if $T_1*=T_2*=t$, we can obtain that

$$\text{if and only if } 2S-Dt^2(h+cI_d)=0 \tag{12}$$

Furthermore, we let $\Delta=-2S+Dt^2(h+cI_d)$. Then, we can obtain following results.

Theorem 1

A. If $\Delta>0$, then $T*=T_2*$.
B. If $\Delta<0$, then $T*=T_1*$.
C. If $\Delta=0$, then $T*=T_1*=T_2*=t$.

Theorem 1 has been discussed in Chung[2].

Teng's model: We discuss Teng's model[14] which modified the Goyal's model[1] by considering the difference between unit price and unit cost. So, Teng[14] define the extra notation and modified above assumption (5) as follows:

Extra notation

p = unit selling price per item, p>c
$\bar{T}$*= the optimal cycle time of $\bar{Z}(T)$

Amended assumption: During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. At the end of this period, the retailer pays off all units sold, keeps profits and starts paying for the interest charges on the items in stocks.

Algebraic modeling: Based on the above arguments and easy interpreting, we use following notation to express Teng's model[14].

$$\bar{Z}(T)=\begin{cases}Z_3(T) & \text{if } T\geq t \quad (13a)\\ Z_4(T) & \text{if } 0<T\leq t \quad (13b)\end{cases}$$

where

$$Z_3(T)=\frac{2S+Dt^2(cI_c-PI_d)}{2T}+\frac{DT(h+cI_c)}{2}-DctI_c \tag{14}$$

and

$$Z_4(T)=\frac{S}{T}+\frac{DT(h+pI_d)}{2}-DptI_d \tag{15}$$

$Z_3(t)$ and $Z_4(t)$ are similar to $Z_1(t)$ and $Z_2(t)$ in Teng [14], respectively. Since $Z_3(t)=Z_4(t)$, $\bar{Z}(T)$ is continuous for T>0. Then, we can rewrite

$$Z_3(T)=\frac{2S+Dt^2(cI_c-pI_d)}{2T}+\frac{DT(h+cI_c)}{2}-DctI_c$$
$$=\frac{D(h+cI_c)}{2T}\left\{T^2+\frac{2S+Dt^2(cI_c-pI_d)}{D(h+cI_c)}\right\}-DctI_c$$
$$=\frac{D(h+cI_c)}{2T}$$
$$\left\{T^2-2T\sqrt{\frac{2S+Dt^2(cI_c-pI_d)}{D(h+cI_c)}}+\left[\frac{\sqrt{2S+Dt^2(cI_c-pI_d)}}{D(h+cI_c)}\right]^2\right\}$$
$$+\left\{\sqrt{D(h+cI_c)[2S+Dt^2(cI_c-pI_d)]}-DctI_c\right\}$$
$$=\frac{D(h+cI_c)}{2T}\left[T-\sqrt{\frac{2S+Dt^2(cI_c-pI_d)}{D(h+cI_c)}}\right]^2$$
$$+\left\{\sqrt{D(h+cI_c)[2S+Dt^2(cI_c-pI_d)]}-DctI_c\right\} \tag{16}$$

Equation (16) represents that the minimum of $Z_3(T)$ is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value T_3* is

$$T_3* = \sqrt{\frac{2S+Dt^2(cI_c-pI_d)}{D(h+cI_c)}} \quad \text{if } 2S+Dt^2(cI_c-pI_d)>0 \tag{17}$$

Therefore, equation (16) has a minimum value for the optimal value of T_3* reducing $Z_3(T)$ to

$$Z_3(T_3*) = \left\{\sqrt{D(h+cI_c)\left[2S+Dt^2(cI_c-PI_d)\right]} - DctI_c\right\} \tag{18}$$

Similarly, we can derive $Z_4(T)$ without derivatives as follows:

$$Z_4(T) = \frac{S}{T} + \frac{DT(h+pI_d)}{2} - DptI_d$$

$$= \frac{D(h+pI_d)}{2T}\left\{T^2 + \frac{2S}{D(h+pI_d)}\right\} - DptI_d$$

$$= \frac{D(h+pI_d)}{2T}\left\{T^2 - 2T\sqrt{\frac{2S}{D(h+pI_d)}} + \left[\sqrt{\frac{2S}{D(h+pI_d)}}\right]^2\right\}$$

$$+ \left\{\sqrt{2SD(h+pI_d)} - DptI_d\right\}$$

$$= \frac{D(h+pI_d)}{2T}\left[T - \sqrt{\frac{2S}{D(h+pI_d)}}\right]^2$$

$$+\left\{\sqrt{2SD(h+pI_d)} - DptI_d\right\} \tag{19}$$

Equation (19) represents that the minimum of $Z_4(T)$ is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value T_4* is

$$T_4* = \sqrt{\frac{2S}{D(h+pI_d)}} \tag{20}$$

Therefore, equation (19) has a minimum value for the optimal value of T_4* reducing $Z_4(T)$ to

$$Z_4(T_4*) = \left\{\sqrt{2SD(h+pI_d)} - DptI_d\right\} \tag{21}$$

T_3* and T_4* are similar to T_1* and T_2* in Teng [14], respectively.

Decision rule of the optimal cycle time $\bar{T}^*$: From above equation (17) implies that the optimal value of T for the case of $T \ge t$, that is $T_3*>t$. We substitute equation (17) into $T_3*>t$, then we can obtain that

$$\text{if and only if } 2S-Dt^2(h+pI_d)>0 \tag{22}$$

Similarly, equation (20) implies that the optimal value of T for the case of $T \le t$, that is $T_4*< t$. We substitute equation (20) into $T_4*< t$, then we can obtain that

$$\text{if and only if } 2S-Dt^2(h+pI_d)<0 \tag{23}$$

Of course, if $T_3*= T_4*= t$, we can obtain that

$$\text{if and only if } 2S-Dt^2(h+pI_d)=0 \tag{24}$$

From above arguments, we can summarize following results.

Theorem 2

A. If $2S-Dt^2(h+pI_d)>0$, then $\bar{T}*=T_3*$

B. If $2S-Dt^2(h+pI_d)<0$, then $\bar{T}*=T_4*$

C. If $2S-Dt^2(h+pI_d)=0$, then $\bar{T}*=T_3*=T_4*=t$

Theorem 2 has been discussed in Teng[14].

Furthermore, we let $\Delta_1=-2S+Dt^2(h+pI_d)$. Then, we can modify Theorem 2 to following results.

Theorem 3

A. If $\Delta_1>0$, then $\bar{T}*=T*_4$

B. If $\Delta_1<0$, then $\bar{T}*=T*_3$

C. If $\Delta_1=0$, then $\bar{T}*=T*_3=T*_4=t$.

Theorem 3 is an effective procedure to find the optimal cycle time $\bar{T}*$ by easy judgment Δ_1. If p=c, the Δ_1 will be equal to Δ in Theorem 1. Then Theorem 3 will be reduced to Theorem 1. So, Theorem 1 is a special case of Theorem 3.

In real-life business transactions, the supplier offers the trade credit policy to stimulate the demand of the retailer. The present study adopts the easy algebraic procedure to reinvestigate Goyal's model[1] and Teng's model[14] to find the optimal cycle time under permissible delay in payments. Using the algebraic approach is a more accessible approach to ease the learning of basic inventory theories for younger students who lack the knowledge of differential calculus.

The proposed model can be extended in several ways. For instance, we may generalize the model to allow for shortages, quantity discounts, time value of money, finite time horizon, finite replenishment rate and others.

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