An EOQ Model Under Trade Credit Linked to Order Quantity Using Algebraic Method

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Abstract: In 1985, Goyal investigated the inventory problem under permissible delay in payments independent of the order quantity. In the present study, the restrictive assumption of the trade credit independent of the order quantity is relaxed. The present study investigates the retailer’s inventory problem under permissible delay in payments dependent of the order quantity within the economic order quantity (EOQ) framework. In addition, we adopt the algebraic method to determine the retailer’s optimal ordering policy under minimizing the annual total relevant cost was adopted. Then, two theorems are developed to efficiently determine the optimal cycle time and optimal order quantity for the retailer. Finally, numerical examples are given to illustrate these theorems obtained in the present study.

Key words: EOQ, inventory, trade credit, permissible delay in payments, algebraic method

INTRODUCTION

Several studies have carried out previously treat inventory problems with varying conditions under the consideration of permissible delay in payments.


Goyal[1] is well known when the inventory systems under conditions of permissible delay in payments and implicitly makes the following assumption. Supplier credit policy offered to the retailer where credit terms are independent of the order quantity. That is, whatever the order quantity is small or large, the retailer can take the benefits of payment delay. Under this condition, the effect of stimulating the retailer’s demand may be reduced. So, the present study will adopt the following assumption to modify the Goyal’s model[1]. To encourage retailer to order a large quantity, the supplier may give the trade credit period only for a large order quantity. In other words, the
retailer requires immediate payment for a small order quantity. This viewpoint can be found in Khouja and Meherz[21] and Chang et al.[22].

In addition, in previous study which have been derived using differential calculus to find the optimal solution and the need to prove optimality condition with second-order derivatives. The mathematical methodology is difficult to many younger students who lack the knowledge of calculus. In recent study, Grubbström and Erdem[24] and Cárdenas-Barrón[23] showed that the formulae for the economic order quantity (EOQ) and economic production quantity (EPQ) with backlogging could be derived without differential calculus. They mentioned that this approach must be considered as a pedagogical advantage for explaining the EOQ and EPQ concepts to students that lack knowledge of derivatives, simultaneous equations and the procedure to construct and examine the Hessian matrix. Therefore, we want to adopt the algebraic method to investigate the effect of trade credit policy depending on the order quantity within the economic order quantity (EOQ) framework. Then, two theorems are developed to efficiently determine the optimal cycle time and optimal order quantity for the retailer. Finally, numerical examples are given to illustrate these theorems obtained in the present study.

Algebraic model formulation

Notation

- **Q**: Order quantity
- **D**: Annual demand
- **W**: Minimum order quantity at which the trade credit is permitted
- **A**: Cost of placing one order
- **c**: Unit purchasing price per item
- **h**: Unit stock holding cost per item per year excluding interest charges
- **I₀**: Interest charges per $ investment in inventory per year
- **Iᵢ**: Interest which can be earned per $ per year
- **M**: The trade credit period
- **T**: The cycle time
- **TRC(T)**: The annual total relevant cost when **T**>0
- **T***: The optimal cycle time of TRC(T)
- **Q***: The optimal order quantity=DT*

Assumptions

1. Demand rate is known and constant
2. Shortages are not allowed
3. Time period is infinite
4. Replenishments are instantaneous with a known and constant lead time

5. If Q< W, i.e. T<W/D, the trade credit is not permitted. Otherwise, fixed trade credit period M is permitted. Hence, if Q<W, pay cQ when the order is received. If Q≥ W, pay cQ M time periods after the order is received
6. During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When T≥ M, the account starts paying for the higher interest charges on the items in stock. When T≤ M, the account is settled at T=M and the buyer does not need to pay any interest charge
7. Iᵢ ≥ I₀

The annual total relevant cost consists of the following elements. Two situations arise. (I) **M**≥ W/D and (II) **M**< W/D

Case I: Suppose that **M**≥ W/D

1. Annual ordering cost = A/T
2. Annual stock holding cost (excluding interest charges) = DTh'/2
3. There are three cases in terms of cost of interest charges for the items kept in stock per year.
   (I) : 0<T<W/D
   In this case, the retailer must pay cDT when the order is received since the trade credit is not permitted. Therefore, Cost of interest charges for the items kept in stock per cycle = c₁DT²/2
   Cost of interest charges for the items kept in stock per year = c₁DT²/2
   (ii) : W/D ≤ T ≤ M
   In this case, the fixed trade credit period M is permitted since Q≥ W. According to assumption (6), no interest charges are paid for the items kept in stock.
   (iii) : M≥ T
   In this case, the fixed trade credit period M is permitted since Q≥ W. According to assumption (6), Cost of interest charges for the items kept in stock per cycle = c₁DT(M-T)²/2
   Cost of interest charges for the items kept in stock per year = c₁D(T-M)²/2T
4. There are three cases in terms of interests earned per year.
   (I) : 0<T<W/D
   In this case, no interest earned since the trade credit is not permitted.
(ii) \( W/D \leq T \leq M \)

In this case, the fixed trade credit period \( M \) is permitted since \( Q \leq W \). According to assumption (6), interests earned per cycle

\[
= cI \left[ \frac{DT^2}{2} + DT(M-T) \right] = DTcI(M - \frac{T}{2})
\]

Interests earned per year = \( DcI(M - \frac{T}{2}) \)

(iii) \( M \leq T \)

In this case, the fixed trade credit period \( M \) is permitted since \( Q \leq W \). According to assumption (6),

Interests earned per cycle = \( cI \int_0^M D(t)dt - \frac{DM^2cI_p}{2} \)

Interests earned per year = \( \frac{DM^2cI_p}{2T} \)

From the above arguments, the annual total relevant cost for the retailer can be expressed as:

\[
TRC(T) = \text{ordering cost} + \text{stock-holding cost} + \text{interest payable-interest earned}.\]

It is shown that the annual total relevant cost, \( TRC(T) \), is given by

\[
TRC(T) = \begin{cases} TRC_1(T) & \text{if } 0 < T < W/D \\ TRC_2(T) & \text{if } W/D \leq T \leq M \\ TRC_3(T) & \text{if } M \leq T \\ \end{cases}
\]

where

\[
TRC_1(T) = \frac{AT}{2} + \frac{cI_DDT}{2},
\]

\[
TRC_2(T) = \frac{AT}{2} - DcI(M - \frac{T}{2})
\]

and

\[
TRC_3(T) = \frac{AT}{2} + \frac{cI_D(DT-M)^2}{2T} - \frac{DM^2cI_p}{2T}.
\]

Since \( TRC_1(W/D) = TRC_2(W/D) = TRC_3(M) = TRC_3(M) \), \( TRC(T) \) is continuous except \( T=W/D \).

Then, we can rewrite

\[
TRC_1(T) = \frac{AT}{2} \left[ DT(h+cI_p) \right] = \frac{DT(h+cI_p)}{2T} \left[ \sqrt{\frac{2A}{D(h+cI_p)}} + \sqrt{2AD(h+cI_p)} \right].
\]

Equation (5) represents that the minimum of \( TRC_1(T) \) is obtained when the quadratic non-negative term, depending on \( T \), is made equal to zero. Therefore, the optimum value \( T_1^* \) is

\[
T_1^* = \sqrt{\frac{2A}{D(h+cI_p)}}.
\]

Therefore, equation (5) has a minimum value for the optimal value of \( T_1^* \) reducing \( TRC_1(T) \) to

\[
TRC_1(T_1^*) = \sqrt{2AD(h+cI_p)}.
\]

Similarly, we can derive \( TRC_3(T) \) without derivatives as follows.
\[ T_{RC_2}(T) - \frac{A}{T} \cdot \frac{DT(h+cel)}{2} - DcM_\theta \]

\[ = \frac{DT(h+cel)}{2T} \left[ T - \sqrt{\frac{2A}{D(h+cel)}} \right]^{2} + \sqrt{2AD(h+cel)} - DcM_\theta \]. \tag{8}

Equation (8) represents that the minimum of TRC_2(T) is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value \( T_{2,*} \) is

\[ T_{2,*} = \sqrt{\frac{2A}{D(h+cel)}} \]. \tag{9}

Therefore, equation (8) has a minimum value for the optimal value of \( T_{2,*} \) reducing TRC_2(T) to

\[ TRC_2(T_{2,*}) = \sqrt{2AD(h+cel)} - DcM_\theta \]. \tag{10}

Likewise, we can derive TRC_3(T) algebraically as follows:

\[ TRC_3(T) = \frac{2A + DcM^2c(I_p-I_e)}{2T} + \frac{DT(h+cel)}{2} - DcM_\eta \]

\[ = \frac{DT(h+cel)}{2T} \left[ T - \sqrt{\frac{2A + DcM^2c(I_p-I_e)}{D(h+cel)}} \right]^{2} + \sqrt{D(h+cel)} [2A + DcM^2c(I_p-I_e)] - DcM_\eta \]. \tag{11}

Equation (11) represents that the minimum of TRC_3(T) is obtained when the quadratic non-negative term, depending on T, is made equal to zero. Therefore, the optimum value \( T_{3,*} \) is

\[ T_{3,*} = \sqrt{\frac{2A + DcM^2c(I_p-I_e)}{D(h+cel)}} \]. \tag{12}

Therefore, equation (11) has a minimum value for the optimal value of \( T_{3,*} \) reducing TRC_3(T) to

\[ TRC_3(T_{3,*}) = \sqrt{D(h+cel)} [2A + DcM^2c(I_p-I_e)] - DcM_\eta \]. \tag{13}

**Case II: Suppose that M<W/D:** If M<W/D, equations 1(a, b, c) will be modified as

\[ TRC(T) = \begin{cases} 
TRC_1(T) & \text{if } 0 < T < W/D \\
TRC_3(T) & \text{if } W/D \leq T 
\end{cases} \]

Since TRC(W/D)<TRC(W/D), TRC(T) is continuous except T=W/D.

**Determination of the optimal cycle time \( T^* \)**

**(1) When M≥W/D:** Above equation (6) implies that the optimal value of T for the case of 0<T<W/D, that is 0<T,*<W/D. We substitute equation (6) into 0<T,*<W/D, then we can obtain the optimal value of T.
if and only if \( 0 < 2A < \frac{W^2}{D} \) \((h+eL_p)\). \(\text{(14)}\)

Similarly, equation (9) implies that the optimal value of \(T\) for the case of \(W/D \leq T \leq M\), that is \(W/D \leq T \leq M\). We substitute equation (9) into \(W/D \leq T \leq M\), then we can obtain the optimal value of \(T\) if and only if \(W^2/D\) \((h+eL_p) \leq 2A \leq DM(h+eL_p)\). \(\text{(15)}\)

Finally, equation (12) implies that the optimal value of \(T\) for the case of \(T \geq M\), that is \(T \geq M\). We substitute equation (12) into \(T \geq M\), then we can obtain the optimal value of \(T\) if and only if \(2A \geq DM^2(h+eL_p)\). \(\text{(16)}\)

Furthermore, we let

\[
\Delta_1 = -2A + \frac{W^2}{D}(h+eL_p),
\]
\(\text{(17)}\)

\[
\Delta_2 = -2A + \frac{W^2}{D}(h+eL_p),
\]
\(\text{(18)}\)

and

\[
\Delta_3 = -2A + DM^2(h+eL_p).
\]
\(\text{(19)}\)

From equations (17), (18) and (19), we can easily obtain \(\Delta_1 \geq \Delta_2\). In addition, we know \(TRC_1(W/D) > TRC_2(W/D)\), \(TRC_2(M) = TRC_3(M)\), \(TRC(T)\) is continuous except \(T = W/D\) from equations (2), (3) and (4). Then, we can summarize above arguments and obtain following results:

**Theorem 1**

1. If \(\Delta_1 > 0\), \(\Delta_2 > 0\) and \(\Delta_3 > 0\), then \(TRC(T^*) = \min\{TRC_1(T^*), TRC_2(W/D)\}\). Hence \(T^*\) is \(T^*_1\) or \(W/D\) associated with the least cost.
2. If \(\Delta_1 > 0\), \(\Delta_2 < 0\) and \(\Delta_3 < 0\), then \(TRC(T^*) = \min\{TRC_1(T^*), TRC_2(T^*_1)\}\). Hence \(T^*\) is \(T^*_1\) or \(T^*_2\) associated with the least cost.
3. If \(\Delta_1 > 0\), \(\Delta_2 < 0\) and \(\Delta_3 < 0\), then \(TRC(T^*) = \min\{TRC_1(T^*), TRC_2(T^*_1)\}\). Hence \(T^*\) is \(T^*_2\) or \(T^*_1\) associated with the least cost.
4. If \(\Delta_1 < 0\), \(\Delta_2 > 0\) and \(\Delta_3 > 0\), then \(TRC(T^*) = TRC_2(W/D)\) and \(T^* = W/D\).
5. If \(\Delta_1 < 0\), \(\Delta_2 < 0\) and \(\Delta_3 < 0\), then \(TRC(T^*) = TRC_2(T^*_1)\) and \(T^* = T^*_1\).
6. If \(\Delta_1 < 0\), \(\Delta_2 < 0\) and \(\Delta_3 < 0\), then \(TRC(T^*) = TRC_2(T^*_2)\) and \(T^* = T^*_2\).

**Theorem 2**

1. If \(\Delta_1 > 0\) and \(\Delta_3 > 0\), then \(TRC(T^*) = \min\{TRC_1(T^*), TRC_2(W/D)\}\). Hence \(T^*\) is \(T^*_1\) or \(W/D\) associated with the least cost.
2. If \(\Delta_1 \leq 0\) and \(\Delta_3 \leq 0\), then \(TRC(T^*) = TRC_1(T^*_1)\) and \(T^* = T^*_1\).
3. If \(\Delta_1 > 0\) and \(\Delta_3 \leq 0\), then \(TRC(T^*) = \min\{TRC_1(T^*), TRC_2(T^*_2)\}\). Hence \(T^*\) is \(T^*_1\) or \(T^*_2\) associated with the least cost.
4. If \(\Delta_1 \leq 0\) and \(\Delta_3 > 0\), then \(TRC(T^*) = TRC_2(W/D)\) and \(T^* = W/D\).

**Numerical examples:** To illustrate all results obtained in the present study, let us apply the proposed method to efficiently solve the following numerical examples.
Table 1: The optimal cycle time and optimal order quantity with various values of W and c
Let A=$200/order, D=5000 units/year, h=$5/unit/year, I=$0.15$/year, Ic=$0.05$/year and M=0.1 year

<table>
<thead>
<tr>
<th>c=$unit</th>
<th>$200 units/year</th>
<th>$400 units/year</th>
<th>$600 units/year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1  A2  A3  A4  T*  Q*</td>
<td>A1  A2  A3  A4  T*  Q*</td>
<td>A1  A2  A3  A4  T*  Q*</td>
</tr>
<tr>
<td>30</td>
<td>&lt;0   &lt;0   &lt;0   T*=0.10761 538</td>
<td>&lt;0   &lt;0   &lt;0   T*=0.10761 538</td>
<td>&gt;0   &gt;0   &gt;0   W/D=0.12 600</td>
</tr>
<tr>
<td>50</td>
<td>&lt;0   &lt;0   &lt;0   T*=0.10198 510</td>
<td>&lt;0   &lt;0   &lt;0   T*=0.10198 510</td>
<td>&gt;0   &gt;0   &gt;0   W/D=0.12 600</td>
</tr>
<tr>
<td>70</td>
<td>&lt;0   &lt;0   &gt;0   T*=0.09701 485</td>
<td>&lt;0   &gt;0   &gt;0   T*=0.09701 485</td>
<td>&gt;0   &gt;0   &gt;0   W/D=0.12 600</td>
</tr>
</tbody>
</table>

To study the effects of the minimum order quantity to obtain the permissible delay, W and unit purchasing price per item, c, on the optimal cycle time and optimal order quantity for the retailer derived by the proposed method, we solve the example in Table 1 with various values of W and c. The following inferences can be made based on Table 1. When W is increasing, the optimal cycle time and optimal order quantity for the retailer will decrease. It implies that the retailer will order more quantity to take the benefits of trade credit as much as possible when the minimum order quantity to obtain the trade credit is higher. When c is increasing, the optimal cycle time and optimal order quantity for the retailer are not increasing. This result implies that the retailer will not order more quantity to take the benefits of the trade credit more frequently.

The supplier offers the trade credit policy to stimulate the demand of the retailer in general. For reaching the effect of trade credit, the supplier may give the trade credit period only for a large order quantity. In other words, the retailer requires immediate payment for a small order quantity. This situation is very reasonable in the real business transactions. We develop the retailer’s inventory model in this situation. These results are very helpful to the inventory replenishment decision-makers.

Future study may further incorporate the proposed model into more realistic assumptions, such as allowable shortages, limited storage capacity and a finite rate of replenishment.

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