A Note on an EOQ Model for Deteriorating Items under Trade Credits

1Chih-Sung Lai, 1Yung-Fu Huang and 2Hung-Fu Huang  
1Department of Business Administration,  
2Department of Electrical Engineering, National Cheng Kung University, Taiwan, Republic of China

Abstract: This study deals with the optimal inventory decisions for the customer under the permissible delay in payments and cash discount offered by the supplier. We develop an alternative approach to find the optimal replenishment time for the customer to improve the Ouyang et al.'s model. In Ouyang et al.'s model, they use Taylor's series approximation to obtain the explicit closed-form solution of the optimal replenishment time. We modify this approximation method to exact solution and develop an easy-finding solution theorem to help the decision-maker to decide the optimal replenishment policy.

Key words: Inventory, EOQ, permissible delay in payments, cash discount, trade credit

INTRODUCTION

Recently, Ouyang et al.11 developed a model to determine the optimal replenishment time for the customer under the permissible delay in payments and cash discount offered by the supplier. They use Taylor's series approximation to obtain the explicit closed-form solution and provide an easy-to-use algorithm to find the optimal order quantity and replenishment time.

We know many researchers assume that x is sufficiently small to simplify the process of the solution procedure. They use Taylor's series approximation, let e^x = 1 + x + x^2/2, to deal with the complex solution procedure. But x is not necessarily small. At this condition, the approximation approach may cause significant errors and penalties. Therefore, this note will modify this approximation method to exact solution to find the optimal replenishment time for the customer. In addition, we develop an alternative approach to find the optimal replenishment time for the customer to improve the results of Ouyang et al.11. For convenience, we use notation and assumptions similar to Ouyang et al.11. Ouyang et al.11 developed the following model for the total relevant cost per year. At first, we define the new notation:

\[ Z(T) = \begin{cases} Z_1(T) & \text{if } T \geq M_1 \\ Z_2(T) & \text{if } T < M_1 \end{cases} \]

Where:

\[ Z_1(T) = \frac{S}{T} + \frac{D}{\theta^2 T} \left[ e^{\theta T} - 1 \right] - \frac{e^{\theta T} - 1}{\theta} - \frac{D}{\theta} - \frac{D}{\theta^2 T} \left[ e^{\theta T} - 1 \right] \]

(1)

and

\[ Z_2(T) = \frac{S}{T} + \frac{D}{\theta^2 T} \left[ e^{\theta T} - 1 \right] - \frac{e^{\theta T} - 1}{\theta} - \frac{D}{\theta} - \frac{D}{\theta^2 T} \left[ e^{\theta T} - 1 \right] \]

(2)

At T = M_1, we find \( Z_1(M_1) = Z_2(M_1) \). Hence, \( Z(T) \) is continuous for T > 0.

When the payment is paid at time M_1, the total relevant cost per year is:

\[ Z_1(T) = \begin{cases} Z_1(T) & \text{if } T \geq M_2 \\ Z_2(T) & \text{if } T < M_2 \end{cases} \]

Where:
\[
Z_1(T) = \frac{S}{T} + \frac{D(h + c\theta)}{\theta^2 T} (e^{\theta T} - 1) - \frac{hD}{\theta} + \frac{cI_0 D}{\theta^2 T} (e^{\theta T} - 1) - \frac{eI_0 D}{\theta T} (T - M_2) - \frac{pI_0 D}{2T} M_2^2
\]

and

\[
Z_2(T) = \frac{S}{T} + \frac{D(h + c\theta)}{\theta^2 T} (e^{\theta T} - 1) - \frac{hD}{\theta} - I_0 D(M_2 - \frac{T}{2})\]

At \( T = M_2 \), we find \( Z_2(M_2) = Z_2(M_2) \). Hence, \( Z(T) \) is continuous for \( T > 0 \).

Ouyang et al.\(^{[1]} \) proved all \( Z_i \) (\( i = 1, 2, 3 \) and 4) are convex functions. We do not use Taylor's series approximation to find the optimal solution. We derive \( Z_i'(T_i) = 0 \) if \( T > 0 \) for all \( i = 1, 2, 3 \) and 4. According to the convexity of \( Z_i(T)(i = 1, 2, 3 \) and 4), the Newton's method can be used to locate \( T_i \) if \( T_i \) exists for all \( i = 1, 2, 3 \) and 4. By the convexity of \( Z_i \) (\( i = 1, 2, 3 \) and 4), we see:

\[
\begin{align*}
Z_1'(T) &< 0 \quad \text{if} \quad T < T_1 \\
Z_1'(T) &= 0 \quad \text{if} \quad T = T_1 \\
Z_1'(T) &> 0 \quad \text{if} \quad T > T_1
\end{align*}
\]

Equation A1, A7, A10 and A14 in Ouyang et al.\(^{[1]} \) yield that:

\[
Z_1'(M_2) = Z_1'(M_3) = \frac{-2h^2 + 2D(h + c\theta)(1 - r)(\theta M_2 e^{\theta T} - e^{\theta T} + 1) + (\theta M_2)^2 pI_0 D}{2(\theta M_2)^2}
\]

and

\[
Z_1'(M_2) = Z_1'(M_3) = \frac{-2h^2 + 2D(h + c\theta)(1 - r)(\theta M_3 e^{\theta T} - e^{\theta T} + 1) + (\theta M_3)^2 pI_0 D}{2(\theta M_3)^2}
\]

Furthermore, we let:

\[
\Delta_1 = -2h^2 + 2D(h + c\theta)(1 - r)(\theta M_1 e^{\theta T} - e^{\theta T} + 1) + (\theta M_1)^2 pI_0 D
\]

and

\[
\Delta_2 = -2h^2 + 2D(h + c\theta)(1 - r)(\theta M_2 e^{\theta T} - e^{\theta T} + 1) + (\theta M_2)^2 pI_0 D
\]

Then, the optimal cycle time \( T^* \) and optimal payment time (\( M_i \) or \( M_j \)) can be obtained as following theorem.
Theorem 1:
(A) If $\Delta_1 > 0$ and $\Delta_2 > 0$, then $Z(T^*) = \min\{Z_1(T_1), Z_2(T_2)\}$. Hence $T^*$ is $T_1$ or $T_2$ and optimal payment time is $M_1$ or $M_2$ associated with the least cost.
(B) If $\Delta_1 \leq 0$ and $\Delta_2 \leq 0$, then $Z(T^*) = \min\{Z_1(T_1), Z_2(T_2)\}$. Hence $T^*$ is $T_1$ or $T_2$ and optimal payment time is $M_1$ or $M_2$ associated with the least cost.
(C) If $\Delta_1 \leq 0$ and $\Delta_2 > 0$, then $Z(T^*) = \min\{Z_1(T_1), Z_2(T_2)\}$. Hence $T^*$ is $T_1$ or $T_2$ and optimal payment time is $M_1$ or $M_2$ associated with the least cost.
(D) If $\Delta_1 > 0$ and $\Delta_2 < 0$, then $Z(T^*) = \min\{Z_1(T_1), Z_2(T_2)\}$. Hence $T^*$ is $T_1$ or $T_2$ and optimal payment time is $M_1$ or $M_2$ associated with the least cost.

Proof:
(A) If $\Delta_1 > 0$ and $\Delta_2 > 0$, then we have $Z_1'(M_1) = Z_2'(M_1) > 0$ and $Z_1'(M_2) = Z_2'(M_2) < 0$. Equation 5a-c imply that:

(i) $Z_1(T)$ is increasing on $[M_1, \infty)$.
(ii) $Z_2(T)$ is decreasing on $(0, T_2]$ and increasing on $[T_2, M_2]$.
(iii) $Z_1(T)$ is increasing on $[M_2, \infty)$.
(iv) $Z_2(T)$ is decreasing on $(0, T_1]$ and increasing on $[T_1, M_1]$.

Combining (i)-(iv), we have that $Z(T)$ is decreasing on $(0, T_1]$ and increasing on $[T_1, \infty)$ and $Z(T)$ is decreasing on $(0, T_2]$ and increasing on $[T_2, \infty)$. Therefore, $Z(T^*) = \min\{Z_1(T_1), Z_2(T_2)\}$. Consequently, $T^*$ is $T_1$ or $T_2$ and optimal payment time is $M_1$ or $M_2$ associated with the least cost.

(B) If $\Delta_1 \leq 0$ and $\Delta_2 \leq 0$, then we have $Z_1'(M_1) = Z_2'(M_1) < 0$ and $Z_1'(M_2) = Z_2'(M_2) > 0$. Equation 5a-c imply that:

(i) $Z_1(T)$ is decreasing on $[M_1, T_1]$ and increasing on $[T_1, \infty)$.
(ii) $Z_2(T)$ is decreasing on $(0, M_2)$.
(iii) $Z_1(T)$ is decreasing on $[M_2, T_2]$ and increasing on $[T_2, \infty)$.
(iv) $Z_2(T)$ is decreasing on $(0, M_2)$.

Combining (i)-(iv), we have that $Z(T)$ is decreasing on $(0, T_1]$ and increasing on $[T_1, \infty)$ and $Z(T)$ is decreasing on $(0, T_2]$ and increasing on $[T_2, \infty)$. Therefore, $Z(T^*) = \min\{Z_1(T_1), Z_2(T_2)\}$. Consequently, $T^*$ is $T_1$ or $T_2$ and optimal payment time is $M_1$ or $M_2$ associated with the least cost.

Combining the above arguments, we have completed the proof of Theorem 1.

Theorem 1 immediately determines the optimal cycle time $T^*$ and optimal payment time ($M_1$ or $M_2$) after computing the numbers $\Delta_1$ and $\Delta_2$. Theorem 1 is really very simple.

A special case: Here, we want to deduce one previously published model as a special case.

Huang and Chung's model: Suppose that the selling price per unit is equal to the unit purchasing price and the deterioration is ignored. Let $p = c$, we have:

$$\lim_{n \to \infty} Z_n(T) = \frac{S}{T} + \frac{DTh}{2} + c(1-r)D$$

$$+ \frac{(c(1-r)D)^2 - cD^2M}{2T^2}$$

$$\lim_{n \to \infty} Z_n(T) = \frac{S}{T} + \frac{DTh}{2} + c(1-r)D - DcM\left(\frac{T}{2}\right)^{1/2}.$$
\[
\lim_{n \to \infty} Z_1(T) = \frac{S}{T} + \frac{DTh}{2} + cD \\
+ \frac{cL_D(T - M_2)^2}{2T} - \frac{cL_D M_2}{2T} 
\]  

Equations 10-13 will be consistent with Eq. 1(a,b) and 4(a,b) in Huang and Chung\textsuperscript{[3]}, respectively. Hence, Huang and Chung\textsuperscript{[3]} will be a special case of Ouyang et al.\textsuperscript{[1]} model.

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