An Optimal Geometry for Power Energy and Data Transmission in the Inductive Link

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Abstract: This study presents the inductive link and the stability of the mutual inductance between various coils of different forms to guarantee a transfer of unidirectional energy communication between the implantables electronic devices. Study rested on the increase of the efficiency allocated by the tolerance to the lateral and longitudinal displacement between the external coil (transmitter) and the implanted coil (receiver). This study consisted in looking for the geometry of coils the least sensitive to the displacement and the most stable in mutual inductance. The mutual inductance was treated analytically by mathematical simulations using the Neumann equation for some geometries, to guarantee the maximum of power transfer energy in electronic systems implantables. This study treated the optimal geometry, the experimental measure of the mutual inductance for the energy transmission by various implantables systems in the human body.

Key words: Inductive link, mutual inductance, coil geometries, sensibility displacements

INTRODUCTION

In the medical devices, an external and internal electronic parts are found, the last find requires a mating by inductive link wrapping supply and the meditative data from the external part. Present study consists in studying the sensibility of the mutual inductance between the internal and the external part to obtain the maximum of system’s stability. This sensibility depends of miss placement due to the bad emplacement of primary with respect to the secondary. Communication between the internal and the external part of the implant is done by emitter and receiver rolling-ups[3]. To realise an efficient coupling to transmit the signal and the energy to implantable part, the principle of the mutual inductance is used to make link between the two parts.

A bad coupling of two coils decreases efficiency. This is due to a lot of factors on which depends the implant, namely the material nature, the shape, the dimension, the location (the anatomy of the place where system is implanted) and in major party the relative lateral and longitudinal displacements[3]. Consequently, the following study describes the priority factors required by the system.

For aesthetics reasons and surgical constraints the maximum radius of two systems must not overtake 15 mm[2]. Furthermore, in favour of the obtaining an elevated mutual inductance, to guarantee the energy maximum transmission in the implantable system, an external coil is used that doesn’t exceed a radius of 15 mm (a<15 mm).

The purpose of this work is to present simplified calculation methods of the mutual inductance for simple geometry’s such as two circular coils and more complex geometry’s such as between two solenoids, between two Archimedes coils and between solenoid and Archimedes coil[3]. Several studies are made to discover the case presenting the maximum of mutual inductance[4].

Influential parameters on this mutual inductance are geometrical forms, environment and principally lateral displacement caused by the bad placement of the external part by report the internal one of the system[5,6]. A comparison of the mutual inductance treated analytically is presented by mathematical simulations of the Neumann equation and its practical measure for two unspecified geometries. This measure puts also in evidence the influence of equipment on experimental calculations.

Besides, because the inductances thickness used has also an impact on the estimate of the experimental mutual inductance, it results from it a light shift between the theory and experiment. For the practical measure of the mutual inductance two possible methods can be used:

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ARCHITECTURE BETWEEN
TWO CIRCULAR COILS

The M mutual inductance of the classic case is studied between two circular coils, the first coil is browsed by magnetizing current $i$. $a$ and $b$ are the radii of the emitter and receiver, respectively. $x$ and $z$ are the laterally and longitudinally separate distance coils, respectively (Fig. 2).

By using the classic Neumann equation, the mutual inductance between these two coils is given by:

$$M = \frac{\phi_i}{i}$$

(2)

The law of electromagnetic inductance in the space established by Faraday, permits us to write the flux:

$$\phi_{ik} = \frac{\mu_0 i}{4\pi} \oint\frac{dl_1 \cdot dl_2}{r}$$

(3)

With $dl_1$ and $dl_2$ are the crosses elements placed, respectively in $P_1$ and $P_2$ position that describe mathematically the geometry of circuits to treat, whereas $r$ is the relative distance of these separation and $\mu_0$ is the permeability in the vacuum.

The mutual inductance will be:

$$M = \frac{\mu_0 i}{4\pi} \oint\frac{dl_1 \cdot dl_2}{r}$$

(4)

It is really necessary to write the Cartesian coordinates of these two points $P_1$ and $P_2$ and their elements of displacements $dl_1$ and $dl_2$:

$$P_1 = (a \cos \theta, a \sin \theta, 0)$$

$$P_2 = (x + b \cos \varphi, b \sin \varphi, z)$$

(5)

(6)

Fig. 2: Disposition of not coaxial two circular turns distant of $z$ longitudinally and $x$ laterally.
Although, the simplicity of the functions of the Cartesian coordinates and displacement's elements, the system to resolve stay always complex as gives evidence the equation of the mutual inductance (9).

$$M = \frac{\mu_0}{4\pi} \int_{0}^{2\pi} \int_{0}^{\pi} \frac{ab \cos(\theta - \varphi) d\theta d\varphi}{\sqrt{a^2 + b^2 + z^2 + 2bc \cos(\theta - \varphi)}}$$

Regrettably, this equation has not analytical solution since elliptic integrals are complicated to resolve. For this, the mathematical software MATHCAD is used. Before passing in results, it's necessary to indicate that the mutual inductance value evolves in same sense that the dimension of the emitter and the receiver. Regrettably, some constraints of aesthetics and surgery limit the dimensions of these circuits. According to Gauthier\[20\], the maximum radius of these two systems must be limited to 15 mm. The first result (Fig. 3) corresponds to the study of the mutual inductance in function to lateral displacements (x) obtained by simulation of the Eq. 9 for various longitudinal displacements z.

The longitudinal displacements z must cover the variation domain between 4 and 10 mm (space understanding the corporal tissues and the protective envelope) and the lateral displacements x vary between 0 and 10 mm corresponding to average displacements due to the bad emplacement of the transmitter system.

Several calculations are made to determine the optimal radius which can supply the maximum of the mutual inductance. The emitter radius is fixed at a and for various longitudinal displacements the radius b is extracted. So to have the maximum of mutual, results show that b increases with z.

Other calculations are made in the same purpose, by looking for the optimal b radius for the same value of the mutual inductance and for various of longitudinal displacements z values and null lateral displacement (x=0). Table 1 is obtained for M=22nH and various radius a of the first circuit.

To guarantee the even mutual, these results show that the longitudinal displacement and the a and b radii, grow in the same way.

In this part we are going to study the stability of the mutual inductance for the case of two circular turns of radius a and b. Knowing that the complexity of the mutual inductance function depends on some variables, the sensibility to lateral displacements must be established for the fixed variables z and a. The sensibility calculation\[21\] will take account of longitudinal displacement for the external fixed coil radius. Sensibility in lateral displacements, for a radius a of the external turn and for a longitudinal displacement z constants, is defined by the following relation:

$$S = \frac{\Delta M}{\Delta x} \cdot \frac{1}{a} = \text{cte}, z = \text{cte}$$

Where, $\Delta M$ is the variation element of the mutual inductance and $\Delta x$ is the variation element of the lateral displacement.

The determination of the sensibility requires the calculation of the partial derivative of the mutual inductance with regard to the lateral displacement (Eq. 11). It can be done only if the denominator of the mutual
inductance Eq. 9 is different to zero in the integral’s borders. This is true in our case since z represents spacing understanding the corporal tissues and the protective envelope.

\[
S = \left[ \frac{\partial M}{\partial x} \right]_{a = \text{cte}, z = \text{cte}}
\]

(11)

This derivative could be difficult to compute since the elliptic integrals in the mutual. Since variables a, b and z are considered as constants, the derivative will be made inside the integrals, that permits to have the Eq. 12:

\[
S = \frac{1}{4\pi} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} \frac{ab\cos(\theta - \varphi) - x \cos(\theta - \varphi) + b \sin(\theta - \varphi)}{(x^2 + a^2 + b^2 + z^2 + 2bx \cos \varphi})^{3/2} \, d\theta \, d\varphi
\]

(12)

This equation does not have an analytical solution because of its complexity and the presence of elliptic integrals. For the resolution, the mathematical simulations are necessary. These simulation results of the mutual inductance stability are given according to the x lateral displacement, for different values of the b radius, with a 15 mm emitter radius and a 6 mm longitudinal average distance (Fig. 2).

Figure 4 shows a less pronounced sensibility for the weak b radius values of the interturn S(b,x). This permits to have a best mutual stability since this one is inversely proportional to the sensibility. This simulation does not give any information on the mutual inductance value. Other simulations for different b values radius showed that the mutual inductance will be more stable when this radius decreases (the mutual inductance decreases also).

In order to put in evidence the stability effect of the mutual inductance according to the lateral displacements, the Eq. 9 of the mutual inductance was resolved with a 15 mm radius of the emitter turn and for two different b radius values of the receiver one. The simulation results of the mutual inductance M and M, relative to the cases of 15 mm and 10 mm radius, are represented on the Fig. 5.

The reduction of the second turn radius will give further evidence of the stability phenomenon. But the mutual inductance decreases about a factor 2 for a reduction of a factor 0.66 of the second turn radius by report to the first.

The device of the Fig. 5 presents, more pronounced stability for the case of two circular coils possessing radiuses of 10 and 15 mm, respectively that the case of two identical coils of 15 mm, but this improvement is obtained only by sacrificing a great part of the mutual inductance M.
ARCHITECTURE BETWEEN TWO SOLENOIDS

Two not coaxial solenoids are considered as represents the Fig. 6. The calculation of the mutual inductance can be done either rigorously or by making an estimate which consists in neglecting the thickness of conductor leads and in multiplying M (case of two circular coils) by the number of turns used in the solenoids. This estimate will be less distinguished in the case of a reduced number of turns. Indeed, all the turns are considered at the same level. In the contrary case, a staggering on the mutual inductance will be observed because more the number of turns increases, the relative longitudinal displacement to these turns increases. This staggering will be much more remarkable than the diameter of the leads is bigger.

For landing in this problem, the detailed calculation must be made by taking into account the diameter of the leads and the relative position of two circuits. Indeed the borders of integral will be modified as well as the thickness of the two coils, this idea is the perspective object of Marc LECLOIR in his thesis memory.

From equation of the mutual inductance of two circular coils (including the coils diameters and relative position), the Eq. 13 from the mutual inductance is given by:

\[
M = \sum_{\mu=0}^{n-1} \sum_{\nu=0}^{m-1} \frac{\mu \nu}{4 \pi} \int \frac{\cos(\theta - \psi) \cos \phi}{\left( x^2 + y^2 + z^2 + \left( z + \frac{E_1}{2} + \frac{E_2}{2} + iE_1 + jE_2 \right)^2 \right)^{3/2}} \, dx \, dy \, dz
\]

Where, \( N_1, N_2 \) are the turns number and \( E_1, E_2 \) are the diameter leads respectively of the circuits 1 and 2.

The simulation results of this equation is given by the Fig. 7 that represents the mutual inductance noted by \( M_i \) for the case of 5 turns in the emitter and 2 turns in the receiver. It have all 15 mm of radius and 5/10 mm for lead's diameter. Figure 7 included too, the mutual inductance noted by \( M \) when the result of the two turns of the same radius (15 mm) but multiplied by 10.

Other simulations are made for various lead's diameters and various turn's number showed that more the lead's diameter increases more the gap between the exact and approached calculations will raise. It is important to note that the reduction of a solenoid's radius generates a relative diminution of mutual inductance while assuring a better stability.

Fig. 6: Disposition of two solenoids separated from a longitudinal distance \( z \) and lateral \( x \) and having 5 turns in the emission and 2 turns in receipt

Fig. 7: Mutual inductance for various longitudinal displacements between two circular solenoids of 15 mm radius and 5/10 mm of leads diameter with \( N_1 = 5 \) and \( N_2 = 2 \)

The sensibility can be calculated rigorously or by making an approached calculation by neglecting the section of the wires and multiplying the case of two circular turns by the number of turns used in two solenoids (as the mutual inductance). In the rigorous
The numeric solutions of this equation are represented in Fig. 8. The emitter includes 5 turns of 15 mm radius, whereas the receiver has only 2 turns. The used wires have a section of 5/10 mm. So for this case the mutual sensibility is all the more weak that the radius of the receiving coil is small. Which permits to have a remarkable stability for the weak values of this radius. In order to see the effect of the mutual stability in function of the lateral displacements, the mutual inductance Eq. 13 is calculated for a 15 mm radius of the emitting coil and for two different values from the radius of the receiving coil. The simulation results of mutual inductance M and M_i relative to cases of a 15 mm radius and 10 mm are represented on the Fig. 9. From these graphs, the mutual stability for lateral displacements improves when the receiver radius decreases.

ARCHITECTURE BETWEEN TWO ARCHIMEDES COILS

These coils present a more complicated geometry than that of the circular coils (Fig. 10), indeed a and b radius increase with the angle under the shape:

$$a = \frac{\theta}{2\pi}$$  \hspace{1cm} (15)

and

$$b = \frac{\phi}{2\pi}$$  \hspace{1cm} (16)

a and b radii describe surface part of the coil (the radius grows of 1 mm in every turn).

By taking into account the Eq. 4, ..., 8, 15 and 16 the mutual inductance will be given by the relation 17:

$$M = \frac{\mu_0}{4\pi} \int_{\text{tr}1}^{\text{tr}2} \int_{\text{tr}1}^{\text{tr}2} \text{d}\theta\text{d}\phi$$

$$\sqrt{x^2 + (t_1\theta)^2 + (t_1\phi)^2 + x_t^2 + x_\phi^2 \cos \varphi - 2t_1x_t \cos \theta - 2t_1x_\phi \cos (\theta - \phi)}$$

\hspace{1cm} (17)
Fig. 10: Disposition of two Archimedes coils of a and b radiiuses, separated of z longitudinally and x laterally and having 5 turns in the emitter and turns in the receiver

Fig. 11: Mutual inductance for different longitudinal displacements between two Archimedes coils of external radius 15 mm and having 5 turns in the emitter and 2 turns in the receiver.

So, the optimisation of the surface, by Archimedes coils, allows increasing the mutual inductance in comparison with the case of two circular coils for identical external radius. On the other hand, the circuit formed by Archimedes coils is much more sensitive in lateral displacements. To compare the results of the mutual inductance of Archimedes coils with those of the solenoids having both an identical radius (15 mm), calculations must be done with 5 turns in the emitter and 2 turns in the receiver.

Fig. 12: The mutual inductance sensibility for two Archimedes coils of 15 mm radius separated by 6 mm

2 turns in the receiver (N₂=5 and N₂=2). The mutual inductance relation (17) for this last case is given by the Fig. 11. It shows a light difference between the obtained results and the rigorous calculation of the Fig. 7.

The relations (10) and (11) permit to reduce the mutual sensibility equation which is given by the relation (18).

\[
S = \frac{\mu_0}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} [z^2 + (r_1 \varphi)^2 + z^2 + 2r_1 x \varphi \cos \varphi - 2r_1 x \theta \cos \Theta - 2r_1 \theta \varphi \cos (\theta - \varphi)]^{1/2} \, d\varphi \, d\theta
\]

(18)

Having 5 turns and 15 mm of external radius for the emitting coil, the first integral borders must cover the interval of [10.2\pi, 15.2\pi]. To have the same conditions as that of a solenoid, the integral borders of the receiving coil of two turns must cover the interval [(b-2), 2\pi, b, 2\pi].

The sensibility simulation results of this equation are given by the Fig. 12 for different values of the external receiving coil radius. The sensibility varies in the same sense that the receiving coil radius. The best stability is
Fig. 13: Mutual inductance stability of two internal Archimedean coils with 15 and 10 mm radiuses

thus obtained for the weak values of this radius but to the detriment of the mutual. Broadly speaking, this mutual stability optimization is based on the reducing of one of two geometries. The choice to reduce the internal coil geometry (radius b) explains itself by the fact that the dimension of the implanted part must be as small as possible.

In order to put, as previously, the evidence of the mutual stability effect in function the lateral displacements, the Eq 17 is resolved for 15 mm radius of the transmit coil and for two radius values of the receiving one. The mutual inductance simulation results M and M_i relative to 15 mm and 10 mm radius are represented at the Fig. 13.

So the set of two Archimedean coils presents a better mutual stability for the weak receiver radius values. This system is more sensitive to the displacement than a system formed by two circular turns. The reduction of the second turn radius will further put in evidence the stability phenomenon. But the mutual inductance decreases about a factor 2 for a reduction of a factor 0.66 of the second turn radius by report the first.

**ARCHITECTURE BETWEEN ARCHIMEDEAN COIL AND SOLENOID**

This combination between two geometries is studied to see stability and mutual inductance between the two circuits. Case of two Archimedean coils is adopted as the optimal case because it presents a remarkable mutual. But actually, by considering rigorous calculations in the case of two solenoids, the stability is more pronounced than that of two Archimedean coils with a light decrease of the mutual. To have a good compromise between the stability and the mutual interaction the case of a solenoid with Archimedean coil will be studied in this section (Fig. 14).

While taking account of Eq. 4, ..., 8 and Eq. 16 the mutual inductance will be given by the relation 19:

$$M = \sum_{i=1}^{n} \frac{H_i}{4\pi r_{2i}} \int_{0}^{2\pi} \int_{r_i}^{r_{i+1}} \left[ x^2 + a^2 + (r_i \phi)^2 + \left( \frac{z + \frac{E_i}{2} + iE_i}{2} \right)^2 + \left( 2(r_i \phi)x \cos \varphi - 2ax \cos \theta - 2a(r_i \phi) \cos \theta \varphi \right) \right] d\phi dp \right)$$

The mutual inductance is represented together with the experimental results at the Fig. 15 and 16.

To put in evidence the effect of the mutual stability according to lateral movements, the simulation results and the practice, represented by the Fig. 15, will be considered for a solenoid of 5 turns of 15 mm radius in the emission and Archimedean coil of 2 turns of 10 mm outside radius in the reception. While the simulation results and the practice of the Fig. 16 are obtained for Archimedean coil of 5 turns of 15 mm outside radius in the emission and a solenoid of 2 turns of 10 mm radius in the reception.

The Fig. 15 and 16 present a well agreement between the theoretical and practical results. On the other hand it is possible to conclude that the first case (solenoid in the emission and Archimedean coil in the reception) presents a remarkable stability and a mutual interaction slightly less pronounced than the second case (Archimedean coil in the
emission and the solenoid in the reception). Of more, it should note that it is more simple to implant Archimedes coil of which the surface shape that a solenoid of which the voluminal shape.

CONCLUSIONS

The results of simulation show that the variations of the mutual inductance are stable for lateral displacements going until \( x=10 \) mm for the case of the association between circular coil and Archimedes coil. The case of two circular coils allows to assure some stability of the mutual inductance with a remarkable loss of the last. The case of two solenoids presents a important mutual inductance but a bad stability. The optimization of the surface, by Archimedes coil, allows to increase the mutual inductance. The optimal case is obtained by joining two geometrical forms, a circular coil and Archimedes coil, since it presents a good compromised between stability and mutual. Raised practical results showed a good approach with theoretical results especially for combinations Archimedes coil solenoid. On the other hand, the numeric and practical mutual inductance solutions showed that the optimization of the surface in the case of two Archimedes coils allows to assure almost the same values of the mutual inductance in comparison with the case of two solenoids for identical external radius and of the same number of turns.

It is possible to end as well as the case of a solenoid and Archimedes coil presents a slightly remarkable stability and a mutual interaction less pronounced than the case of Archimedes coil and a solenoid. But because the abstract aspect of the medical implant could be more adequate not according to a shape voluminal but under a shape surface, intuitively, the choice can be made for the case of a solenoid in the emission and Archimedes coil in the reception.

REFERENCES


