A Simulation Technique For Engineering Control Systems

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Abstract: A constructable simulation scheme whose framework does not require a great mathematical sophistication is introduced for control systems. The material provided is intended to serve as a basis for developing the primary skills needed to apply and validate the scheme to flexibly meet the simulation requirements. The introduced scheme can be used to simulate a wide range of continuous and nonlinear systems maintaining the fidelity required for meaningful analysis. It allows the engineer to apply his expertise in the selection and refinement of solution technique best suited to his design problem.

Key words: Simulation, computer aided design, model assessment, computer applications

INTRODUCTION

The participation in the simulation of a specialized engineering system\(^\text{[1-4]}\) provided a useful background to inspire the present general approach as applicable technique with an important impact on many areas of control engineering design.

Compared with the basic techniques available for simulation, the present method has a flexible model that can handle complex system configurations, which are not necessarily reducible to a mathematical model of a single equation.

A highly accurate model for a system may be obtained easily without using domains other than time or adding special element, where the system characteristic may be altered to suit some operator method\(^\text{[9]}\).

The present method shares with the available simulations languages the common goal of increasing the utilization of digital computers for simulation and design activities and is intended to project forward a number of key advantages at the research and design levels. These are:

(i) It allows for a wider physical feel and participation in the simulation process. (ii) It is amenable to a systematic procedure for model development, which will guarantee successful simulation and design refinement. (iii) It can be conducted in terms of a user-oriented language. (iv) It is oriented towards the time domain and the topological format of the system. (v) It increases the utility of simulation in the newly developed fields of the subject.

A thorough description of the model formulation and implementation is introduced. The model assessment and validation is studied in respect of the accuracy and convergence of the digital computer solution of the models is provided for various configurations of the system.

MODEL FORMULATION

This section will be concerned with the introduction of a hypothesis which formulate a computer model and a procedure which is necessary to set forth the solution for a given control system.

Any control system is composed of a combination of various types of elements namely, linear, nonlinear and time delay elements that are interconnected directly or through various feedback and summing junctions to its terminals. The computational aspects across each type of the elements are explained separately, as this is primarily needed for the computer model, which will be proposed.

The dynamics of a linear element can always be described by a set of first order differential equations. A stepwise integration scheme can be carried out by one of the many numerical methods for the solution approximation.

An appropriate incremental relations for the integration for a set of \(m\) first order simultaneous differential equations has the form

\[
X_j(t_\ast) = \varphi \left[ X_1(t_{i,1}), \ldots, X_m(t_{i,m}), R(t), \Delta t \right] \quad (1)
\]

Where, \(X_j(0) = X_{0j}\) as initial conditions

for \(j = 1, 2, \ldots, m\) and \(t = r \Delta t\)

Where, \(r\) is the time increment index.
This way the linear continuous-time element is approximated on pivotal values along a single increment of time with the input and output signals \( R(t) \) and \( X(t) \) respectively, as discrete-time functions. However, if desired, an alternative approximation may involve several pivotal values.

Nonlinear input-output characteristics, which are normally sufficient to describe its effect as a magnitude dependent and not a time dependent behavior depict the nonlinear element. In general, the nonlinear characteristics are represented in a piece-wise linear fashion by straight-line segments. The segments are terminated at the intersections of the straight lines of adjacent segments. These are, in fact, points of the nonlinear characteristics. Of course the more the segments the better the approximation. If projection is made, for the magnitude of the input signal from the \( x \)-axis on a segment of the nonlinear characteristics, the magnitude of the output signal is obtained from the corresponding \( y \)-axis of the projection. This can be described by the symbolic equation.

\[
X_{ni}(r \Delta t) = G_{ni} [R(r \Delta t), (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)]
\]  

(2)

Where, \( R \) and \( X_{ni} \) are the input and output magnitudes respectively and \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\), define the termination points or the segments of the nonlinear characteristics.

The behavior of time delay elements can be handled accurately by the computer as it can store the discrete values of the signals and present it again after the delay. If the time delay is an integer number \( r_{i} \) of the time increment \( (\Delta t) \) then, one way for accounting for the time delay is to use a procedure having two arrays for storing the discrete values of the input and output signals respectively. Each array will be subscripted in terms of the number \( r \), which indicates the time order of the sequence of the discrete values of the variables. The output variable \( X_r \) is then related to that of the input \( R \) through the following equation

\[
X_r(r \Delta t) = G_r [R(r \Delta t)]
\]  

(3)

Where

\[
G_r [R(r \Delta t)] = 0
\]

for \( r \leq r_{i} \)

\[
G_r [R(r \Delta t)] = R[(r-r_{i}) \Delta t]
\]

for \( r > r_{i} \)

This means that the input samples are held and read out \( (r_{i} \Delta t) \) later.

The equation of the error discriminator element, that produces the actuating signal. Therefore, the error actuating signal \( X_e \) is taken as a function of the input signal \( R \) and the feedback signal \( F \) as

\[
X_E(r \Delta t) = G_e [R(r \Delta t), F(r-1) \Delta t]
\]  

(4)

The feedback signal is taken one step behind the error to account for the close loop operation. A general function form is used for the error discriminator as this may function as a subtractor, as the case in a delta modulation system, or a multiplier, for example, a nonlinear phase locked loop system.

In the following, it will be shown how, the desired overall model can be formulated and organized by exploiting the sequence of computable relations which

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Fig. 1a: Model formulation for a typical multiple loop control system

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158
Fig. 1b: Model formulation for a typical multiple interconnection control system

characterize its respective elements. To allow the exposition of the separate facets of the model formulation it is instructive to relate the simulation procedure of a particular system to its block diagram notation. This will portray the model computational sequence to the composition and interconnections of the system.

Fig. 1c: Model formulation for an inter coupling system

As the block diagram of control systems may have several topological formats, depending on the type of control system and on the objective of the design, the block diagrams shown in Fig. 1a, b and c, are chosen to illustrate how the models are formulated for these arrangements. These choices are appropriate as the existence of the typical multiple loop shown in Fig. 1a, multiple interconnections shown in Fig. 1b, or intercoupling system shown in Fig. 1c, are very common in the field of automatic control. In addition, the concepts for the model formulation to these arrangements are particularly instructive to develop the models of other systems with various configurations. For generality, an
element type is not specified. However, it is assumed that there exist some elements in the systems loops that have nonlinear characteristics, which forbid an analytical reduction of the systems.

The corresponding flowcharts of Fig. 1 portray graphically the systematic procedure, which simulates the respective control system shown. It relates the actual time sequence of the computed parameters to the various elements that are fitted in the control system and the signal flow directions in various loops.

Each flowchart uses short indicative relations where the discrete output signal for the i-th element is expressed as a function G or H, or the discrete input signal and the current time t, irrespective of the element type which is deliberately left unspecified to preserve the generality of the concepts. Attention should be directed towards the need for providing an updated values for the simultaneous processing through the summing junction of Fig. 1 and the need to group the computed parameters of the two loops to provide an updated values for the inter-coupling elements of Fig. 1c.

The time steps of the simulation procedure are synchronized with the timing of the integration steps boundaries. The subscript (r) indicates the present value of the parameter, whereas (r-1) indicates their value at a single step earlier. Each traversal of the loop represented by a continuous line advances time by an amount Δt.

Thus, the fixed step size Δt is to be chosen of a size that is small enough to deal with the smallest time constant of the linear elements of the system, as these elements are responsible for the transient behavior of the system. However, the model can be made to automatically adjust the step size to meet a prespecified error criterion and is thus able to use large step size (for rapid computation) when the response is changing slowly, but will switch to smaller steps when rapidly changing variables require it.

The loops drawn with broken lines in the flowchart of the algorithms are actually to secure a refinement of the computed parameters for the simulated system. This allows a closer approximation to the original system. Each iteration modifies the feedback signal values and recompute the elements output values again, as dictated by their trace on the particular loop of the system. A modified value of the feedback signal is obtained as the arithmetic mean of the previous value and the new computed value of this signal. Thus in an assignment statement form the modified feedback signal value is given by

\[ F((r-1) \Delta t) = [F((r-1) \Delta t) + F(r \Delta t)]/2 \]  (5)

Of course other iteration schemes for accuracy improvement may be carried out. These have to be suggested and validated before implementation.

**MODEL ASSESSMENT**

A series of experimental simulation has been conducted on a range of control systems, which mainly are chosen to have an analytical solution. This way a comparison between the simulation results and those of the corresponding analytical solution can be presented, as a quantitative approach to the problem of estimating the proposed model accuracy for its assessment and validation.

The simulation is developed for an increased number of pivotal points, that is shorter time increments, along a finite range of the time response, usually until the steady state is reached. The error is calculated as the sum of the squared deviation from the true solution at a number of points, which are uniformly distributed in the region of interest of the solution. The results marked A are recorded for the model with the accuracy refinement loops being disabled, whereas those for the complete model are marked B.

A display of the error variations for a nonlinear control system, a system with a time delay element and a minor feedback loop system are shown in Fig. 2a. The corresponding curves are marked N, T and M respectively. The analytical solutions used for comparison with the present simulation results of those systems are given in Torby[9]. Further refinement of

![Fig. 2a: Error variations for validating continuous control system models](image_url)
A cross coupling problem whose details are given in Doebele[9] is simulated by the present method. The model development is based on the ideas stressed for the model formulation and organization exposed for the system shown in Fig. 1c. The two cases namely the two loops having significant interaction and one loop having little effect on the other are tried.

A combined observer controller scheme whose detail is given in Kailath[10] is simulated by the present method. The model development is based on the ideas stressed for the model formulation and organization exposed for the system shown in Fig. 1b and it may be described as follows; For each integration step the reference signal (which is the difference of the outputs of the original system and that of the observer) is calculated. The calculations involve the previous values of the terms proportional to the errors in the state estimates of the observer and the previous values of the controller terms, which effect a state variable feedback to the input of the system and usually not amenable to analysis is presented. Thus the constructable modal may be built-up to provide a simulation on the basis of sufficiently detailed elements characteristics, which are abstracted from the actual system components.

All the steps necessary in the implementation of the simulation process are integrated into one unified methodology with all the computations and sampling instants being synchronized with respect to the integration step boundaries.

The numerical experimentation has provided the computed errors of the simulation results and established the scientific validity of the method. The validation of a technique, which upgrades the accuracy of the models for the continuous systems by liquidating the errors in the equations for the junctions of the feedback loops, is also included. This enhances the potential power of the feedback principle in following commands and rejecting disturbances for the model.

In considering the results of the present simulation method, they are found to be indistinguishable from those produced by other techniques and displayed by the respective references for these systems when using the same integration step size.

**CONCLUSION**

An invaluable simulation scheme for the design of realistic control systems, which in general have an additional nonlinearity in the continuous part of the control system theory. The numerical experimentation is used to validate the models with respect to their fundamental features.
In considering the numerical experimentation and the various implementable integration schemes for the linear part of the model, the present method can provide a large trade off between the accuracy and the number of calculations (speed). However, the suitability of the present method to real time simulation requires further exploration. Thus, while the available simulation methods emphasize some particular characteristics rather than the others, the present method introduces a balanced blend of these desired characteristics. Therefore, the present method has much to offer and can produce a fundamental shift in altitude and in practice to the design of control system.

REFERENCES