Fracture Mechanics Modelling of Cracked Aluminium Panel Repaired with Bonded Composite Circular Patch

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Abstract: The process of repair of the structures by using bonded composite patch is an effectiveness and economical method to increase the service life of damaged structures. In this study, the finite element method is used to analyze the behaviour of a crack repaired by a patch by computing the stress intensity factor at the crack tip in mode I and mixed mode. The effects of the mechanical and geometrical properties of the patch on the fracture parameters are highlighted. The obtained results show that there is a considerable reduction of the asymptotic value of the stress intensity factor at the crack tip in the case of the use of the double symmetric patch compared with the single patch. Moreover, the good orientation of fibres having the highest mechanical characteristics compared to the crack growth influences considerably the reduction of the stress intensity factor. For the boron/epoxy patch, the orientation of fibres perpendicularly compared to the crack is four times larger than the orientation parallel with the crack. The adhesive properties must be optimized to increase the performance of the repaired structure by the reinforcement.

Key words: Bonded composite patch, repaired crack, Stress Intensity Factor (SIF), Stress Concentration Factor (SCF), gain

INTRODUCTION

Repair of cracked metallic structures with bonded composites is finding increases in service due to the several advantages offered by the repair concept. This method has been shown to be very promising owing to the light weight, high stiffness and strength of the composite. The stress intensity factor is then reduced by the presence of the patch. Several authors showed that the mode I stress intensity factor of repaired crack, exhibits an asymptotic behaviour as the crack length increases (Ting et al., 1999; Callinan et al., 1997; Jones and Chiu, 1999; Achour et al., 2003; Schubbe and Mall, 1999, Quinas et al., 2003; Bachir et al., 2002). Bachir et al. (2002) affirmed that the adhesive properties must be optimized in order to allow the transmission of the stresses toward the patch and to avoid the adhesive failure. Numerous models have been developed for the analysis of symmetric repairs using a wide variety of calculation techniques including the location method (Dowrick et al., 1980), boundary element method (Young et al., 1992), finite element method (Quinas et al., 2003; Bachir et al., 2002; Baker and Joher, 1988; Ratwani et al., 1977; Chue et al., 1994), finite element alternating method (Park et al., 1992) and approximate analytical solutions (Baker and Joher, 1988).

A study was carried out on the repair of a crack emanating from semicircular notch by a semicircular composite patch (Quinas et al., 2004a). Quinas et al. (2005a, 2006) showed the effect of the disbond on the amplification of the stress concentrations at the notch root and of the SIF on the cracks tip.

In this study, one studies the analysis of the behaviour of a reinforced cracked aluminium plate in mode I and mixed mode by the finite element method. The composite patch subjected to the analysis is the boron/epoxy used with much success in aeronautical repair. In practice the parameters influencing the performance of repair are the properties of the patch and the adhesive (Ting et al., 1999; Jones and Chiu, 1999). For that, the effects of the adhesive shear modulus, the adhesive thickness and the patch thickness on the variations of the stress intensity factor will be examined. One shows the influence of the orientation of fibres of composite patch on the reduction of the stress intensity factor. One compares the values of the stress intensity factors of repaired cracks with single and double patches. The gain of the thickness eventually obtained by the use of the double symmetric patch is determined.

GEOMETRICAL MODEL

To study the effect of the bonded composite patch on the rupture parameters, the values of and were calculated by the modified crack closure technique
configuration we employ a circular patch. The transfers of tensions and deformations of the plate and the patch are continuous.

The modified crack closure technical are used to calculate the stress intensity factor. An automatic two-dimensional finite element mesh generation is employed to create adequate finite element mesh (FRANC2D/L., 1998).

FINITE ELEMENT MODELLING

A two-dimensional finite element code named FRacture ANalysis Code for 2-D Layered structure was used in the numerical modelling work. This code was developed at Cornell University and was modified for multi-layers structures at Kansas University (FRANC2D/L., 1998). The code is based on the theory of linear and non-linear fracture mechanics. In this study, simplified assumptions are adopted in order to solve the problem. These assumptions are:

- Each layer is considered to be a two-dimensional structure on the stress plane.
- The layers may be assembled with an adhesive layer.
- The adhesive layer is homogeneous, linear, elastic and isotropic.
- The adhesive works only in shear and the deformation is uniform throughout the adhesive thickness.

Shear stresses in the adhesive are given by the relation:

\[ \tau = \frac{G_s}{e_s} (u_1 - u_2) \]  \hspace{1cm} (1)

where \( u_1 \) are displacements in layers 1 and 2, respectively for the plate and the patch.

The total structure (plate and patch) is meshed using standard eight noded serendipity elements with quadratic shape functions. These elements perform well for elastic analysis and have the advantage that the stress singularity at the crack tip can be incorporated in the solution by moving the eight nodes to the quarter point locations (Henshel and Shaw, 1975).
PURE MODE I

Stress concentration factor on the level of the patch: The stress concentrations of shear and delamination are localised on the level of the extremities involving a maximum deformation of the medium (Ouinas et al., 2005b). For illustrate the effect of the stress absorbed by the bonded patch, we represented on Fig. 2 the variation of the stress concentration factor for various sizes of the repaired crack. The patch diameter is equal to \( d_e = 60 \text{ mm} \).

We note that an increase in the crack length involves an increase in the deformation energy created in its tip. The stress concentration factor increases proportionally with increase the length of the crack. The maximum stress are strongly localised around the crack and decrease proportionally while moving away from this one. The minimal stresses are located around of the patch extremities.

![Fig. 2: Variation of the SCF of the patch according to the crack size](image)

The most intense stress are localised on the crack lips and are small on the circumference of the patch. The stress concentration factor increases indefinitely with the increase in the size of the crack repaired by only one patch; for the double boron/epoxy patch the stabilization of the factor \( K_t \) appears from the ratio \( a/d_e = 0.25 \).

The conception of the assemblies of repair by patch will aim at making the distribution of the stress as uniform as possible; so the fact of using the couple materials plate/patch the same rigidities improves the mechanical resistance of repair (Ouinas et al., 2003, 2004b). However it should be noted that the stress concentration transmitted of the crack to the patch through the adhesive are localised on the level of the interface (Ouinas et al., 2005b).

Influence of adhesive shear modulus on the factor \( K_t \) of the patch: The adhesive shear modulus is a significant characteristic influencing the mechanical properties of the assembly or the repair of the cracks. On Fig. 3 we represented the effect of the adhesive shear modulus of thickness \( e_v = 0.127 \text{ mm} \) on the evolution of the stress concentration factor of the patch for various materials. The analysis of Fig. 3 indicates that the stress concentration factor is strongly depend on the shear adhesive modulus when this last reaches the value \( G_v \geq 200 \text{ MPa} \). Beyond this value the factor \( K_t \) is stable for two materials constituting the patch and its capacity for absorption increase while thus leading to the widening of the maximum stress zone. The minimal stress tends to being negligible when the adhesive shear modulus increases.

![Fig. 3: Variation of the SCF according to the adhesive shear modulus \( G_v \)](image)

Effect of the fibres orientation of composite patch: To highlight the effect of the fibres direction of the composite patch on the repair of the central crack, we considered two cases. The first consists in taking a fibres having the higher mechanical characteristics in the crack direction (orientation 1). In the second case, these fibres are perpendicular to the crack (orientation 2).

The results obtained are represented on Fig. 4. It is noticed that the application of same composite material of the patch in the process of repair of crack gives different results. The curve of the stress intensity factor of the crack repaired by the patch of orientation (2) converges quickly from \( 2a \geq 10 \text{ mm} \) compared to the patch of orientation (1). These results are confirmed by Ouinas et al. (2005a).

The reduction of the stress intensity factor is much better when the orientations of the stiffest fibres are perpendicular to advanced crack. The maximum reduction of the composite patch of orientation (2) is about 30%
Table 2: Evaluation of the gain for $2a = 10$ mm

<table>
<thead>
<tr>
<th>$\sigma_1$(mm)</th>
<th>$K_1$ (MPa.m)</th>
<th>$\sigma_2$(mm)</th>
<th>$K_2$ (MPa.m)</th>
<th>Gain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>double patch</td>
<td>$K_1$</td>
<td>single patch</td>
<td>$K_2$</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>6.394</td>
<td>0.2925</td>
<td>6.393</td>
<td>31.620</td>
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<tr>
<td>0.40</td>
<td>4.898</td>
<td>0.542</td>
<td>4.897</td>
<td>26.199</td>
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<td>0.80</td>
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<td>1.020</td>
<td>3.417</td>
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<td>2.444</td>
<td>1.801</td>
<td>18.167</td>
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<tr>
<td>4</td>
<td>0.997</td>
<td>4.856</td>
<td>0.997</td>
<td>17.628</td>
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Table 3: Evaluation of the gain for $2a = 20$ mm

<table>
<thead>
<tr>
<th>$\sigma_1$(mm)</th>
<th>$K_1$ (MPa.m)</th>
<th>$\sigma_2$(mm)</th>
<th>$K_2$ (MPa.m)</th>
<th>Gain (%)</th>
</tr>
</thead>
<tbody>
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<td>double patch</td>
<td>$K_1$</td>
<td>single patch</td>
<td>$K_2$</td>
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<tr>
<td>0.20</td>
<td>6.766</td>
<td>0.181</td>
<td>6.769</td>
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<td>0.40</td>
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<td>0.377</td>
<td>5.093</td>
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<tr>
<td>0.80</td>
<td>3.507</td>
<td>0.77</td>
<td>3.507</td>
<td>48.052</td>
</tr>
<tr>
<td>1</td>
<td>3.043</td>
<td>0.568</td>
<td>3.042</td>
<td>48.347</td>
</tr>
<tr>
<td>2</td>
<td>1.835</td>
<td>1.96</td>
<td>1.835</td>
<td>48.979</td>
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<tr>
<td>4</td>
<td>1.014</td>
<td>3.99</td>
<td>1.013</td>
<td>49.875</td>
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</table>

Table 4: Evaluation of the gain for $2a = 40$ mm

<table>
<thead>
<tr>
<th>$\sigma_1$(mm)</th>
<th>$K_1$ (MPa.m)</th>
<th>$\sigma_2$(mm)</th>
<th>$K_2$ (MPa.m)</th>
<th>Gain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>double patch</td>
<td>$K_1$</td>
<td>single patch</td>
<td>$K_2$</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>6.925</td>
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<td>47.644</td>
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<td>0.40</td>
<td>5.148</td>
<td>0.391</td>
<td>5.145</td>
<td>48.849</td>
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<tr>
<td>0.80</td>
<td>3.513</td>
<td>0.791</td>
<td>3.512</td>
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<tr>
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<td>0.993</td>
<td>3.044</td>
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<td>51.124</td>
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<tr>
<td>4</td>
<td>1.026</td>
<td>4.382</td>
<td>1.026</td>
<td>54.358</td>
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</table>

The relative maximum reduction of the SIF $K_1$ is about 68 and 78% respectively for simple and double patch. Jones and Chiu (1999) give a mean value of 60% for the boron/epoxy whereas Bachir et al. (2002) give a value of 80% for the same material. A comparison of the SIF between a simple patch and double patches metallic circular is the order 22% (Ouanas et al., 2003). One can be seen in Fig. 5 that the SIF for patched crack exhibits an asymptotic behaviour as the crack growth. This tendency was quoted by several investigators among which Ting et al. (1999), Bachir et al. (2002) and Ouanas et al. (2004b, 2005a, c).

**Comparison between single and double patched crack:** To make a comparison between a simple and a double patch, we determined the stress intensity factor according to the thickness of the patch. The patch thickness for a double patch is half of that used for a single patch.

Table 2-4 present the gain of thickness obtained by the use of the double symmetric patch. This gain is defined as the relative difference in percent between the thickness of double symmetric and single patch when the SIF is the same for the two cases. Table 2-4 are established for three different crack lengths. It can be noted that the gain may be considerable and it reaches about 55% for $2a = 40$ mm, which permits us to confirm that while adding the gain obtained by the elimination of the bending effect, the total gain obtained by the use of
Fig. 6: Influence of $e_a$ on the variation of the SIF

Fig. 7: Effect the patch thickness on the variation of the SIF

the double patch can largely exceed 50%. The repair of a crack length $2a = 40$ mm by a symmetrical double patch thickness $e_a = 1$ mm one will have a gain which reaches 50% compared to a single patch of the same thickness. For a crack length four times minus this gain will be about 21%.

It can also be seen in Table 2 that the gain decreases as the patch thickness increases. This is due to the fact that the transfer of the stress towards the patch is less important for high values of the patch thickness, then the difference of the transferred stresses between a single and double symmetric patch is less important. On Table 3 and 4 one notices that the gain increases as the patch thickness increases. This is due to advanced crack towards the patch extremities.

Effect of the adhesive thickness: Figure 6 shows the variation of the SIF according to the adhesive thickness ($e_a$) for three crack lengths $a = 10$, 20 and 30 mm. One can see in this figure that a reduction of the adhesive thickness decreases the value of the SIF, which means that low adhesive thickness is desirable to repair the cracks. This effect was highlighted by Turaga and Ripudaman (1999). A great thickness reinforces the adhesion but reduced the capacity for absorption of the stress field of the patch.

The values of the SIF increases indefinitely with the increase of the adhesive thickness. A low adhesive thickness increases the transfer of the load to the patch but increases the risk of the fracture of adhesion (Ouinas et al., 2005c, d). Theoretically, it is thus preferable to employ adhesives having a higher shear modulus for repair of the cracks or the defects (Bachir et al., 2003).

Effect of the patch thickness: This analysis treats the influence of the patch thickness on the amplitude of the fracture parameters. Figure 7 shows the effect of the composite patch thickness on the variation of the SIF. The stress intensity factor increases exponentially with the decrease of the composite patch thickness.

The elastic energy relaxed by the cracked plate will be absorbed by the patch by the means of the adhesive. One can see that the increase of the patch thickness reduces the SIF at crack tip in a way proportional. This confirms that the choice of the thick patch improves their performances. For a better distribution of the stresses, it is preferable to use a multiple layers of bonded composite patch for repairing cracks (Bachir et al., 2002).

MIXED MODE

To determine the effect of the presence of the circular patch on the performance of the repair of crack by the procedure of bonded in mixed mode, we represented on

Fig. 8: Variation of the SIF in mode I according to crack inclination
Fig. 9: Variation of the SIF in mode II according to crack inclination

Table 5: Comparison of the SIF of uncracked crack

<table>
<thead>
<tr>
<th>Angle θ</th>
<th>Smith (1988) Mode I</th>
<th>Mode II</th>
<th>FRANC 2DL Mode I</th>
<th>Mode II</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19.52 MPa√m</td>
<td>0</td>
<td>23.34 MPa√m</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>18.94 MPa√m</td>
<td>2.34 MPa√m</td>
<td>23.64 MPa√m</td>
<td>3.97 MPa√m</td>
</tr>
<tr>
<td>20</td>
<td>17.42 MPa√m</td>
<td>2.28 MPa√m</td>
<td>21.47 MPa√m</td>
<td>7.51 MPa√m</td>
</tr>
<tr>
<td>30</td>
<td>14.64 MPa√m</td>
<td>2.45 MPa√m</td>
<td>18.25 MPa√m</td>
<td>8.435 MPa√m</td>
</tr>
<tr>
<td>45</td>
<td>9.76 MPa√m</td>
<td>9.76 MPa√m</td>
<td>12.2 MPa√m</td>
<td>9.771 MPa√m</td>
</tr>
<tr>
<td>60</td>
<td>4.88 MPa√m</td>
<td>8.45 MPa√m</td>
<td>6.06 MPa√m</td>
<td>8.454 MPa√m</td>
</tr>
<tr>
<td>70</td>
<td>2.28 MPa√m</td>
<td>6.28 MPa√m</td>
<td>2.77 MPa√m</td>
<td>6.279 MPa√m</td>
</tr>
<tr>
<td>80</td>
<td>0.59 MPa√m</td>
<td>3.34 MPa√m</td>
<td>0.66 MPa√m</td>
<td>3.346 MPa√m</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 10: Reduction of the SIF $K_I$ according to the crack inclination

Fig. 11: Reduction of the SIF $K_{II}$ according to the crack inclination

$$K_I = \sigma \sqrt{a} \sin \alpha \sin \beta \alpha$$  \hspace{1cm} (2)

$$K_{II} = \sigma \sqrt{a} \sin \alpha \cos \beta \alpha$$ \hspace{1cm} (3)

Where $\sigma$ is applied stress, $a$ is the half-central crack and $\alpha$ is the angle located between the crack and the stress direction.

The comparative study between the results obtained by the finite element method (Fig. 8 and 9) and those of Smith (1988) obtained by another method of calculation gives a fork of error varying between 4% and 7% (Table 5). This shows that these two methods are in concord. This difference can result from the model considered, indeed, we took into account a finished model, whereas that of Smith is of infinite dimensions.

On Fig. 10 and 11 we represented the variation of the dimensionless stress intensity factor reduction according to the crack inclination. This SIF reduction factor is defined by:
\[ K^* = 1 - \frac{K_c}{K_i} \]

Where \( K_c \) and \( K_i \) are the SIF for the patched and unpatched crack plate, respectively.

The results show that the use of the patch is very effective and makes to slow down considerably the criteria of the crack growth. In comparison with unpached crack, the SIF in mode I of the patched crack is reduced approximately 48 to 78%. This reduction varies with the variation thickness of used patch. The stress intensity factor in mode I decrease with the crack inclination increases. This reduction is accentuated by the increase of the patch thickness. An inverse effect occurs when the crack is oriented with an angle higher than 75°, the SIF in mode I increases when the patch thickness increase.

The SIF of the crack repaired in mode I is not null with the value \( \pi /2 \). This phenomenon exists because of the presence of deformation due to the bending which causes the non-linear behaviour of material and geometry. The stress intensity factor in mode II of the reinforced plate is reduced approximately 48 to 80%. Thus one can conclude that, the effect of the thicker patch becomes even stronger and the SIF does not increase infinitely when the patch thickness increases because of the deformation generated by the anti-plane bending.

CONCLUSIONS

In this study, we treated the influence of the materials properties on the values of the fracture parameters calculated for a plate subjected to tensile load containing a central crack. The quality of the patch material and its dimensional geometry play an essential role in the distribution of the stress concentrations at the crack tip. The advantages of the use of the bonded symmetrical composite patch for repairing crack were highlighted. The following conclusions can be drawn from the present study:

- The stress intensity factor to the crack tip is inversely proportional to the increase of the patch rigidity and its geometrical characteristics. However, the double patch reduces appreciably the SIF to single patch.
- The choice of the adhesive properties for the repair of cracks with the bonded patch must be optimized.
- In mixed mode, the reduction of stress intensity factor in opening mode is more important than that in shear mode.
- The choice of the patch thickness is one of the best means to increase the performance of the repaired structures.
- The relative maximal reduction of the SIF \( K_i \) is of the order of 68 and 78%, respectively for the simple patch and double.
- In the mixed mode, the circular composite patch reduces both \( K_i \) and \( K_{ii} \), but its effect on mode I stress intensity factor is more considerable.
- The maximum reduction of composite patch of orientation (2) is about 30% with regard to the metallic patch.
- The gain in thickness increases with the crack length increase and it decreases when the thickness of the patch increases for big crack length. For small length this gain decreases with the patch thickness increase.

REFERENCES


