Integration of Geological Data Sets Using Fussy Logic

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Abstract: Fussy set theory provides a simple framework for integrating and analyzing qualitative and quantitative data irrespective of their sources. This study introduces fussy logic and demonstrated its application for predicting hydrothermal mineral occurrences in northwestern part of Yunnan. Three geovariables were considered and their membership functions were determined subjectively. Combination of the three fussy evidence layers; geochemical, lineament and lithology resulted into a favorability map for the occurrences of hydrothermal Cu, Ag, Au and Pb-Zn.

Key words: Fussy logic, GIS, hydrothermal mineralization, factors, fussy sets, membership scores

INTRODUCTION

The role being played by Geographic Information System commonly known a GIS can not be over emphasized. It provides a quick access to; (a) information extraction; (b) ranking various geo-variables (c) integrating diverse information; (d) reducing data processing time; and (e) enhancing decision making processes. Most of the statistical approaches based on regression and characteristics or canonical correlation analysis transform the multi-layer to binary or ternary form. Such techniques may be limited in the type of information that can be quantified. Bayesian methods and weight of evidence may be limited by conditional independence requirements for the data used. Belief and plausibility functions may in practice, be difficult to interpret and assess. The limitations of the above approaches are that, they can be somewhat inflexible in expressing the different degrees of favorability of the mineral occurrence for each of the individual geo-variables considered.

Fussy sets provide an alternative framework that could improve upon some of the limitations in previous techniques. A fussy set-based formulation is used by An et al. [3] to integrate geological and geophysical data from Farley lake area, Canada. The study uses the algebraic-sum and aggregation operators to outline favorable areas for base metal and iron deposits, based on an approach also advocated by Chung and Agterberg[8]. Although a notable development, the approach use combined geo-information does not take into account the individual relative importance of geo-variables in quantifying favorability for mineral occurrence. In addition relative, the approach is knowledge-driven and as such does not include objectives data driven criteria in the combination of the geo-information used. The purpose of this study is to test the implementation of fussy logic in the northwestern Yunnan by way of predicting favorable hydrothermal mineral occurrences.

Fuzzy sets: Geological information and data interpretations in mineral exploration are inherently ambiguous. The quantitative expressions like relative high, fair, low and relatively, fairly favorable, unfavorable for the mineral occurrences as well as the grey areas between these expressions, is difficult to define. Fussy set theory[3] and[6] provides a mathematical framework to represent the linguistic and ambiguities frequently encountered in mineral exploration geological information analysis and interpretation. The theory formally associates any statement with a quantifiable measure indicating the degree of possibility of the statement.

Definition and examples:
If X is a collection of objects denoted by x, fussy set in X is the set of ordered pairs,

\[ A = \{ (x, \mu_A(x)) | x \in X \} \]

(1)

Where, is \( \mu_A(x) \) termed the membership function or membership grade of x in A[3]. \( \mu_A(x) \) maps X to the membership space M. When M contains only two points 0 and 1, A is a non fuzzy set and \( \mu_A(x) \) is identical to the characteristics function of a regular, non-membership and expresses full membership.

In mineral potential mapping, the individual spatial objects on a map are considered as members of the fussy set and the set is defined as areas containing a specific type of mineral deposit. The degree of membership is expressed with respect to some attribute of interest. The
level of measurement of mapped variables can be
categorical, ordinal or interval. Generally the attribute
of interest is measured over discrete intervals and the
membership function can be expressed as a table relating
to map classes to the membership values. Thus individual
objects or classes of objects can be evaluated regarding
their membership in a fuzzy set of geologic objects.

Determining the initial fuzzy membership function
critically depends on the exploration target and related
geological deposits characteristics. In general initial fuzzy
memberships are assigned subjectively based on the
expert opinion. This may sometimes be very qualitative,
however, subsequent steps, including the integration of
multilayered data using various fuzzy operators are
quantitatively precise. Grade of membership is usually
represented by a membership function that need not to be
linear or even continuous. Indeed many fuzzy sets have
extremely nonlinear membership functions\textsuperscript{[4]} or fuzzy
membership can be thought of as in terms of support for
proposition or hypothesis. In mineral exploration, the
proposition is favorable location for mineral deposits\textsuperscript{[7]}. In
this study the goal is to identify areas favorable for
mineral deposits. The maps to be used as evidence to
support the proposition are all expressed as a common
information metric—namely fuzzy membership on the
\{0,1\} scale.

Fuzzy sets can be combined using fuzzy operators.
Zimmerman\textsuperscript{[8]} and Zadeh\textsuperscript{[9]} discuss more than two dozen
fuzzy operators An et al\textsuperscript{[5]} discuss 5 of the various fuzzy
operators as the more basic operators found to be useful
for combining exploration datasets, namely fuzzy AND
OR, fuzzy algebraic product, fuzzy algebraic sum and
fuzzy gamma operators.

**Fussy OR:** is equivalent to the Boolean OR (logical
union) where the output membership values are controlled
by the maximum values of any of the input maps for any
particular location. The fuzzy OR is defined as:

\[
\mu_{\text{combination}} = \text{MIN}(\mu_A, \mu_B, \mu_C, \ldots, \mu_D) \tag{3}
\]

Using this operator, the combined membership value at
any location is limited only by the most suitable of the
evidence maps. This operator in some circumstances
might be reasonable for mineral potential mapping, where
favorable indicators of mineralization are rare and the
presence of any positive evidence may be sufficient to
suggest favorability.

Using the fuzzy AND or fuzzy OR, a membership of
a single piece of evidence controls the output value. The
following operators combine the effect of two or more
pieces of evidence in a blended result so that each data
source has some effect on the output.

**Fuzzy algebraic product:** The combined membership
function is defined as:

\[
\mu_{\text{combination}} = \prod_{i=1}^{n} \mu_i \tag{4}
\]

Where, \( \mu_i \) is the fuzzy membership function for the \( i \)th
map and \( i = 1, 2, \ldots n \) maps to be integrated. The combined
fuzzy membership values tend to be very small with this
operator, due to the effect of multiplying several numbers
less than 1. The output is always smaller than, or equal to
the smallest contributing membership value and is
therefore decrease.

**Fuzzy algebraic sum:** This operator is complementary to
fuzzy algebraic product and defined as:

\[
\mu_{\text{combination}} = 1 - \sum_{i=1}^{n} (1 - \mu_i) \tag{5}
\]

The result is always larger (equal to) the largest
contributing fuzzy membership value. The effect is there
increative. Two pieces of evidence that both favor a
hypothesis reinforce one another and the combined
evidence is more supportive than either piece of evidence
taken individually. The increase effect combining
several favorable pieces of evidence is automatically
limited by maximum value of 1, 0 which can never be
exceeded.

**Fuzzy gamma operation:** This is defined in terms of the
fuzzy algebraic product and fuzzy algebraic sum as:

\[
\mu_{\text{combination}} = \left( \prod_{i=1}^{n} \mu_i \right)^{\gamma} \left( \prod_{i=1}^{n} (1 - \mu_i) \right)^{\gamma} \tag{6}
\]
Where, $\gamma$ is a parameter chosen in the range zero to one. When $\gamma$ is 1, the combination is the same as the fuzzy algebraic sum and $\gamma$ is 0, the combination equals the fuzzy algebraic product. Judicious choice of $\gamma$ produces output values that ensure a flexible compromise between the increasing tendencies of the fuzzy algebraic sum and the decreasing effects of the fuzzy product (Fig. 1).

APPLICATIONS OF FUZZY LOGIC IN NORTHWESTERN YUNNAN

Yunnan province is a geological complex region endowed with various forms of mineral formations. At least five different styles of lead-zinc-silver mineralisation occur in Yunnan.\textsuperscript{[30]}

- With copper in sedimentary exhalative (Sedex) deposits (e.g., Tertiary Jingdian/Laping deposit) and Mississippi Valley Type (MVT) deposits (e.g., Carboniferous Qilinchang deposit).
- With copper in volcanic-hosted massive sulphide (VHMS) deposits (e.g., Carboniferous Laocang deposit and the recently discovered Carboniferous Dapingzhang deposit).
- With copper and tin in lead-zinc skarn deposits (e.g., Triassic Dadongchang and Hongshan deposits).
- With precious metals in veins and porphyry deposits (e.g., Beiya deposit).
- With tin and copper in Sn-W vein and skarn deposits (e.g., Mengzimao).

Table 1: Fuzzy membership scores for the various factors

<table>
<thead>
<tr>
<th>Lithology</th>
<th>Copper</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>T31 BS+L</td>
<td>0.90</td>
<td>0.91</td>
</tr>
<tr>
<td>C-11</td>
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<td>0.20</td>
</tr>
<tr>
<td>D2-3</td>
<td>0.19</td>
<td>0.30</td>
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<tr>
<td>E1-2</td>
<td>0.80</td>
<td>0.50</td>
</tr>
<tr>
<td>E7</td>
<td>0.40</td>
<td>0.55</td>
</tr>
<tr>
<td>H1-2</td>
<td>0.55</td>
<td>0.60</td>
</tr>
<tr>
<td>RKS1b</td>
<td>0.40</td>
<td>0.70</td>
</tr>
<tr>
<td>K1</td>
<td>0.85</td>
<td>0.80</td>
</tr>
<tr>
<td>KFe1/2</td>
<td>0.40</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Silver | Lead | Lineament

<table>
<thead>
<tr>
<th>Classes</th>
<th>Score</th>
<th>Classes</th>
<th>Score</th>
<th>Classes</th>
<th>Score</th>
</tr>
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<tbody>
<tr>
<td>19.56</td>
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<td>19.56</td>
<td>0.01</td>
<td>549.31</td>
<td>0.00</td>
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<td>28.34</td>
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<td>47.90</td>
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<td>0.65</td>
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<tr>
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<tr>
<td>1970.92</td>
<td>0.90</td>
<td>1970.92</td>
<td>0.90</td>
<td>15514.38</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Fuzzy geo-variables for predicting hydrothermal mineral occurrences: The following are the geological criteria considered for the study;

Lineament: Extracted from Lansat images of the area.

Lithology: Reclassified based on the geological time scale (Fig. 2).

Geochemical data: Stream sediment sample result of Cu, Au, Ag, Pb-Zn.

Fuzzy membership scores

Lithology: The membership scores for lithologies were determined subjectively based on their relationship to the mineralization. The highest score was given to the Triassic Gocchung series and the rest of the scores assigned to others shown on Table 1.

Lineament: The lineament map consists of both ring structures and linear features. The line-density was
Fig. 2: Reclassified lithology map of NW Yunnan. The symbols adopted by Yunnan Geological Bureau. The classification for prediction purposes only

Fig. 3: The evidence map for the various factors
Fig. 4: The map for the various operations adopted by An et al.[11]

Fig. 5: Fuzzy favorability map for the occurrence of Au, Ag, Cu, Pb-Zn overlaid with known mineral occurrence calculated using MapGIS software developed by CUG and after, exported to Arc GIS and contoured using universal Kriging. The various scores were assigned to them based on their relationship to the known mineralization shown on Table 1.

Geochemical: The geochemical data of the various elements were also contoured using the same universal Kriging function and the contour values depict their assay values from stream sediment analysis. The membership values were assigned with respect to these values where the highest membership scores were assigned to the highest assay values also shown Table 1. Fuzzy evidence maps: The evidence maps for the various factors were extracted shown in Fig. 3.

Integration of fuzzy sets: With the various scores assigned to the fuzzy sets, the various operators were used to integrate them representing intermediate stages. The Fig. 4 shows the various combinations.

Fuzzy favorability map for hydrothermal Cu, Au, Ag Pb-Zn: The Fig. 5 below shows the inference network for producing the favorability map for the occurrences of the above mineralization. Two of the intermediate stages Sum and PRD were combined with a gamma value of 0.9 and this resulted in producing favorable areas of

Fig. 6: Inference network for producing hydrothermal Au, Ag, Cu, Pb-Zn, in northwestern Yunnan
the above mineralization. Combination of the OR and AND also produced the same result with the same gamma value but not shown due to lack of space.

The overlay of the known occurrences with the predicted result has demonstrated a successful implementation of fuzzy logic in northwestern Yunnan for producing favorable location of hydrothermal mineralization of Cu, Au, Ag and Pb-Zn (Fig. 6). The result covers more than 70% of the known mineral occurrences and has also predicted new area for follow up.

REFERENCES