A New Model for Coupled Rock-coal Deformation and Gas Leak Flow in Mining Engineering

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Abstract: Underground coal seams are mostly distributed in adjacent multi-layers in most countries of the world, especially in China. Many problems in mining engineering and rock engineering remain unsolved and these problems, in essence, are linked with rock mass deformation and gas leak flow by Sun. From the viewpoint of interaction mechanics for solid and gas, a coupled mathematical model is presented for coal/rock visco-elastic-deformation and gas leak flow in parallel deformable coal seams. The new mathematical model consists of equations for gas leak flow, equations for solid coal/rock mass visco-elastic-deformation and boundary/initial conditions.

Key words: Coupled model, visco-elastic-deformation, gas leak flow, mining safety, parallel coal seams

INTRODUCTION

Underground coal seams are mostly distributed in multiple adjacent layers in most countries of the world, especially in China. Many hard problems in mining engineering remain unsolved such as prediction and control of methane gas emission from coal faces during mining, safety mining area prediction issues related to coal and gas outbursts in protective layer mining, prediction of methane gas drainage rate near to the mining seam and from original coal seams, as well as prediction of gas flow rate in multiple coal seams. These problems, in their essence, are intimately linked with solid deformation and gas leak flow by Sun (1998, 2002, 2003a,b 2004a,b, 2005).

Under the action of a pore gas pressure gradient in the multi-coal-seam system, gas flow emission occurs near to the mining seam, which in turn leaks through the layer of intercalation with lower permeability and flows into the mining coal faces or gas drainage bores. This is referred to as gas leak flow by Sun (1998). As gas flows during coal mining and gas drainage, the effective stress of the solid skeleton of the visco-elastic-coal/rock mass changes and the pore pressure in the coal seam or gas drainage bores also changes, which lead to a gas flow through a network made of micro-pores and micro-cracks. Moreover, the gas flowing through coal/rock cracks also alters with the variance of the gas permeability in Sommerton et al. (1975), Sun (1990) and Zhao (1994a, b). Therefore, the interaction between the coal/rock mass and gas must be taken seriously into account for simulating the action of methane gas flow.

It is well established that the description of the interaction between the visco-elastic-coal/rock mass and gas in the gas leak flow system is difficult. Nonetheless, the problem of gas leak flow in multiple coal seams must be studied for the sake of coal mining safety and coal gas draining. In this paper, the gas leak flow systems for coupled coal/rock mass deformation and gas seepage in double coal seams are studied with theory of solid mechanics and fundamentals of coal gas seepage. A coupled mathematical model for visco-elastic deformation and gas leak flow in multiple coal seams is presented for processing gas-leak-flow-related problems in mining engineering, gas drainage engineering and mining safety engineering.

PHYSICAL ASSUMPTIONS

The theory of coupled solid visco-elastic-deformation and gas leak flow involves seepage mechanics, rock mechanics and mining science, etc. In this paper, the gas leak flow through multiple coal seams is assumed to be a compressible, mixed, time-dependent flow with permeability and diffusion through anisotropic and heterogeneous deformable visco-elastic-media containing pores and crevices, subject to varying mechanical fields. The following assumptions are applied for establishing our coupled mathematical model.

We consider the interaction mechanism between coal/rock mass visco-elastic-deformation and gas leak flow through double parallel coal seams. The direction of gas leak flow through a weak permeability intercalation is vertical and the direction of gas flow in coal seams is horizontal, as shown in Fig. 1.
The gas flow process is divided into two stages. Firstly, gas flows from micro-pores into crevices, which can be formulated with Eq. 1 by Fick's law of diffusion in Bear (1972). Secondly, gas flows from crevices into free space of underground mining, which can be formulated with Eq.2 by Darcy’s law of nonlinear seepage in Zhao et al. (1994). Thus we have:

\[ Q_{m} = D_{m} \nabla W_{m}, \quad (m = 1, 2) \]  
\[ Q_{m} = T_{m} \nabla P_{m}, \quad (m = 1, 2) \]  
\[ T_{m} = a_{m} e^{c_{m} s_{m}^{\gamma_{m}}}, \quad (m = 1, 2, 3) \]

Where, the subscript m denotes coal seams or the intercalation level with m=1 representing the bottom coal seam, m = 2 standing for the top one and m = 3 symbolizing the intercalation level; \( Q_{m} \) and \( Q_{m} \) are the gas diffusion and the gas seepage, respectively; \( D_{m} \) is the gas diffusion coefficient; \( W_{m} \) describes the desorbable gas content; \( P_{m} \) is the gas permeability and \( P_{m} \) is the pore pressure. It should be pointed out that the equation for the gas permeability (Eq. 3) is an empirical model built from our real experimental data with the \( a_{m} \) (j = 0, 1, 2, 3) being fit constants determined by real experiments and \( \Theta_{m} \) denoting the total effective stress which need be calculated by Sun (1998, 2002) and Zhao (1994 a,b).

The coal gas is treated in an ideal sense and its flow is considered as an isothermal process. To be more concrete, the gas in the coal seam exists in two different states: the adsorption state and the desorption one. In other words, the gas is composed of both free (desorbable) and adsorbed gas.

The adsorbed gas content is formulated with Eq. 4 by Langmuir’s isothermal law by Sun (1998).

\[ M_{m} = \frac{a_{m} b_{m} \rho_{m}}{1 + b_{m} \rho_{m}} \quad (m = 1, 2) \]

Where, \( M_{m} \) is the adsorbed gas content, \( b_{m} \) is the adsorption gas coefficient and \( a_{m} \) is the maximum adsorption gas coefficient and \( P_{m} \) is the pore pressure and \( \rho_{m} \) is the methane gas density.

The two different states of the gas are not mutually exclusive. In fact, the free gas may be adsorbed into the adsorbed one and vice versa.

The average desorbable gas content in the coal seam \( m = 1, 2 \) is regarded as the difference between the initial gas content and the adsorbed one by Sun (1998, 2002), that is

\[ M_{m} = M_{m} - \int_{t_{i}}^{t} \frac{d(M_{m} - M_{m})}{dt} R(t) dt \]

Where, \( M_{m} \) is the average desorbable gas content, \( M_{m} \) is the gas content at initial time \( t_{i} \) and \( R(t) \) describes the gas desorbing rate from the adsorption state in coal.

The coal seam is saturated with single phase of methane gas in Jaeger and Cook (1979). Thus we have

\[ M_{m} = n_{m} \rho_{m} \]

Where, \( M_{m} \) is the free gas content and \( n_{m} \) is the porosity; \( \rho_{m} \) is the methane gas density.

Methane gas is considered as ideal gas. The seepage of methane gas can be dealt with isothermal process. Hence, the equation of the state of methane gas is

\[ \rho_{m} = p_{m}/RT \]

Where RT is methane gas constant.

The coal/rock mass is linearly visco-elastic so that the solid visco-elastic-deformation can be formulated with Eq. 7 and 8 by Sun (1998, 2002):

\[ U_{m,ij} * d[\lambda_{m}(t) + G_{m}(t)] + U_{m,ij} \]
\[ dG_{m}(t) + F_{m} + (\alpha_{m} P_{m}) = 0 \]
\[ \varepsilon_{m} = \frac{U_{m,ij}}{E_{ij}} ; (m = 1, 2; i, j = 1, 2, 3) \]

Where, \( U_{m,ij} \) is the displacement function, \( \lambda_{m} \) is the Lame-constant, \( G_{m} \) is the shear modulus, \( a_{ij} \) is the effective stress coefficient, \( F_{m} \) is the free term for the stress and \( \varepsilon_{m} \) is the deformation rate. Moreover, for the mathematical symbol * is convolution integral, so we have the definition for this convolution integral as following by Sun (1998).

\[ U_{m,ij} * dG_{m}(t) = U_{m,ij}(t) G_{m}(t) + \int_{0}^{t} U_{m,ij}(t) \]
\[ (t - \xi)(dG_{m}(\xi)/d\xi) d\xi \]
\[ t \in (-\infty, +\infty) \]
Where, $t$ is time and $a_m$ is named as the effective stress coefficient, which is fit with the Eq. 9 by experimental data by Sun (1998).

$$a_m = b_m - b_m \Theta_m + b_m P_m - b_m \Theta_m P_m$$  \hspace{1em} (m = 1,2)  \hspace{1em} (9)

Where, $\Theta_m$ is the total stress and $b_{mq}$ are fit constants.

By substituting $\Theta_m$ with $\Theta_m'_{eq}$ the total effective stress (Eq. 10), we have Eq. 11.

$$\Theta_m = \Theta_m'_{eq} + 3a_m P_m$$ \hspace{1em} (m = 1,2)  \hspace{1em} (10)

$$a_m = \frac{b_m P_m - b_m \Theta_m P_m}{1 + 3b_m + 3b_m P_m^2}$$ \hspace{1em} (m = 1,2)  \hspace{1em} (11)

The deformation of the coal/rock mass with pores and crevices saturated by gas is equal to the deformation of the solid skeleton plus the deformation of pores and crevices by Bear (1972), i.e.,

$$\alpha_{mb} = (1 - n_m \alpha_m + n_m \alpha_m P_m)$$  \hspace{1em} (m = 1,2)  \hspace{1em} (12)

Where, $\alpha_{mb}$ is the bulk strain, $\alpha_m$ and $\alpha_m P_m$ are respectively the solid mass deformation rate and the pore deformation rate and $n_m$ is the porosity.

Generally, the deformation of the solid skeleton is far less significant than the deformation of pores and crevices; therefore, the deformation of the coal/rock mass is often represented by Eq. 13.

$$\alpha_{mb} = n_m \alpha_m P_m$$ \hspace{1em} (m = 1,2)  \hspace{1em} (13)

The continuity equations for gas leak flow are formulated as following by Sun (1998):

$$\text{div} (\rho_g \vec{Q}_g) = -T_{eq} \frac{T_{eq}}{2b_m b_j} (P_j^2 - P_j^3)$$

$$\frac{\partial M_{mf}}{\partial t} + \frac{\partial M_{md}}{\partial t}, (m = 1,2)$$  \hspace{1em} (14)

Where, $\rho_g$ is the methane gas density; $Q_g$ is the gas flow rate; $b_j$ is the average thickness of the coal seam $m$, with $b_j$ represents for the thickness of the intercalation level; $M_{mf}$ is the free gas content; $M_{md}$ is the desorbable average gas content in the coal seam $m$ and $t$ is the gas flow time.

**SOLID-GAS COUPLED MATHEMATICAL MODELS**

Based on the above assumptions, we propose a coupled mathematical model for the interaction between coal/rock mass and gas in the gas leak flow system. The model consists of three components: (1) equations for gas leak flow; (2) equations for coal/rock mass deformation; and (3) boundary and initial conditions describing the system.

**Equations for gas leak flow:** The continuity equations for gas leak flow are formulated by Eq. 14. By substituting Eq. 2, 5 and 6 into Eq. 14, we have Eq. 15, which is expressed in tensor notation in the Cartesian coordinate system by Sun (1998). In Eq. 15, the deformable porosity is considered as $n_m = n_{inf} - e_m$ where $n_{inf}$ is initial porosity, $e_m$ is porosity deformation rate in the coal seam $m$.

$$T_{eq}^2 P_m \frac{\partial P_m}{\partial t} - 2P_m \frac{\partial e_m}{\partial t} = \text{Sun}(P_m, t)$$  \hspace{1em} (15)

$$\text{Sun}(P_m, t) = n_m \frac{a_m b_j (1 - e_m - e_m^2)}{P_m (1 + b_m P_m)^2}$$  \hspace{1em} (16)

$$\kappa_m = \frac{12}{a_m} \frac{D_p}{\sqrt{n_m} P_m}$$ \hspace{1em} (m = 1, 2, i, j = 1, 2, 3)  \hspace{1em} (17)

Where, Sun($P$, $t$) is a state function to describe methane gas flow, which is named as Sun function. $d_i$ is the average coal micro-grain size; $P$ equals one atmosphere and $D_p$ is the gas diffusion coefficient when the pore pressure equals $P_m$. In general, the gas flow grows fast as the value of the Sun function increases and vice versa by Sun (1998). The full deduction detail of the Sun function is given in Sun (1998, 2002, 2005).

**Equations for coal/rock mass deformation:** The equations for coal/rock mass visco-elastic-deformation are obtained by Sun (1998).

$$U_{m,ji} \delta \lambda_m (t) + G_{m,ij} (t) +$$

$$U_{m,ji} \delta G_m (t) + F_m + (a_m P_m)_{inf} = 0$$  \hspace{1em} (18)

$$e_m = U_{m,ji}$$ \hspace{1em} (m = 1, 2, i, j = 1, 2, 3)  \hspace{1em} (19)

Where, $U_{m,ij}$ is the displacement function; $\lambda_m$ is the Lame constant of the coal seam, $G_m$ is the shear modulus of the coal seam, $a_m$ is the effective stress coefficient, $F_m$ is the free term for the stress and $e_m$ is the deformation rate.

**Solid-gas coupled models for the gas leak flow system:**

With the continuity equations for gas leak flow and the equations for coal/rock mass visco-elastic-deformation provided, the coupled mathematical model
for the interaction between visco-elastic-coal/rock mass and gas in the gas leak flow system can be described with the boundary and initial conditions (Eq. 20).

Where, \( P_m \) is the initial pore pressure; \( \rho_m \) is the pressure in free space; \( c_m \) is the initial deformation rate; \( U_m \) is the boundary value for displacement and \( Q_{m} \) is the quality vector for gas outflow from boundary surface.

It can be gotten from Eq. 20 that the Eq. 21 of the coupled model for coal/rock mass visco-elastic-deformation and gas leak flow when the deformation for up-seam and down seam is the same one and the gas desorption due to pore pressure is also completed in time by equations for gas leak flow.

Moreover, it can be gotten from Eq. 21 that the Eq. 22 of the coupled model for coal/rock mass visco-elastic-deformation and gas leak flow when the deformation for up-seam and down seam is the same one and the gas desorption due to pore pressure is also completed in time by equations for gas leak flow.

\[
(T_p P_{11}, \lambda) + T_p \frac{P_{11} - P_{11}}{\beta_{m1}} = \frac{\partial P_{11}}{\partial t} - 2P_{11} \frac{\partial c_{i1}}{\partial t} + Q_{11}
\]

\[
(T_p P_{12}, \lambda) - T_p \frac{P_{12} - P_{12}}{\beta_{m2}} = \frac{\partial P_{12}}{\partial t} - 2P_{12} \frac{\partial c_{i2}}{\partial t} + Q_{12}
\]

\[
U_{m,i} \frac{d[x_{m}(t) + c_{m}(t)]}{dt} + U_{m,i} \frac{dG_{m}(t) + F_i}{(a_m P_m)_i} i = 0
\]

\[
e_n = U_{m,i} (m = 1, 2, i, j = 1, 2, 3)
\]

\[
U_{m,i} \frac{dG_{m}(t)}{dt} = U_{m,i} (m)(G_m(0) + \int_0^t U_{m,i} (t - t) dG_m(\xi) d\xi)
\]

\[
t \in (0, +\infty)
\]

\[
P_m(x, y, z, 0) = P_m(x, y, z) P_m(x, y, z, 0) \mid_{t=0} = P_s
\]

\[
T_m \frac{\partial P_m}{\partial n} (x, y, z, 0) \mid_{t=0} = Q_m(x, y, z, t)
\]

\[
e(x, y, z) \mid_{t=0} = e_0
\]

\[
U_i(x, y, z) \mid_{t=0} = 0
\]

\[
U_i(x, y, z) \mid_{t=0} = \Delta_i(t)
\]

(20)

Where, it can be obtained from Sun function (16) that the simplification formula (23) of Sun function when \( t \to +\infty \).

\[
\text{Sun}(P_m) = \frac{n_m a_m}{P_m (1 + b_m P_m)}
\]

(23)

Furthermore, it can be derived from the equation (22) that the numerical coupled modeling for coal/rock mass elastic-deformation and gas leak flow by Sun (1998, 2002, 2004, 2005) and Sun and Wan (2004).

CONCLUSIONS

In this study, a mathematical model for coupled solid coal/rock mass visco-elastic-deformation and gas leak flow in parallel coal seams is presented from a new perspective of solid-gas interaction. The model consists of equations for gas leak flow, equations for solid coal/rock mass deformations and boundary/initial conditions. It is characterized by:

- The main consolidation of gas leak flow through visco-elastic-coal/rock mass due to gas drainage or coal mining is described by linear-visco-elastic deformation in the coal/rock mass;

- The mechanism for gas leak flow with both seepage and diffusion is given by equations of gas leak flow.
and the gas description due to pore pressure is also given by equations for gas leak flow and

- The interaction between gas leak flow and coal/rock mass visco-elastic-deformations is also described by the coupled model.

The interaction between visco-elastic deformation of coal/rock mass and gas leak flow has been thoroughly studied with the built equations. And the coupled modeling for elastic deformation of coal/rock mass and gas leak flow can be derived from the equations (20-22). Numerical simulation of this modeling has been performed with the SIP method with robustness and efficiency by Sun (1998, 2002, 2004) and Sun and Wan (2004).

Our solid-gas coupled model can be applied to a wide range of gas-leak-flow-related problems in mining engineering, gas drainage engineering and mining safety engineering. With the help of numerical simulation results, we can analyze the effective protective range at the down-proximity seam quantitatively rather than qualitatively. We can also predict the change of pore pressure along the run of the coal seams and the distribution tendency in the down-proximate seam at any time when the up-protective layer is being exploited. This quantitative analysis can guide safety exploitation for mining and protective measure for outburst, etc.

**NOMENCLATURE**

- $Q_{m}$: Gas diffusion in the coal seam m
- $Q_{n}$: Gas seepage in the coal seam m
- $D_{m}$: Gas diffusion coefficient in the coal seam m
- $T_{m}$: Gas permeability coefficient in the coal seam m
- $w_{m}$: Desorbable gas content in coal seam m
- $P_{m}$: Pore pressure in the coal seam m
- $\Theta_{m}$: Total effective stress in the coal seam m
- $a_{m}$: Crystals (j = 0, 1, 2, 3) for the coal seam m
- $\Delta$: Gradient
- $M_{m0}$: Desorbable average gas content in the coal seam m
- $M_{m}$: Absorbed gas content in the coal seam m
- $M_{m0}$: Gas content in the coal seam m at initial time $t_{0}$
- $M_{m}$: Free gas content in the coal seam m
- $a_{m}$: Maximum adsorption gas coefficient in the coal seam m
- $b_{m}$: Adsorption gas coefficient in the coal seam m
- $R(t)$: Gas desorbing rate from adsorption state in coal
- $e_{m}$: Deformation rate in the coal seam m
- $e_{n}$: Initial deformation rate in the coal seam m
- $\lambda_{m}$: Lame-constant of the coal seam m
- $G_{m}$: Shear modulus of the coal seam m
- $\sigma_{m}$: Effective stress coefficient of pore pressure in the coal seam m
- $\Theta_{m}$: Overall stress in the coal seim m
- $b_{m}$: Fit constants ($j = 1, 2, 3, 4$)
- $\sigma_{n}$: Bulk strain in the coal seam m
- $\sigma_{m}$: Solid mass deformation rate in the coal seam m
- $\sigma_{m}$: Pore deformation rate in the coal seam m
- $n_{m}$: Porosity in the coal seam m
- $n_{m0}$: Initial porosity in the coal seam m
- $\rho_{m}$: Methane gas density in the coal seam m
- $Q_{m}$: Gas flow rate in the coal seam m
- $T_{m}$: Gas permeability of the coal seam m, with $T_{j}$ for intercalation
- $b_{m}$: Average thickness of the coal seam m, with $b_{j}$ for intercalation
- $t$: Time
- $D_{m}$: Gas diffusion coefficient at pore pressure $P_{m}$ with $P_{a} = 1$ atmosphere
- $d_{m}$: Average coal micro-grain size of the coal seam m
- $F_{m}$: Free terms for stress in the coal seam m
- $U_{m}$: Displacement function in the coal seam m
- $P_{m}$: Initial pore pressure in the coal seam m
- $P_{a}$: Pressure in free space
- $\rho_{a}$: One standard atmosphere
- $U_{m}$: Boundary value for displacement in the coal seam m
- $Q_{m}$: Quality vector for gas outflow from boundary surface in the coal seam m
- $\gamma_{m}$: Consistency of coal in the coal seam m
- $M_{m}$: Gas content in the coal seam m
- $q_{m}$: Overburden stress

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