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Influence of Aspect Ratio on Natural Convection in a Partially Porous Enclosure

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Abstract: The objective of this research is the study the influence of aspect ratio A_1 of fluid portion compared to the height of cavity, on the natural convection in a partially porous enclosure. A cylindrical heat source maintained at a uniform temperature is introduced into porous portion. The equations which describe the thermal transfer and the hydrodynamic flow are described by the Navier-Stokes equations modified by using the extension of Darcy-Brinkman and the energy equation. The results obtained are in the form of isotherm's distribution, stream's functions, the local and the average Nusselt numbers and the pressure. Correlations between these parameters and the aspect ratio are considered.

Key words: Natural convection, porous, saturated, enclosure, fluid, Darcy, Brinkman, aspect ratio

INTRODUCTION

Natural convection in porous media is of practical interest in several sciences, engineering, agriculture, energy's stocking system, building and geothermal science. In the theoretical domain, several works related to stability examination were carried out by Nield (1991), Cheng (1978) and Combarous and Bories (1977). We also find the problems of thermal sources flooded in a porous media studied by Bejan (1979), Hickox (1981) and Polikakos and Bejan (1983). An important number of studies of natural convection in confined and semi-confined porous media, especially those of Bejan and Khair (1985), Weber (1975) and Masuouka *et al.* (1981). Convective processes of fluid flow and associated heat transfer in porous cavities have been studied extensively. These studies focus on the thermal convection performance within a heated porous cavity for different geometrical parameters (aspect ratio), heating mode (isothermal). The two dimensional free convection within a porous square cavity heated on one vertical side and cooled on the opposite side, while the horizontal walls are adiabatic, is currently considered a reference or benchmark solution for verifying other solution procedures. For a list of the basic references concerning this subject, we refer to the review articles by Ingham and Pop (2005), Vafai (2000), Pop and Ingham (2001), Bejan and Kraus (2003), Ingham *et al.* (2004) and Bejan *et al.* (2004). The Brinkman-extended Darcy model has been considered by Tong and Subramanian (1985) and Lauriat and Prasad (1987) to examine the buoyancy effects on free convection in a vertical cavity.

The bibliographical study shows that the works of natural convection in partially porous media are less numerous.

In the present research we study how aspect ratio A_1, influences the thermal transfer and the hydrodynamic flow in partially porous cavity where a cylindrical heat source is introduced into the porous portion. Our objective is to visualize which mode of transfer is favored in two portions of cavity, with the variation of aspect ratio A_1.

PHYSICAL MODEL AND FORMULATION

We suggest studying the natural convection in an enclosure, in presence of heat source submerged in porous region subjected to a high uniform temperature T_h. The bottom and sides of enclosure are assumed to be adiabatic and the top is maintained to low uniform temperature T_c. This study is the one application object to the creation of a climate favorable to pushed plants in an agricultural greenhouse in an arid region.

Hypotheses: We considered the following simplifying:

- The porous media is assumed saturated.
- The porous-fluid interface is supposed impermeable.
- The flow is two-dimensional, laminar and incompressible.
- The fluid is Newtonian and the physical properties are constant.
- Boussinesq approximation is adapted.

Mathematical formulation: The flow field is governed by the Navier-Stokes equations in the fluid region, the

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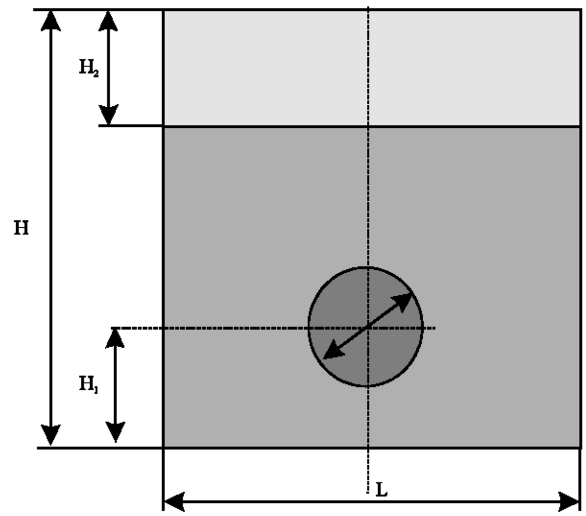

Fig. 1: Physical model

Navier-Stokes equations modified by using the extension of Darcy-Brinkman (Tanmay *et al*., 2006) in the porous region and the thermal field by the energy equation.

In Cartesian Coordinates and considering the above simplifying hypotheses and the symmetry of the model (Fig. 1), the nondimensional equations are:

Fluid region

Mass conservation equation

$$\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}=0 \tag{1}$$

Momentum conservation equation

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{\partial P}{\partial x}+\Pr\nabla^2 U \tag{2}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{\partial P}{\partial y}+\Pr\nabla^2 V+\mathrm{Ra}\Pr T \tag{3}$$

Energy conservation equation

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial y}=\nabla^2 T \tag{4}$$

Porous region

Mass conservation equation

$$\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}=0 \tag{5}$$

Table 1: Appropriate boundary conditions

Faces	U	V	T
Inferior	0	0	$\frac{\partial T}{\partial y}=0$
Superior	0	0	0
Side	0	0	$\frac{\partial T}{\partial x}=0$
Source	0	0	1
Interface (fluid-porous)	0	0	$-\lambda_p\frac{\partial T_p}{\partial y}=\lambda_f\frac{\partial T_f}{\partial y}$, $T_p=T_f$

Momentum conservation equation

$$\varepsilon\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{\partial P}{\partial x}-\frac{\Pr}{\mathrm{Da}}\varepsilon^2 U+R_v\Pr\varepsilon^2\nabla^2 U \tag{6}$$

$$\varepsilon\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{\partial P}{\partial y}-\frac{\Pr}{\mathrm{Da}}\varepsilon^2 V +R_v\Pr\varepsilon^2\nabla^2 V+\mathrm{Ra}\Pr\varepsilon^2 T \tag{7}$$

Energy conservation equation

$$\sigma\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial y}=k_r\nabla^2 T \tag{8}$$

The governing equations are made dimensionless by adopting the following nondimensional quantities:

$$x=\frac{x^*}{H}\ ;\ y=\frac{y^*}{H}\ ;\ U=\frac{H}{\alpha}U^*\ ;\ V=\frac{H}{\alpha}V^*,$$
$$t=\frac{\alpha}{H^2}t^*;\ P=\frac{\varepsilon^2H^2}{\rho_f\alpha^2}P^*;\ T=\frac{T^*-T_0}{T_H-T_C}$$

In the above system, the following nondimensional parameters appear:

$\Pr=(\nu/\alpha)_f$; $\mathrm{Ra}=g\beta H^3\Delta T/\nu_f\alpha_f$; $\mathrm{Da}=K/H^2$

The initial conditions With t =0 are:

U = V = 0 and T = 0

The appropriate boundary conditions are shown in the Table 1.

NUMERICAL METHOD

Numerical results are obtained by solving the system of unsteady differential Eq. 1-8, with appropriate boundary and initial conditions, using the finite volume method, Patankar (1980). The discretized equations are solved using the SIMPLE algorithm with a fully implicit alternating-direction Gauss-Seidel method and the efficient tri-diagonal matrix Thomas algorithm. The convergence criterion imposes a relative error inferior to 10^{-4}.

RESULTS AND DISCUSSION

To describe the influence of aspect ratio A_1 on the flow structure of natural convection in the partially porous cavity, the following parameters are fixed:

$A = 4$, $Da = 0.002$, $Pr = 0.71$, $Ra = 10^5$, $\varepsilon = 0.513$, $k_r = 1$

Figure 2 represents the distribution of isotherms for various values of aspect ratio A_1. These isotherms change pace with the variation of A_1 and become parallel when this last increases. This shows that the convective transfer mode in the fluid zone is dominating when A_1 decrease and tends towards a conductive transfer mode with the growth of this last. On the other hand, in the porous zone the convective transfer mode is more favored when the value of A_1 decreases. In the vicinity of the heat source and higher wall, the isotherms are parallel what explains the domination of the conductive transfer mode. The influence of aspect ratio also appears on temperature's values.

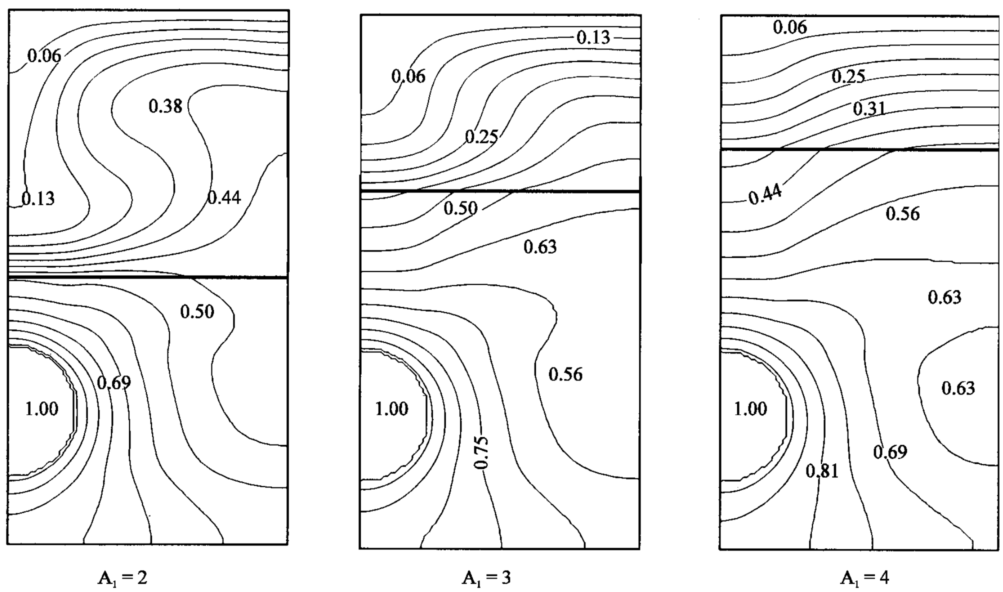

Fig. 2: Isotherms for $Ra = 10^5$, $Pr = 0.71$, $A = 4$, $Da = 0.002$, $\varepsilon = 0.513$, $kr = 1$

Figure 3 represents the distribution of streamlines for various values of A_1. In general these lines are formed in two principal cells. For $A_1 = 2$ and $A_1 = 3$, one of these cells is located in porous zone and the other in fluid zone. The latter starts to decrease with the growth of aspect ratio, until complete disappearance and is formed another zone of circulation in porous portion near the interface and of with dimensions vertical of heat source. The latter occurs starting from $A_1 = 3.5$ and develops as the aspect ratio increases until obtaining a second principal cell in the porous portion.

This distribution of streamlines shows the instability of flow development in the porous zone and the variation of aspect ratio in descending order favored the convective transfer mode in fluid region. This mode decreases with the increase in A_1 until complete domination of conductive transfer mode, which explains the distribution of the isotherms. The influence of aspect ratio also appears on the stream functions values.

Figure 4 and 5 represent the evolution of streamlines and the stream functions in two zones for various values of A_1.

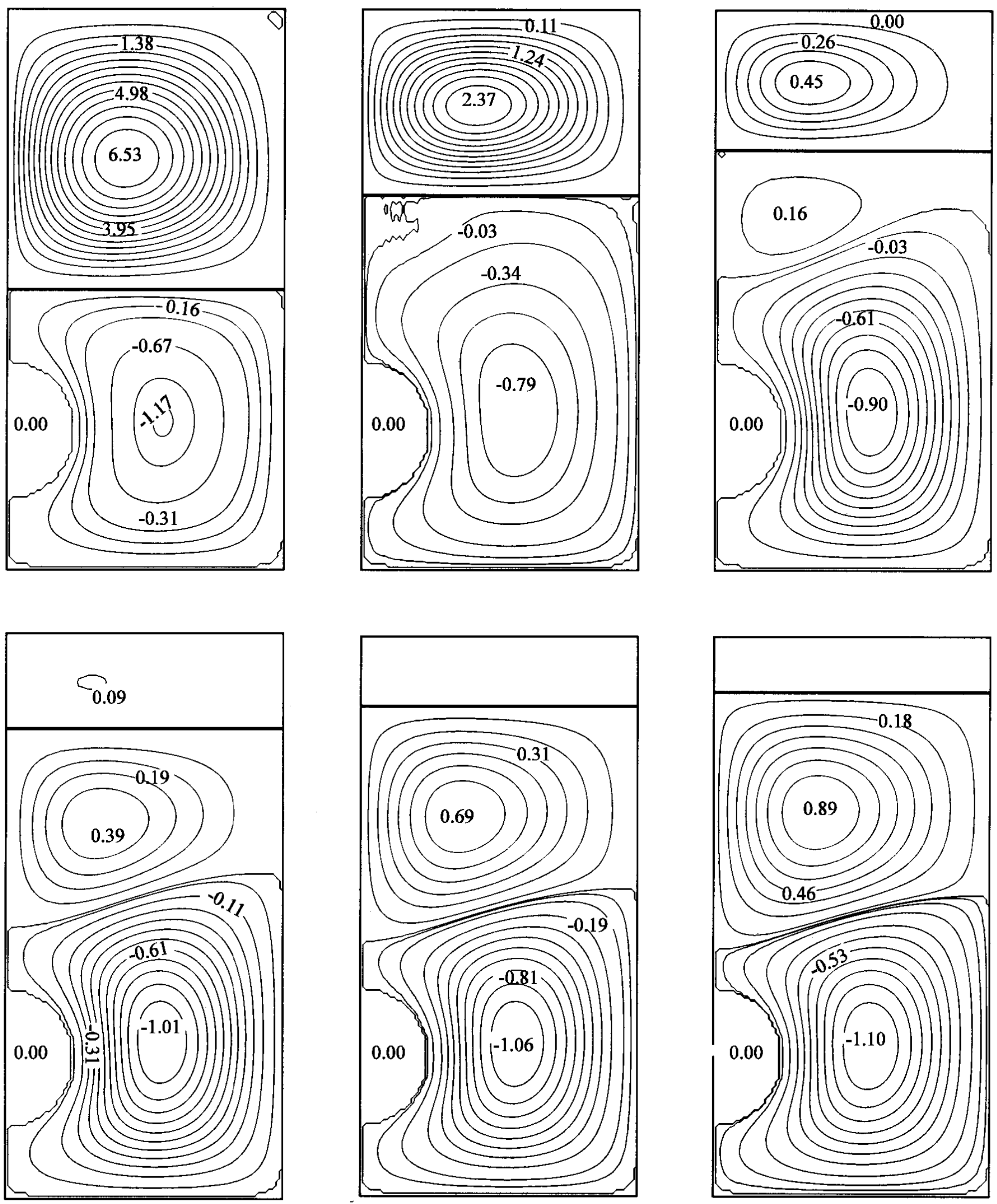

Fig. 3: Stream lines for Ra = 10^5, Pr = 0.71, A = 4, Da = 0.002, ε = 0.513, kr = 1

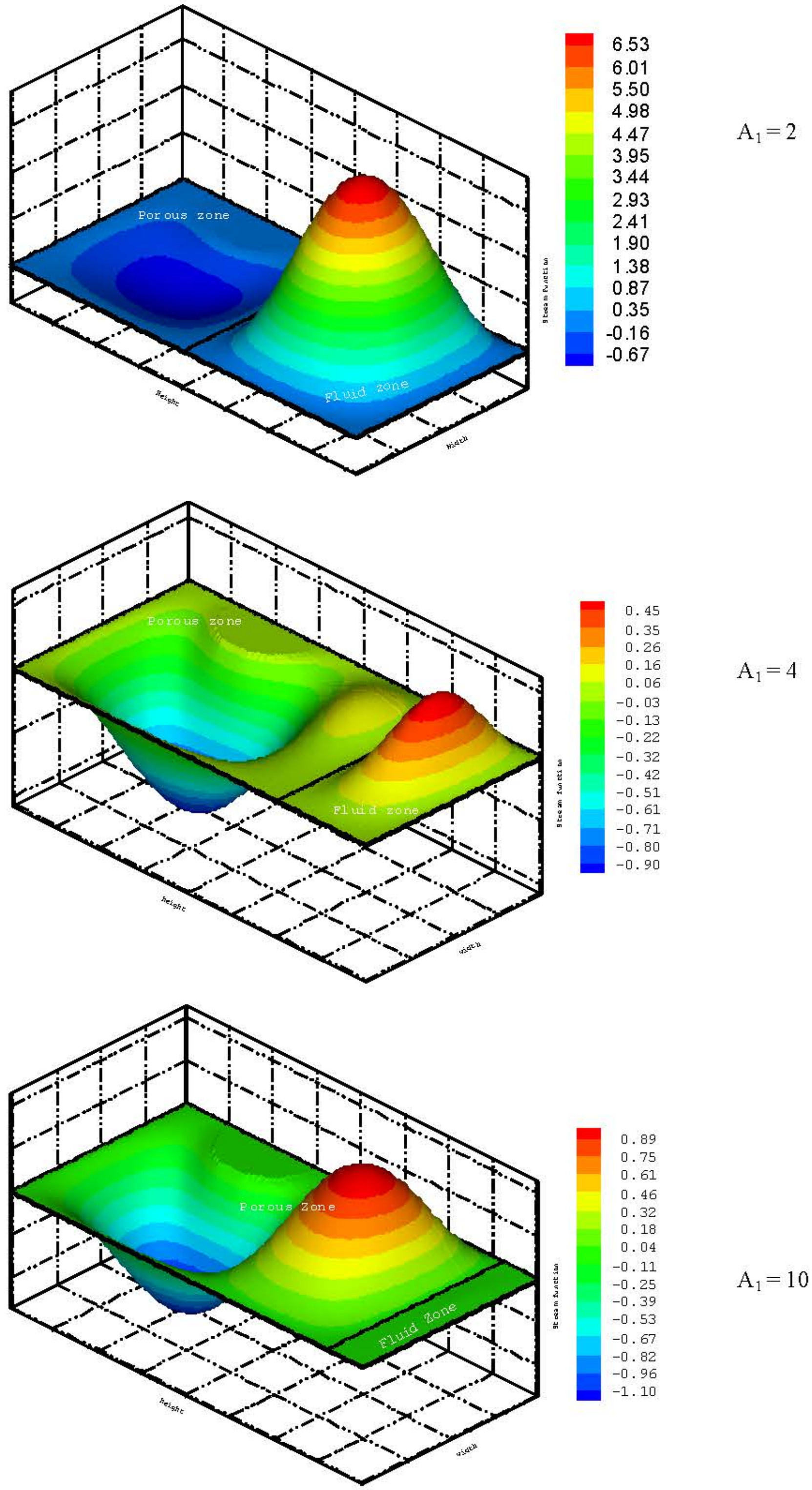

Fig. 4: Evolution of stream lines for $Ra = 10^5$, $Pr = 0.71$, $A = 4$, $Da = 0.002$, $\varepsilon = 0.513$, $kr = 1$

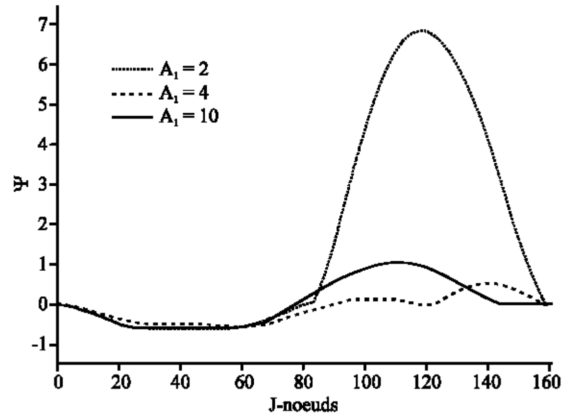

Fig. 5: Evolution of stream function for Ra = 10^5, Pr = 0.71, A = 4, Da = 0.002, ε = 0.513, kr = 1, x1 = 0.18

For A_1 = 2, streamlines develop in two principal cells distributed on two zones and the maximum values of stream functions are located in fluid region, allowing the domination of convective transfer mode in this region.

For A_1 = 4, the cell of stream lines in fluid region starts to decrease and is formed a secondary cell near to impermeable interface. The maximum value of stream functions in absolute value is located in the principal cell of porous zone, which supports the convective transfer mode in the latter. In the fluid portion, this mode decreased by a rate of about 90%.

For A_1 = 10, one observes a complete disappearance of the cell of the fluid portion and development of the second cell which is located in the porous portion until becoming principal. The conductive transfer mode is completely dominating in the fluid region and the convective mode is favored in second cell of porous region. The maximum value of stream functions is located in the latter. In this case, the convective transfer mode is completely transformed.

Figure 6 represents the evolution of the maximum stream function in absolute value according to the aspect ratio in fluid zone. This value decreases as A_1 increases. From the value of A_1 = 6, it becomes almost null. According to this evolution, we propose a correlation between maximum stream function and the aspect ratio A_1in the form:

$$|\psi|_{max} = 61.15\ e^{-1.06A_1}$$

With a correlation coefficient R = 0.99

Figure 7 represents the evolution of maximum stream function in absolute value according to the factor of form in porous zone. For $A_1<3.5$, this value varies in a decreasing way. Beyond this value, it varies in an

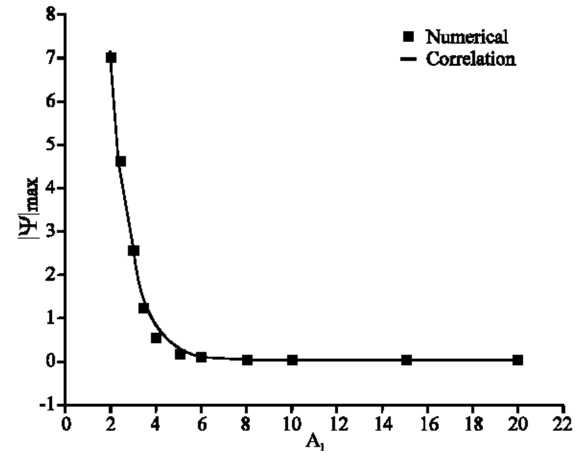

Fig. 6: Evolution of the maximum absolute value stream function in the fluid zone for: Ra = 10^5, Pr = 0.71, A = 4, Da = 0.002, ε = 0.513, kr = 1

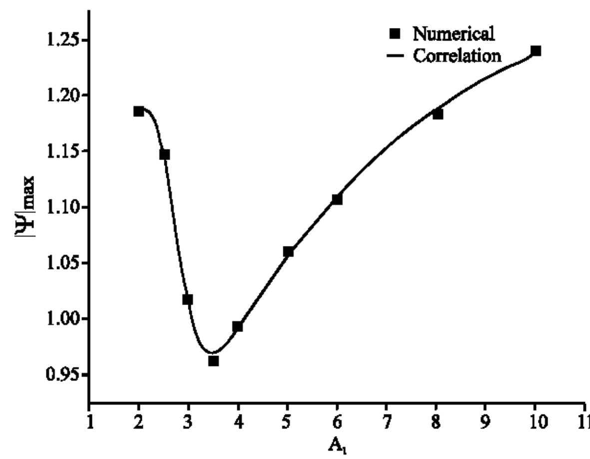

Fig. 7: Evolution of the maximum absolute value stream function in the porous zone for: Ra = 10^5, Pr = 0.71, A = 4, Da = 0.002, ε = 0.513, kr = 1

increasing way what justifies the development of the second cell of porous zone and that the maximum value of the function of current is in this cell. The extremism of maximum stream function in absolute value (3.5, 0.95) shows the birth of secondary cell. A correlation was proposed according to this evolution, in the following form:

$$|\psi|_{max} = -048-\frac{0.57}{1+1.66\,10^{(1.74A_1-5)}}+\frac{0.56}{1+0.66\,10^{-0.12A_1}}$$

With a correlation coefficient: R = 0.99

Figure 8 represents the variation of the local Nusselt number according to the width of the cavity along the interface for various values of aspect ratio. Local Nusselt

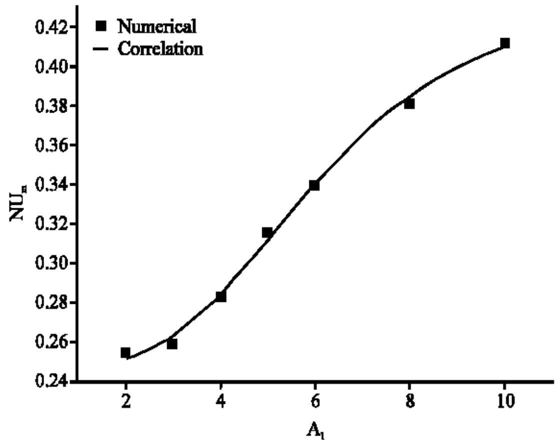

Fig. 8: Evolution of the Nusselt number for: Ra $= 10^5$, Pr = 0.71, A = 4, Da = 0.002, ε = 0.513, kr = 1

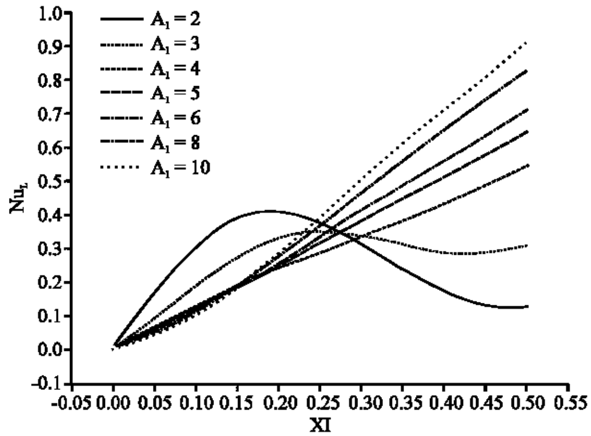

Fig. 9: Evolution of the average Nusselt number for: Ra $= 10^5$, Pr = 0.71, A = 4, Da = 0.002, ε = 0.513, kr = 1

varies in a linear and increasing way for various values of A_1. Except for the cases or A_1<4, this variation loses its linearity and becomes a curve which increases up to the Xl = 0.20 value and starts to decrease beyond this value and is stabilized near to the side wall of the with dimensions right of the cavity. For XL<0.2 and A_1 = 4, the aspect ratio does not have almost any influence on local Nusselt number.

Figure 9 represents the evolution of the average Nusselt number according to the aspect ratio. This evolution is increasing and average Nusselt varies in a way sigmoïdale in the following form:

$$Nu_m = 0.45 + \frac{0.20}{1 + 0.0023\ A_1^{3.32}}$$

With a correlation coefficient R = 0.99

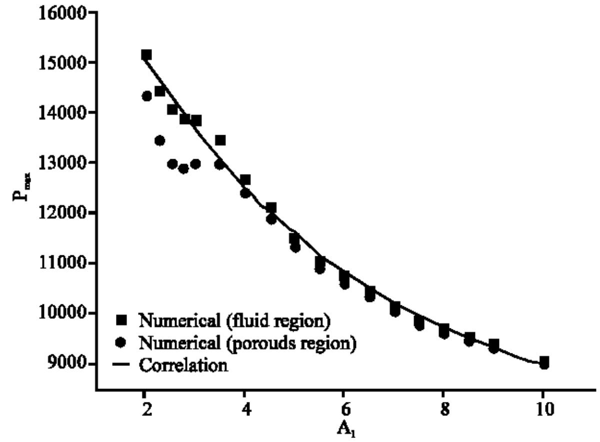

Fig. 10: Evolution of the maximum value pressure for Ra $= 10^5$, Pr = 0.71, A = 4, Da = 0.002, ε = 0.513, kr = 1

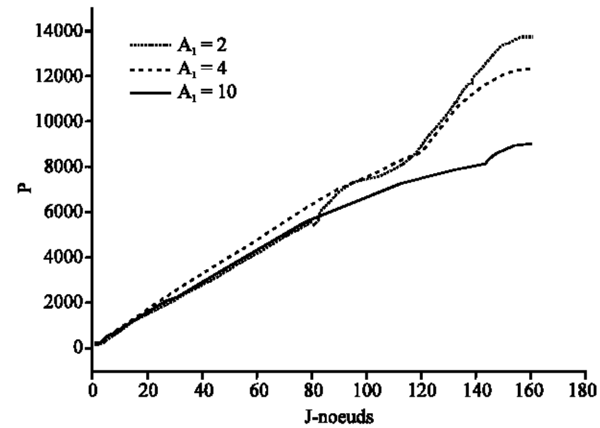

Fig. 11: Evolution of the pressure for Ra $= 10^5$, Pr = 0.71, A = 4, Da = 0.002, ε = 0.513, kr = 1, x1 = 0.18

Figure 10 represents the evolution of maximum pressure according to the aspect ratio, in the two zones. This pressure decrease with the increase in the aspect ratio. The two numerical curves have the same pace. There is a singularity of the maximum pressure in porous zone for 2<A_1<3.5, which is due to the birth of the secondary cell. The two curves vary exponentially. The following correlation is suggested:

$$P_{max} = 7525.7 + 11306.3e^{-0.20A_1}$$

With a correlation coefficient R = 0.99

Figure 11 represents the variation of the pressure along the height of the cavity for various values of the aspect ratio. The pressure varies in an increasing way. In the lower middle height, this growth is almost linear and the influence of the aspect ratio on the pressure is negligible. In the higher partition of the cavity, the evolution of the curve of the pressure loses its linearity and the influence of aspect ratio is important.

CONCLUSIONS

The natural convection in a partially porous enclosure is considered in this paper with the effect of aspect ratio A_1. A cylindrical heat source maintained at a uniform temperature is introduced into porous portion. The bottom and sides of enclosure are assumed to be adiabatic and the top is maintained to low uniform temperature T_c. The non-dimensional forms of the continuity, momentum equation based on the Navier-Stokes equations modified by using the extension of Darcy-Brinkman and the energy equation. are solved numerically using the finite volume method. The discretized equations are solved using the SIMPLE algorithm with a fully implicit alternating-direction Gauss-Seidel method and the efficient tri-diagonal matrix Thomas algorithm.

The results of this study show that the influence of aspect ratio on the thermal and hydrodynamic transfer is summarized as follows:

- The maximum stream function in absolute value changes position with the variation of aspect ratio.
- The aspect ratio influences directly of convective transfer mode or conductive transfer mode. If one triples the value of aspect ratio, the convective mode is transformed completely into conductive mode. Thus the choice of the value of this factor is based on the objective of the work and the applicability.
- The aspect ratio influences especially the pressure in the fluid zone.

NOMENCLATURE

A	:	Aspect ratio; A = H/D	[-]
A_1	:	Aspect ratio; $A_1 = H/H_2$	[-]
D	:	Diameter of the cylinder	[m]
Da	:	Darcy number	[-]
H	:	Enclosure height	[m]
H1	:	Ordinate of center of cylinder	[m]
H2	:	Height of the fluid region	[m]
K	:	Permeability	[m^2]
k_r	:	Conductivity ratio; $k_r = k_m/k_f$	[-]
L	:	Enclosure length	[m]
Nu_l	:	Local Nusselt Number	[-]
Nu_m	:	Average Nusselt Number	[-]
P	:	Pressure	[$N.m^{-2}$]
Pr	:	Prandlt number	[-]
Ra	:	Rayleigh number	[-]
R	:	Correlation coefficient	[-]
R_v	:	Viscosity ratio; $R_v = \mu_{eff}/\mu_f$	[-]
t^*	:	Time	[s]
T^*	:	Temperature	[K]
U^*	:	Velocity in x-direction	[$m.s^{-1}$]
V^*	:	Velocity in y-direction	[$m.s^{-1}$]
x^*, y^*	:	Dimensional coordinates	[m]

Greek symbols

α	:	Thermal diffusivity	[$m^2.s^{-1}$]
ϵ	:	Porosity	[-]
ν	:	Kinematic viscosity	[$m^2.s^{-1}$]
σ	:	Heat capacity ratio; $\sigma = (\rho C)_m/(\rho C)_f$	[-]
(ρC)	:	Heat capacity	[$j.m^{-3}.K^{-1}$]

Subscripts

f	:	Fluid properties
p	:	Porous media properties
*	:	Dimensional properties

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