The Anisofrequency Parameter Calculation in the Regions of Langmuir Probe Operation-Application to Argon Plasma

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Abstract: The anisofrequency dimensionless parameter $\Omega(\theta, \phi)$ is calculated for low density argon (Ar) plasma which is studied in his conditions of electrical potential $\phi$ and ion temperature $T_i$. However, the varied magnitudes are the potential $\phi$ which is varied in $-10$ to $10$ V range, ion temperature $T_i$ from $0.1$ to $1$ eV range and parameter $\theta = T_e/T_i$ from $0$ to $0.4$. The study is done especially for argon and then relative dielectric function versus parameter $\Omega(\theta, \phi)$ is calculated in different region of Langmuir probe operation. The dispersion relation, medium refractive index and $\varepsilon$ versus $\Omega$ achieved this work.

Key words: Anisofrequency parameter, ion plasma frequency, electron plasma frequency, dielectric function, argon plasma

INTRODUCTION

Low temperature plasmas have wide range of technological applications. Argon (Ar), is among the most used plasmas in experiments and technology applications as ECR plasmas which have been done by Bon-Woong et al. (1999), Minayeva and Hopwood (2003) had studied ICP system, Rousseau et al. (2002) had made planar microwave reactors, microwave diagnostic results from gaseous electronic devices rf had been studied by Overzet (1995).

Langmuir Probe (LP) is a tool inserted in plasma in the aim to determine his characteristics. The sheath is formed near the langmuir probe of cylindrical geometry inserted in the plasma, these experiments were made by Laframboise and Rubinstein (1976) and Morales et al. (2003 and 2004). An electrical wave is an oscillating periodically field which depended on time and frequency $f = \omega/2\pi$ expressed as $E = E_0 \exp(\text{j}ot)$ and no magnetic field case is studied. In order to compare electron plasma frequency $\omega_{pe}$ to ion plasma frequency $\omega_{pi}$, the ratio of $\omega_{pe}$ to $\omega_{pi}$ is then defined and introduced in this study as called anisofrequency parameter $\Omega(\theta, \phi)$. This last depended on ratio $\theta$ and the potential $\phi$. The plasma anisofrequency parameter, the relativ dielectric function $\varepsilon_r$ are the main magnitudes studied in this research.

MATERIALS AND METHODS

Dielectric function in different frequency regions: Often it is necessary to investigate a plasma that changes in time. Famous are the studies of the variations of electron energy distributions function in ionisations waves which done by Rayment and Twiddy (1969) and Sica et al. (1971) or in a stationary afterglow studied by Smith and Plumb (1973). Furthermore, Dvoracek et al. (1990) took an interest in many studies using probe for exploring plasma in single shot systems, such as tokamak. Also many technological applications where plasma are generated by tools of alternatim current or radiofrequency power. The approach to problem differs depending on the possibility to make the studied changes of plasma parameters periodic in time with reasonably short period at least several hertz.

Smith and Plumb (1973) used plasma parameters depending on time $t \sim \omega^{-1}$, stationary afterglow had been studied by Shur’ko (2003), ionisation waves, ac or rf generated plasmas are some examples when the periodicity is realisable. The frequency regions are compared to free charge carriers, as ions and electrons, plasma frequency $\omega_{pi}$ and $\omega_{pe}$.

Five regions of LP operation: There are 5 regions of LP operation as far as the frequency of plasma generation or plasma changes is concerned in the Swift and Schwar (1970) and Winkler et al. (1985).

1st region $\omega \ll \omega_{pi}$: Both electrons and ions are in equilibrium with the periodically varying electrical field. The steady ion as well as the electron current increase due to the rectification effect of the nonlinear probe characteristic.

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Time resolved measurement of plasma parameters are possible. This frequency range concerned low frequency plasma oscillations such as ionisation waves, periodically discharge, for example from 100 to 400 KHz.

2nd region $\omega = \omega_p$: The electrons are in equilibrium with varying electrical field.

The oscillation period is comparable to the ions transit time through the space charge sheath and therefore a small resonance peak in the incremental increase in the steady ion current is observed. Since the resonance effect is difficult to assess it is not recommended to make probe measurements in this frequency range as remarked by Hippler et al. (2000), this frequency range is typical for example plasma generated by a small power signal followed by an amplifier of power.

3rd region $\omega_n < \omega < \omega_p$: Only the electrons are in equilibrium with varying electrical field.

This case is typical for single frequency rf discharges ($f = 13.56, 27.12$ MHZ) used in the study of Hippler et al. (2000).

4th region $\omega = \omega_p$: The period of oscillating is comparable to the electrons' transit time through the sheath. It remarked an increase of electron current.

5th region $\omega > \omega_p$: Neither ions nor electrons are able to respond to the oscillating electrical field. This kind of range is available for magnetron generated microwave plasma ($f > 2.4$ GHz).

Calculation of anisofrequency parameter $\Omega$: Since the langmuir probe technique relies upon the formation of a space charge sheath around the probe, its time resolution cannot be higher than ion plasma frequency $\omega_p$. If the plasma frequency variation as in rf plasma, is higher than $\omega_n$, only measurement of time average values of plasma parameters are possibly obtained by LP. The electron plasma frequency $\omega_p$ is a second interesting with $\omega_n$. In plasmas of relatively low density and temperature equal or under 1 and 10 eV for ions and electrons, respectively is well established by Armstrong and Darrow (1994).

This study was conducted at the Plasmas Physics and Conducting Materials and their Applications Laboratory, University of Sciences and Technology (2006). The anisofrequency parameter $\Omega$ is defined as the ratio of electron plasma frequency $\omega_p$ to ion plasma frequency $\omega_n$ which depends directly on the plasma electrical potential $\phi$, ion temperature $T_i$, and the anisothermicity parameter $\theta$ which is the ratio of ion temperature $T_i$ to electron temperature $T_e$ given by the following expression,

$$\theta = \frac{T_i}{T_e} \tag{1}$$

where $0 \leq \theta \leq 0.4$ is the range chosen in experiments of Morales et al. (2003) and Laframboise et al. (1976). The details are shown in following Table 1.

In first, the electron plasma frequency $\omega_p$, and ion plasma frequency $\omega_n$ are expressed as:

$$\omega_p = \left( \frac{n_e e^2}{m_e \varepsilon_0} \right)^{1/2} \tag{2}$$

$$\omega_n = \left( \frac{n_i e^2}{m_i \varepsilon_0} \right)^{1/2} \tag{3}$$

Where, $n_e, n_i$ are electron and ion density, $m_e, m_i$ are electron and ion mass, respectively. $e$ is the elementary electron charge and $\varepsilon_0$ is the vacuum permittivity, (international system units are used). In our study, the electron and ion density distributions in plasma were maxwellian, respectively (Woong et al., 1999). The anisofrequency parameter is then expressed,

$$\Omega(\theta, \phi) = \frac{\omega_p}{\omega_n} \tag{4}$$

Taking into account the density maxwellian distribution the parameter $\Omega(\theta, \phi)$ became:

$$\Omega(\theta, \phi) = \Omega_0 \exp \frac{\varepsilon_0 (1 + \theta)}{k_b T_i} \tag{5}$$

where, the unperturbed parameter $\Omega_0$ in argon plasma is given as follows:

$$\Omega_0 = \sqrt{m_i 10^{1.5} \approx 258} \tag{6}$$

Dielectric function variation: In the $\omega \tau >> 1$ case, the dielectric function $\varepsilon$, is given as follows:

$$\varepsilon_\phi(\omega) = 1 - \frac{\omega^2}{\omega_p^2} \tag{7}$$

by introducing the anisofrequency parameter, the Eq. 7 was rewritten.
Using the Eq. 8, in the 1st, 3rd and 5th region, the corresponding frequencies were, respectively then 1E5, 13.56E6 and 2.4E9 Hz. Their corresponding relative dielectric function were noted $\varepsilon_r'$, $\varepsilon_r''$ and $\varepsilon_r$' (the exponents $1$, $3$, $5$ were, respectively the number of each region), the plasma frequency $\omega_p$ and the ion density were kept, respectively to the values 0.66E6 Hz, 1E13 m$^{-3}$ in the expression 8. Therefore, the corresponding graphs were shown in Fig. 6-8 when $\Theta = 0.4$. Eq. 8 was then given by the following sets of equations:

$$\varepsilon_r'\left(\Omega\right) = 1 - 43.56\Omega^2$$  
(9)

$$\varepsilon_r''\left(\Omega\right) = 1 - 2.37 \times 10^2\Omega^2$$  
(10)

$$\varepsilon_r\left(\Omega\right) = 1 - 7.56 \times 10^{-2}\Omega^2$$  
(11)

These sets of equations can be expressed as 2nd degree equation, $y = 1 - \varepsilon \, \Omega^2$, $s = 1, 3$ or 5.

** Ion transit time in sheath in collisionless plasma:** In transition regimes, the dimensionless variable $\Omega$ is given as follows: In matrix sheath model where ion density $n_i$ is constant and does not depend on sheath distance d, as described in paper of Chen et al. (2002) and by Boeuf and Carrigués (2003).

$$\tau = \omega_p^{-1} \quad \text{where} \quad n_i = \text{constant}$$

$$\tau = \text{some} \, \omega_p^{-1} \quad \text{otherwise},$$

$$\tau = \text{some} 100 \, \omega_p^{-1}$$

in Child Langmuir CL sheath model which was used Boeuf and Carrigués (2003).

The transit time $\tau$ results were given in Table 2 where the term some is taken in average at 10, namely that ion and electron density are kept at the limit of the low density plasma case. Namely, transit time of free charge carriers is linked to its free mean path $l$ and its velocity as $l = v \tau$, which was cited in Kittel (1983).

** Dispersive relation-refractive index:** The dispersion relation, taking in account ion and electron plasma frequency, is given by following expression which were detailed in Golant et al. (1980) and in Tran (1989).

$$\omega^2 = k^2 c^2 + \omega_p^2 + \omega_e^2$$  
(12)

Where, $k$ is a wave vector, $c$ is the light velocity in vacuum ($v_i$ and $v_e$ are the ion and electron velocity in plasma, respectively).

By introducing the parameter $\Omega$, then the dispersion relation is expressed in following:

$$\omega^2 = k^2 c^2 + \omega_p^2 \left(1 + \Omega^2\right)$$  
(13)

The refractive index $N$ is given as follows:

$$N = \sqrt{\varepsilon_r}$$  
(14)

** RESULTS AND DISCUSSION**

The $\Omega(\Theta,\phi)$ values are plotted in logarithms units. The details were shown in Table 1, 2 and 3 and curves are fitted.

**The anisofrequency parameter dependance on plasma physics variables:** The parameter $\Omega(\Theta,\phi)$ versus the potential $\phi$ at kept value of $T_i = 1$ eV with different $\Theta$. The $\Omega$ shape is illustrated for argon plasma in the Fig. 1 in which the shape could be fitted as two increasing lines which crossed at point of coordinates $(0\,V, 257.24)$ for argon plasma. Therefore, $\Omega(\Theta,\phi)$ increased to maximum of coordinates $(10\,V, 3E8)$ for the value $\Theta = 0.4$. The slope increased as the parameter $\Theta$ increased (Fig. 1) so then

| Table 1: Values of parameter $\Theta$, anisofrequency parameter $\Omega(\Theta,\phi)$ and electron temperature $T_e$ |
|---|---|---|
| $\Theta$ | $\Omega(\Theta,\phi)$ | $T_e$(eV) |
| 0 | $\Omega(\Theta,\phi)$ | $\geq T_e$ |
| 0.1 | $\Omega(0.1,\phi)$ | 10.0 |
| 0.2 | $\Omega(0.2,\phi)$ | 5.0 |
| 0.3 | $\Omega(0.3,\phi)$ | 3.3 |
| 0.4 | $\Omega(0.4,\phi)$ | 2.5 |

| Table 2: Free charge carrier transit time $\tau$ for argon when $n_i = 1E13$, $n_e = 1E15$ m$^{-3}$ |
|---|---|
| $\tau$ (s) | $\tau$ (s) |
| $\tau = \omega_p^{-1}$ | 5.6E-10 |
| $\omega_p^{-1} < \tau \leq 10 \, \omega_p^{-1}$ | 1.5E-6 < $\tau \leq 1.5E-5$ |
| $10^2 \omega_p^{-1} < \tau \leq 10^5 \omega_p^{-1}$ | 1.5E-4 < $\tau \leq 1.5E-3$ |

| Table 3: Different ranges of dimensionless parameter $\omega$ for different frequency regions |
|---|---|
| $\omega$ region | Frequency $f$(Hz) | $\omega$ range |
| 1st | $f = 1E5$-4E5 | 6.5E5-2.5E6 |
| 2nd | $\omega = \omega_p$ | $\omega_p$ |
| 3rd | $f = 13.56$ E6 | 8.5E7-1 |
| 4th | $f = 27.12$ E6 | 1.7E8-1 |
| 5th | $f = 2.4$ E9 | 1.5E10-1 |
the electron energy is then deduced 0.25 eV which led to \(v_e = 10^4\) m s\(^{-1}\) and the same ratio, \(v_e/v_i = 10^{-3}<<1\), was found.

Instead of plasma frequency, the anisofrequency versus ion density is plotted for different electron density in Fig. 3, an important decay is clearly observed mainly for the high electron density which was the top limit value of

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\Omega(0, \phi) \text{ dependence upon } \theta \text{ is important. By exploiting the result, the maximum value of parameter } \Omega(0, \phi) \text{ led to an electrical potential condition: } \phi > -16 \text{ V, the } \phi \text{ range chosen in our study is available with respect to the last condition. } \theta = 0.4 \text{ is the high value of the anisothermcity parameter which corresponded to electron energy of } 2.5 \text{ eV and to mean electron velocity, } v_e = (2k_BT_e/m_e)^{1/2}, \text{ around } 10^6 \text{ m s}^{-1}; \text{ it is easy seen that } v_e \text{ is more less than light velocity } c \text{ in vacuum. About ion argon, the velocity } v_i = (2k_BT_i/m_i)^{1/2}, \text{ which correspond to the fixed ion energy } 1 \text{ eV, is around } 10^3 \text{ m s}^{-1}; \text{ the two velocity values could be compared as follows } v_e/v_i = 10^{-3}<<1.

The parameter \(\Omega(0, \phi)\) versus the ion temperature \(T_i\) whereas the potential is kept at 1 V with different \(\theta\) values. The \(\Omega\) shape is illustrated for argon plasma in the Fig. 2. Therefore, the anisofrequency parameter \(\Omega(0, 1)\) followed an exponential decay function of order 1 as shown in figure and fitted in solid line where the fit expression is \(y = y_0 + A_1 \exp(-x/t_1)\), the function \(y\) can be the anisofrequency parameter \(\Omega\) and \(x\) the ion temperature, the constants are: \(y_0 \text{ offset } y_0 = 6.82 \pm 0.13, \text{ amplitude } A_1 = 17.88 \pm 1.18 \text{ and decay constant } t_1 = 0.14 \pm 0.01\). The maximum correspond to a point of coordinates (0.1 eV, 3E8) only for \(\theta = 0.4\). The maxima are very distinctly located but the minima are disconcerted in an average point of coordinates (1 eV, 1043). The ion energy of 0.1 eV which corresponded to \(\Omega\) maximum value, the ion velocity is then about \(710^5\) m s\(^{-1}\) with \(\theta\) equal to 0.4,
Fig. 4: The dielectric function \( \varepsilon_r \) (○ left), polynomial fitted curve in solid line and refractive index \( N \) ( ● right) polynomial fitted in dash line, versus parameter \( \Omega \) for Argon at \( n_i = 1 \times 10^3 \) m\(^{-3}\), \( \theta = 0.4 \) and \( T_i = 1 \) eV

Fig. 5: The dielectric function \( \varepsilon_r \) ( ● left) and refractive index \( N \) ( □ right), exponential decay of order 3 fitted curve in solid line versus parameter \( \Omega \) for Argon at \( n_i = 1 \times 10^3 \) m\(^{-3}\), when \( \theta = 0.4 \) and \( \phi = 1 \) V

In Fig. 4 where the profile fit equation of \( \varepsilon_r, N \) are approximatively expressed as \( \varepsilon_r = 1 + (1.35E-18) \Omega^2 (7.55E-8) \), \( N = 1 + (3.13E-13) \Omega^2 (3.75E-8) \). The profile of both \( \varepsilon_r \) and \( N \) is decreasing as shown in Fig. 5, the exponential decay of order 3 follows the profile expressed as:

\[
y = y_0 + A_1 \exp(-x/t_1) + A_2 \exp(-x/t_2) + A_3 \exp(-x/t_3)
\]

where \( y \) can be \( \varepsilon_r \) or \( N \). The constants are:

\[
y_0 = 0.99994, A_1 = -5.82242E-5, A_2 = 1.48017E-4, A_3 = -3.15103E-5
\]
\[
t_1 = 12.14939, t_2 = 39.91426, t_3 = -7.183182
\]
The difference between the Fig. 4 and 5 is the plasma physics variables $\phi$ and $T$, which were kept separately. The comparaison of their corresponding energy $e\phi$ and $k_B T$, given more informations for the ion transport in the plasma.

The same shape of dielectric function is observed in 1st, 3rd and 5th region as shown in Fig. 6-8, respectively. The range of $\Omega$ when $\epsilon$, increased is about -10 to 0 and decreased in the range 0-20. In 1st, 3rd and 5th region presented $\epsilon$, as positive magnitude. In Fig. 8, the profile is polynomial fitted in dash line and expressed as: $\epsilon = 1 + (9.94E-19)\Omega - (7.56E-8)\Omega^2$.

OPEN QUESTIONS

- Do the results change for another plasma for example helium?
- Could the dielectric function $\epsilon$, profiles change if the parameters of problems change as potential, ion temperature or even ion density?

CONCLUSIONS

In this study, much magnitudes and parameters have been studied and calculated. The originality of this work is the anisofrequency parameter $\Omega(\theta, \phi)$ calculated for Argon plasma at different values of $\theta$ when the variables were the electrical potential $-10 \leq \phi \leq 10$ V and the ion temperature $0 < T_i < 1$ eV. The optimal case was for $\theta = 0.4$. The originality consists in the parameter $\Omega(\theta, \phi)$ which considered as a tool. This last relies the plasma physics variables as electric potential, ion temperature and density, anisothermalicity parameter to measurableng magnitudes as dielectric function and refractive index. These last are very used in electromagnetic and optical material applications. Considered the results Then the relative dielectric function was expressed in several frequency regions. Then the most important frequencies of each region were calculated and discussed as $1E5$, $1.35E6$ and $2.4E9$ Hz. These were the main frequency examples of each region.

The wave oscillation frequencies identified 3 important regions studied above, 1st, 3rd and 5th region and two boundary regions as known the 2nd and 4th. In which them the plasma frequency of ions and electrons could be compared with wave oscillation frequencies, then the plasma particles will be in equilibrium with superimposed varying electrical field or no. The anisofrequency parameter depended on the density in this study of course, the variation is clearly observed for the high electron density and decreased when the ion density increased.

Transit time is around $1E-10$ s in matrix sheath mode and around $1E-3$ s in CL sheath mode.

The refractive index $N$ values and the wave velocities in these low density plasma could show that they are anisotropic. The second region $\omega = \omega_m$ and the fourth region $\omega = \omega_m$ could be considered as boundary region, respectively between 1st and 3rd in one hand and between 3rd and 5th in other hand. The plasma frequency delimitate the border between two domains, the totally reflection and the transmission of electromagnetic waves in the plasma medium. Any other medium which is characterized by dielectric function, will have a response which depends on frequency, his refractive index will depend then on electromagnetic wavelength.

The variation of magnitudes as $\epsilon$, $N$ versus anisofrequency parameter $\Omega$ with restricted conditions on $\theta$, $\phi$, $T$, and $n_e$ was distinctly studied and observed in Fig. 4-8.

The knowledge of the ions and electrons behaviour in and near the sheath with an electrical wave depending on time is certainly important to anticipate many plasma variables as the ion current, the conductivity etc.

This study could be expanded to a wide range of charge carriers densities $n_i$, $n_e$, the potential range in plasma and ion temperature $T_i$.

REFERENCES


