Analyze Images, the Coefficients of Gabor:
A Simple Method for Calculation of the Coefficients of Gabor

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Abstract: The transform of Gabor is a powerful means of representation of an image. It can be used in an effective way for the analysis, the segmentation and the compression of image. However, the principal problem during its use is that no simple method allowing calculation of the coefficients of Gabor. In this study, like proposed an algorithm based on the FFT; however also show that there are a clear reduction of complexity and thus an increased convergence. At least, a whole of examples are given illustrating present approach.

Key words: Gabor coefficient, analyze images, FFT

INTRODUCTION

The transform of Gabor was in the beginning formulated by Gabor[1]. Gabor suggested making local the analysis of Fourier, while making use of windows. The whole of these transforms of Fourier located, form the transform of Gabor of a signal. Thus, it provides a local frequential analysis[1]. The coefficients of Gabor locate the frequential distribution of a signal or image. They also permit to represent or to approach better a signal or image. Bastiaans[2] determines the analytical expressions of the coefficients of Gabor. A network of neurons was presented by Daugmari[3] finding the coefficients optimal of Gabor of an image. It proves that this method does not allow a satisfactory rebuilding of the image of origin. Gertner and Zeevi[4] use the transform of Zak to calculate the coefficients of an image and the auxiliary function window. This technique is very much used but it presents a too significant load of calculation. Yac[5] used a matrix of Toeplitz for the calculation of the coefficients; this requires carrying out inversions of matrix. Teuver and Hostokak[6] present an adaptive filter with a complex algorithm of the least square LMS. But the filter is stable under certain conditions.

Our proposal in this article is to present a new approach for the calculation of the coefficients of Gabor. Like Wang et al., calculation is done via the FFT, however the approach which we carry out, who combines the discrete transform of Fourier[7] with the application of the auxiliary function window[8] allows a significant reduction of the complexity of the algorithm.

TRANSFORM OF GABOR

We seek to develop a sequence finished by a discrete representation of Gabor. That is say:

\[ x(k) = \sum_{n=0}^{M-1} \sum_{m=0}^{K-1} a_{mn} h(k - mN)e^{i\omega nt} \]  \hspace{1cm} (1)

Where, the coefficients of Gabor are:

\[ a_{mn} = \int_{-\infty}^{\infty} x(k) \overline{g}(k - mN)e^{-i\omega nt} dk \] \hspace{1cm} (2)

m, n are integer and N.W = 2\pi the condition of sampling criticalies[9].

The expression \( h(k-mN)e^{i\omega nk} \) represents the function window of Gabor of order (m, n).

\( \overline{g}(k) \) is the complex combined of the auxiliary function window deduced from the condition of biorthogonality[10] following:

\[ \int_{-\infty}^{\infty} h^*(k - mN)g(k)e^{-i\omega nk} = \delta_{1}\delta_{m} \] \hspace{1cm} (3)

Where, \( h^*(k) \) the complex combined of \( h(k) \) and \( \delta_{k} \) constant of Kronecker defined by:

\( \delta_{k} = 1 \) for \( k = 0 \) and \( \delta_{k} = 0 \) for \( k \neq 0 \).

The evaluation of \( g(k) \) consists in finding solutions in the condition of biorthogonality (3).

ALGORITHM OF THE DISCRETE TRANSFORM OF GABOR WITH ONE DIMENSION

The numerical implementation requires the truncation of the \( K \)-addition in (2) and (3).

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M and N are integer satisfying the equality: \( N_1 = N \cdot M \) and \( x(k) \) a discrete sequence length \( N_1 \). The discrete representation of Gabor \( x(k) \) is:

\[
x(k) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{mn} h(k - mN) e^{j\omega_n k}
\]

(4)

\[
a_{mn} = \sum_{k=0}^{N-1} x(k) g^*(k - mN) e^{-j\omega_n k}
\]

(5)

Where, the auxiliary function \( g(k) \) is the solution of the following equation:

\[
\sum_{k=0}^{N-1} h^*(k - mN) e^{j\omega_n k} g(k) = \delta_n \delta_m
\]

(6)

We refer to Raz\(^{28}\) transform the equation (6) in form matrix:

\[
Hg = v
\]

(7)

With \( v = [1, 0, ..., 0] \) with length \( N_1 \)

\[
g' = [g(0), g(1), ..., g(N_1 - 1)]
\]

And \( H \) is a matrix \( N_1 \times N_1 \), whose elements are blocks of Hankel \( \tilde{N} \times \tilde{N} \) defined by the functions 'window' with order \((m, n)\).

\( G \) is thus the solution of (7) defined by:

\[
g = H^{-1}v
\]

(8)

When the auxiliary function is evaluated, we calculate the coefficients of Gabor by applying the following algorithm:

The discrete transform of Fourier of the sequence \( x(k)g^*(k-mN) \) is:

\[
A_{mn} = \sum_{k=0}^{N-1} x(k)g^*(k-mN)\exp\left(-\frac{j\omega_n k}{M}\right)
\]

(9)

While comparing (5) and (9), we notice that \( A_{mn} = a_{mn} \) when \( r = nM \) for \( n = 0, 1, ..., N-1 \).

To calculate the coefficients of Gabor of a finished sequence \( x(k) \), it is enough to calculate the discrete transform of Fourier of the sequence \( x(k)g^*(k-mN) \) at the points \( r = nM \) with \( n = 0, 1, ..., N-1 \).

If \( N \) of operations carried out by the FFT, the number of operations carried out by this algorithm is \( M^2 N \times N \times N \), multiplications. However, the number of operations carried out by the algorithm proposed by Wang et al.\(^{14}\) is \( M^2 N \times N \times N \) multiplications + \((M^2 - 1)N_1^*N_1\) additions.

**ALGORITHM OF THE DISCRETE TRANSFORM OF GABOR WITH TWO DIMENSIONS**

That is to say a digital image of dimensions \( N_1 \). The discrete representation of Gabor of \( x(k,l) \) is:

\[
x(k,l) = \sum_{m_1=0}^{M_1-1} \sum_{m_2=0}^{M_2-1} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} a_{m_1m_2n_1n_2} g(k - m_1N_1, l - m_2N_2) \exp\{jn_1W_1k + jn_2W_2l\}
\]

(10)

The coefficients of Gabor are calculated by the following expression:

\[
a_{m_1m_2n_1n_2} = \sum_{k=0}^{N_1-1} \sum_{l=0}^{N_2-1} x(k,l)g^*(k - m_1N_1, l - m_2N_2) \exp\{-jn_1W_1k - jn_2W_2l\}
\]

(11)

where, \( m_1, m_2, n_1, n_2, N_1, N_2, M_1, M_2 \) are positive integers with:

\[
N_1 = M_1N_{1b}, N_{1l} = M_1N_1
\]

The functions \( h(k,l) \) and \( g(k,l) \) must satisfy the following equality:

\[
\sum_{k=0}^{N_1-1} \sum_{l=0}^{N_2-1} g(k,l)h^*(k - m_1N_1, l - m_2N_2) \exp\{-jn_1W_1k - jn_2W_2l\} = \delta_{m_1} \delta_{m_2} \delta_{n_1} \delta_{n_2}
\]

(12)

If the function 'window' is separable, \( h(k,l) = h(k)h(l) \), the auxiliary function is also separable.

As in the preceding case, we apply the algorithm of the discrete transform of Fourier to 2 dimensions to rebuild the image of origin.

The number of operations carried out to calculate these coefficients is \( M_1^2M_2^2N \times N_1 \times N_2 \) FFT with two dimensions + \( M_1^2M_2^2N \times N_1 \times N_2 \) multiplications. Knowing that \( N \) of operations carried out by the FFT with two dimensions. On the other hand the algorithm presented requires a number of operation equal to \( M_1^2M_2^2N_1 \times N_2 \) FFT with two dimensions + \( M_1^2M_2^2N_1 \times N_2 \) multiplications + \((M_1^2 + M_1^2)N_1 \times N_2 \) additions.

**RESULTS**

We calculate initially the auxiliary function corresponding to the rectangular window. We notice that they are identical (Fig. 1).
Fig. 1: Profile of rectangular window and its auxiliary $h(k)$ and $g(k)$ for $M = 2$, $N = 32$ and $M = 64$

Fig. 2: Profile of Gaussian window and its auxiliary $h(k)$ and $g(k)$ for $M = 16$, $N = 16$ and $N = 256$

Fig. 3: Profile of Gaussian window with two dimensions

Fig. 4: Profile of auxiliary Gaussian window $g(k, l)$
Where, $K_o = 1.854076$ is a constant of standardization (Fig. 2). The function 'window' and its auxiliary are supposed to be separable.

$$h(k,l) = h(k)h(l) \text{ and } g(k,l) = g(k)g(l)$$

Figure 3 and 4 show the profile of these functions. Figure 5 and 6, respectively show the image of origin and the image rebuilt with the coefficients of Gabor.

In Fig. 7 we show that the image of origin can be rebuilt with some coefficients of Gabor. We thus notice, whom the coefficients of Gabor which we calculated allow a faithful rebuilding of the image of origin.

**CONCLUSION**

This study calculated the coefficients of Gabor using the direct formulation of the auxiliary function. We could reduce the complexity of calculations. A comparison with study of Wang et al. [10] finished showing the interest of the algorithm presented in this research.

**REFERENCES**