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Optimal Strategy for the Integrated Vendor-buyer Inventory Model with Fuzzy Annual Demand and Fuzzy Adjustable Production Rate

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Abstract: In this research we present a stylized model to find the optimal strategy for integrated vendor-buyer inventory model with fuzzy annual demand and fuzzy adjustable production rate. This model with such consideration is based on the total cost optimization under a common stock strategy. However, the supposition of known annual demand and adjustable production rate in most related publications may not be realistic. This paper proposes the triangular fuzzy number of annual demand and adjustable production rate and then employs the signed distance, to find the estimation of the common total cost in the fuzzy sense and derives the corresponding optimal buyer's quantity consequently and the integer number of lots in which the items are delivered from the vendor to the purchaser. A numerical example is provided and the results of fuzzy and crisp models are compared.

Key words: Integrated inventory model, triangular fuzzy number, signed distance

INTRODUCTION

In the current supply chain management environment, Stefan *et al*. (2004) have demonstrated that buyers and vendors can both obtain greater benefit through strategic collaboration with each other.

Ha and Kim (1997) proposed a single-buyer and a single-vendor deterministic model with a single product integrated strategy that sends the first shipment as the product arrives at the transported quantity in a simple JIT environment. Pan and Yang (2004) extended Goyal's model (1988) by relaxing the production assumption and presented an integrated inventory model with controllable lead time. Huang (2002) developed an integrated vendor-buyer cooperative inventory model for items with imperfect quality and assumed that the number of defective items followed a given probability density function. Yang and Pan (2004) developed an integrated inventory model involving deterministic variable lead time and quality improvement investment.

Many researchers have being applied fuzzy theory and techniques to develop and solve production/inventory problems. For example, Park (1987) considered fuzzy inventory costs by using arithmetic operations of the Extension Principle. Chen and Wang (1996) fuzzified the demand, ordering cost, inventory cost and backorder cost into trapezoidal fuzzy numbers in an EOQ model with backorder consideration. Roy and Maiti (1997) presented a fuzzy EOQ model with demand-dependent unit cost under limited storage capacity. Lee and Yao (1998) fuzzified the demand quantity and production quantity per day with EPQ model. Yao *et al*. (2000) proposed an EOQ model where both order quantity and total demand were fuzzified as triangular fuzzy numbers. Chang (2004) applied fuzzy method for both imperfect quality items and annual demand to the EOQ model.

Building upon the work of Yang (2006), this paper proposes the model incorporates the fuzziness of annual demand and adjusted production rate. For the model, Signed distance's ranking method (Yao and Wu, 2000) for fuzzy number is employed to find the estimation of the joint total expected annual cost in the fuzzy sense and the corresponding order quantity of the buyer is derived accordingly.

A FUZZY INTEGRATED INVENTORY MODEL

Consider the model fuzzy D and θ c to triangular fuzzy number $\tilde{D}$ and $\tilde{\theta}$, where $\tilde{D} = (D - \Delta_1, D, D+\Delta_2)$, $0<\Delta_1<D$, $0<\Delta_2$ $\tilde{\theta} = (\theta c - \Delta_3, \theta c, \theta c+\Delta_4)$, $0 < \Delta_3<\theta c$, $0<\Delta_4$, $\tilde{\theta}>1$ and Δ_1, Δ_2, Δ_3, Δ_4 are both determined by decision-makers. Modify Yang's model (2006), the joint total expected annual cost is a fuzzy function and can be expressed as

$$\tilde{J}(Q,m) = \frac{\tilde{D}}{Q}\left(A + \frac{S}{m}\right) + \frac{Q}{2} r[(m(1-\tilde{\theta}) - 1 + 2\tilde{\theta})C_V + C_P] + rC_P k\sigma\sqrt{L} \quad (1)$$

The objective of this problem is to determine the optimal order quantity of the purchaser Q^* and the optimal integer number of lots in which the items are delivered from the vendor to the purchaser such that $\tilde{J}$ (Q, m) achieves its minimum value. Utilizing classical optimization, we take the first and second derivatives of $\tilde{J}$ (Q, m) with respect to Q and obtain

$$\frac{\partial \tilde{J}(Q, m)}{\partial Q} = -\frac{\tilde{D}}{Q^2}(A + \frac{S}{m}) + \frac{r}{2}[(m(1-\tilde{\theta}) - 1 + 2\tilde{\theta})C_V + C_P] \quad (2)$$

and

$$\frac{\partial^2 \tilde{J}(Q, t)}{\partial Q^2} = \frac{2\tilde{D}}{Q^3}(A + \frac{S}{m}) \quad (3)$$

Since $\partial^2 \tilde{J}(Q, m)/\partial Q^2 > 0$, i.e., $\tilde{J}$ (Q, m) is convex in Q and hence the minimum value of $\tilde{J}$ (Q, m) will occur at the point that satisfies $\partial \tilde{J}(Q, m)/\partial Q = 0$. Setting (2) equal to zero and solving for Q, we obtain the optimal order quantity of the purchaser as:

$$Q = \sqrt{\frac{2\tilde{D}(A + \frac{S}{m})}{r[C_V(m(1-\tilde{\theta}) - 1 + 2\tilde{\theta}) + C_P]}} \quad (4)$$

Definition 1: From Kaufmann and Gupta (1991), Zimmermann (1996), Yao and Wu (2000), for any a and 0∈R, define the signed distance from a to 0 as $d_0(a, 0) = a$. If a > 0, a is on the right hand side of origin 0; and the distance from a to 0 is $d_0(a, 0) = a$. If a<0, a is on the left hand side of origin 0; and the distance from a to 0 is $-d_0(a, 0) = -a$. This is the reason why $d_0(a, 0) = a$ is called the signed distance from a to 0.

Let Ω be the family of all fuzzy sets $\tilde{C}$ defined on R, the α-cut of $\tilde{C}$ is $C(\alpha) = [C_L(\alpha), C_U(\alpha)]$, $0 \le \alpha \le 1$, and both $C_L(\alpha)$ and $C_U(\alpha)$ are continuous functions on α ∈ [0,1]. Then, for any $\tilde{C}$ ∈Ω, we have;

$$\tilde{C} = \bigcup_{0\le\alpha\le1} [C_L(\alpha)_\alpha, C_U(\alpha)_\alpha] \quad (5)$$

Besides, for every α ∈[0,1], the α-level fuzzy inter val $[C_L(\alpha)_\alpha, C_U(\alpha)_\alpha]$ has a one-to-one correspondence with the crisp interval $[A_L(\alpha), A_U(\alpha)]$, that is, $[C_L(\alpha)_\alpha, C_U(\alpha)_\alpha] \leftrightarrow [C_L(\alpha), C_U(\alpha)]$ is one-to-one mapping. From definition 1, the signed distance of two end points, $C_L(\alpha)$ and $C_U(\alpha)$ to 0 are $d_0(C_L(\alpha),0) = C_L(\alpha)$ and $d_0(C_U(\alpha),0) = C_U(\alpha)$, respectively.

Hence, the signed distance of interval $[C_L(\alpha),C_U(\alpha)]$ to 0 can be represented by their average, $[C_L(\alpha)+C_U(\alpha)/2$. Therefore, the signed distance of interval $[C_L(\alpha),C_U(\alpha)]$ to 0 can be represented as;

$$\begin{aligned} &d_0([(C_L(\alpha),C_U(\alpha)],0) = \\ &[d_0(C_L(\alpha),0) + d_0(C_U(\alpha),0)/2] \\ &= (C_L(\alpha) + C_U(\alpha))/2 \end{aligned}$$

Further, because of the 1-level fuzzy point $\tilde{0}_1$ is mapping to the real number 0, the signed distance of $[C_L(\alpha),C_U(\alpha)]$ to $\tilde{0}_1$ can be defined as

$$\begin{aligned} &d([(C_L(\alpha)_\alpha, C_U(\alpha)_\alpha], \tilde{0}_1) \\ &= d_0([C_L(\alpha), C_U(\alpha)],0) \\ &= (C_L(\alpha) + C_U(\alpha))/2 \end{aligned} \quad (6)$$

Thus, from (5) and (6), since the above function is continuous on $0\le\alpha\le1$ for $\tilde{C}$ ∈Ω, we can use the following equation to define the signed distance of $\tilde{C}$ to $\tilde{0}_1$ as follows.

Theorem 1:

$$a(\tilde{C}, \tilde{0}_1) = \frac{1}{2}\int_0^1 [C_L(\alpha) + C_R(\alpha)]d\alpha = \frac{1}{4}(2b + a + c) \quad (7)$$

Proof: For a fuzzy set $\tilde{C}$ ∈Ω and α∈[0,1], the α-cut of the fuzzy set $\tilde{C}$ is $C(\alpha) = \{x \in \Omega \mid \mu_C(x) \ge \alpha\} = [C_L(\alpha), C_U(\alpha)]$, where $C_L(\alpha) = a + (b-a)\alpha$ and $C_U(\alpha) = c - (c-b)\alpha$. From definition 1, we can obtain the following equation. The signed distance of $\tilde{C}$ to $\tilde{0}_1$ is defined as

$$\begin{aligned} d(\tilde{C}, \tilde{0}_1) &= \int_0^1 d([C_L(\alpha)_\alpha, C_U(\alpha)_\alpha], \tilde{0}_1)d\alpha \\ &= \frac{1}{2}\int_0^1 (C_L(\alpha) + C_U(\alpha))d\alpha = \frac{1}{4}(2b + a + c). \end{aligned}$$

Substituting the result of (7) into (1) and (4), we have

$$\begin{aligned} \tilde{J}(Q,m) &= \frac{(D + (\Delta_2 - \Delta_1)/4_1}{Q}(A + \frac{S}{m}) \\ &+ \frac{Q}{2}r[(m(1-(\theta + (\Delta_4 - \Delta_3)/4) - 1], \\ &+ \frac{Q}{2}r[2(\theta + (\Delta_4 - \Delta_3)/4))C_V + C_P] \\ &+ rC_P k\sigma\sqrt{L} \end{aligned} \quad (8)$$

and the optimal order quantity of the purchaser as:

$$Q=\sqrt{\frac{2(D+(\Delta_2-\Delta_1)/4)(A+\frac{S}{m})}{r[C_V(m(1-\theta+\frac{\Delta_4-\Delta_3}{4})-1+2(\theta+\frac{\Delta_4-\Delta_3}{4}))+C_P]}} \quad (9)$$

For a particular value of *m*, the joint total expected annual cost in fuzzy sense is described by:

$$\tilde{J}(m)=[2r\tilde{D}(A+\frac{S}{t})(C_V(t(1-\tilde{\theta})-1+2\tilde{\theta})+C_P)]^{1/2} + rC_Pk\sigma\sqrt{L}.$$

We can ignore the terms that are independent of m and take the square of $\tilde{J}(m)$. Then, minimizing $\tilde{J}$ (m) is equivalent to minimizing

$$(\tilde{J}(m))^2=2r\tilde{D}[A(C_P-(1-2\tilde{\theta})C_V)+SC_V(1-\tilde{\theta}) + mAC_V(1-\tilde{\theta})+\frac{S}{m}(C_P-(1-2\tilde{\theta})C_V].$$

Once again, while ignoring the terms that are independent of *t*, the minimization of the problem can be reduced to the minimization of

$$\tilde{Z}(m)=mAC_V(1-\tilde{\theta})+\frac{S}{m}(C_P-(1-2\tilde{\theta})C_V]\cdot$$

The optimal value of $m = m^*$ is obtained when $\tilde{Z}(m^*) \le \tilde{Z}(m^*-1)$ and $\tilde{Z}(m^*) \le \tilde{Z}(m^*+1)$. On substituting relevant values inequality, we get:

$$mC_VA(1-\tilde{\theta})+\frac{S}{m}(C_P-(1-2\tilde{\theta})C_V) \le (m-1)AC_V(1-\tilde{\theta})+\frac{S}{m-1}(C_P-(1-2\tilde{\theta})C_V)$$

and

$$mC_VA(1-\tilde{\theta})+\frac{S}{m}(C_P-(1-2\tilde{\theta})C_V) \le (m+1)AC_V(1-\tilde{\theta})+\frac{S}{m+1}(C_P-(1-2\tilde{\theta})C_V).$$

Then,

$$m^*(m^*-1)\le\frac{S[C_P-(1-2\tilde{\theta})C_V]}{AC_V(1-\tilde{\theta})}$$

and

$$m^*(m^*+1)\ge\frac{S[C_P-(1-2\tilde{\theta})C_V]}{\tilde{A}C_V(1-\tilde{\theta})}.$$

Finally, it is concluded that

$$m^*(m^*-\ 1)\le\frac{S[C_P-(1-2\tilde{\theta})C_V]}{AC_V(1-\tilde{\theta})}\le m^*(m^*+\ 1) \quad (10)$$

Thus, we can use the following procedure to find the optimal values of Q and m.

Step 1: Obtain Δ_1, Δ_2, Δ_3 and Δ_4 from the decision-makers.
Step 2: Compute the optimal integer number of lots in which the items are delivered from the vendor to the purchaser by equation (10).
Step 3: Compute the optimal order quantity of the purchaser by equation (9).
Step 4: The $\tilde{J}$ (Q^*, m^*) is the optimal joint total expected annual cost.

Remark 1. If $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta$, then $\tilde{D}$ and $\tilde{\theta}$ reduces to D and θ c; thus the estimate of the joint total expected annual cost in fuzzy sense (1) is identical to the crisp case . Hence, the crisp average demand per year model is a special case of the fuzzy model presented here. Besides, for the optimal order quantity of the purchaser (9), when $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta$, it reduces to

$$Q=\sqrt{\frac{2D(A+\frac{S}{m})}{r[C_V(m(1-\theta)-1+2\theta)+C_P]}} \quad (11)$$

and the derivation of equation (10) reduces to

$$m^*(m^*-1)\le\frac{S[C_P-(1-\theta)C_V]}{AC_V(1-2\theta)}\le m^*(m^*+1) \quad (12)$$

NUMERICAL EXAMPLES

To illustrate the results of the proposed models, consider an inventory system with data: annual demand D = 1,500 unit/year, production rate θ c = 2, purchaser's ordering cost per order A = $25/order, vendor's set-up cost S = $400/set-up, lead time L = 8weeks, purchase cost C_P = $25/unit, production cost C_V = $20/unit, annual inventory holding cost per dollar invested in stock r = 0.2, safety stock factor k = 2.33, standard deviation σ = 7 unit/week.

To solve for the optimal order quantity of purchaser and find the optimal joint total expected annual cost $J(Q^*, m^*)$ in the fuzzy sense for various given sets of

Table 1: Optimal solutions for the fuzzy integrated vendor-buyer inventory model

Δ_1	Δ_2	Δ_3	Δ_4	$\tilde{D}$	$\tilde{Q}$	m^*	Q^*	$J(Q^*, m^*)$
200	300	1	0	1525	1.75	7	120.0599	\$2,317.61
200	250	1	0.5	1512.5	1.875	7	116.7031	\$2,359.89
200	200	1	1	1500	2	6	127.1868	\$2,392.83
200	150	1	1.5	1487.5	2.125	6	124.7145	\$2,417.74
200	100	1	2	1475	2.25	6	122.5437	\$2,438.88
300	200	2	1	1475	1.75	7	116.8022	\$2,305.49
250	200	1.5	1	1487.5	1.875	7	115.1748	\$2,352.48
200	200	1	1	1500	2	6	127.1868	\$2,392.83
150	200	0.5	1	1512.5	2.125	6	126.2106	\$2,427.75
100	200	0	1	1525	2.25	6	125.4317	\$2,459.78

(Δ_1, Δ_2) and (Δ_3, Δ_4) . Note that in practical situations, Δ_1, Δ_2, Δ_3 and Δ_4 are determined by the decision-makers due to the uncertainty of the problem. The results are summarized in Table 1.

CONCLUSIONS

Uncertainties of annual demand and adjustable production rate are inherented in real supply chain inventory systems. However, in practice, there may be a lack of historical data to estimate the annual demand and adjustable production rate. In this situation, using a crisp value is not appropriate.

This paper proposes a fuzzy model for the integrated vendor-buyer inventory problem. For the fuzzy model, a method of defuzzification, namely the signed distance, is employed to find the estimation of total profit per unit time in the fuzzy sense and then the corresponding optimal m and Q are derived to minimize the total cost. In addition, it is shown that in some cases, the proposed fuzzy model can be reduced to a crisp problem and the optimal order quantity of purchaser in the fuzzy sense can be reduced to that of the classical integrated vendor-buyer inventory model. Although we are not sure the solution obtained from a fuzzy model is better than that of the crisp one, the advantage of the fuzzy approach is that it keeps the uncertainties which always fits real situations better than the crisp approach does.

NOTATION AND ASSUMPTIONS

To develop the proposed model, the following notation is used:

- D : average demand per year;
- θ c : adjustable production rate;
- Q : order quantity of the buyer;
- A : purchaser's ordering cost per order;
- S : vendor's set-up cost per set-up;
- L : length of lead time;
- C_V : unit production cost paid by the vendor;
- C_p : unit purchase cost paid by the purchaser;
- m : an integer representing the number of lots in which the items are delivered from the vendor to the buyer;
- r : annual inventory holding cost per dollar invested in stocks;
- K : safety stock factor.

The assumptions made in this paper are as follows.

- The adjustable production rate, θ c, is given by production rate = θ cD, where θ is greater than 1 and fixed.
- The demand X during lead time L follows a normal distribution with mean μL and standard deviation $\sigma\sqrt{L}$.
- The reorder point (ROP) equals the sum of the expected demand during lead time and safety stock.
- Inventory is continuously reviewed.

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