Effects of the Elastic Strain and Dilation in the Big End Journals

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Abstract: The heating and mechanics effects due to the temperatures and pressures in bearings were largely studied these last years but separately; few studies are devoted to the two phenomena acting together in dynamic mode, such as the crankshaft and rod bearings; where the requirements are very severe. In order to approach reality and confronting with other studies being interested in the thermo-elastohydrodynamic phenomena (TEHD). The studied model takes into account the deformations due to the pressures field for the bearing as well as the differential expansion between that and the crankpin; it takes also into account the reduction of the lubricant viscosity which is included in Reynolds’ equation. Simulation results of the connecting rod bearings of in internal combustion engine are presented and show that the thermal distortion has remarkable effects on the bearings performances; therefore the thermo-elastohydrodynamic analysis is very recommended to predict the performances of the journal bearings in internal combustion engines.

Key words: Thermo-elastohydrodynamic, TEHD, elastic strain, dilatation, compliance matrix

INTRODUCTION

In tribology, the mechanisms impose their constraints and their operating conditions, kinematic, dynamic and thermic to their contacts; the latter, in return can or not support these requests and thus can deteriorate or not the correct operation of these same mechanisms. The interface which is not other than fluid film interposed between surfaces in contact, must support these constraints and ensure relative a correct operation between them. The increasingly severe technological requirements of the mechanisms and engines lead to an increase in dissipated energy; the generated temperature can be very high, then the viscosity is decreases and thus the journal lift, as well as the materials mechanical characteristics. The thermal and mechanical deformations generated, can be considerable and very consequent at such point as they can compromise the working clearance and consequently the minimal film thickness.

After the analysis of the various bibliographies treating the thermo-elastohydrodynamic phenomena in the bearings, we present the theory used for the resolution of the problem TEHD.

The effect of elastic deformation of the bearings, on the performance of connecting rod bearings has been studied by many research workers, this shows that a key factor in the analysis of these bearings, to quote only some, Ferron et al. (1983) note that the effect of dilation of the elements of the journal, compromises the radial clearance under operation. Boncompain et al. (1986) have shown that the variations found between the experimentation and theory were due to the thermoelastic effects, Khonsari and Wang (1991) presented a study including the effect of dilation thermal of the bearing and/or the crankpin and the elastic strain of the bearing due to the pressure field; Piffet and P (2000) presented numerical study for the thermo-elastohydrodynamic comportment of the rod bearing subject at a dynamical loading, they show that for the studies cases, the TEHD modeling does not bring much more precision than isothermal TEHD modeling, like the study realised one year ago by Garnier et al. (1999) in automotive engine with four cylinders in line.

Souchet et al. (2001) Presented a thermo-elastohydrodynamic study, where they analysed the influences of the conditions boundary. During the same time Hoang and Bonneau (2001) undertake an experimental study for the heating effects on the connecting rod bearings, in the same context and in the same year Byung and Kyung (2001) make a (TEHD) study and show the influences of the heating and mechanical effects on the behavior of a big end journal of the Diesel engine Ruston-Hornsby 6 VEB Mk-III, whose study was undertaken before by Oh and Goenka (1985) and thereafter by Kumar et al. (1990).

The taking into account of TEHD analysis, is very recommended in the engines working under severe conditions, this to predict the performances of bearings in internal combustion engines.

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**Governing equations:** The schematic diagram of connecting rod bearing is shown in Fig. 1, both of the value and direction of the load applied to the crankshaft vary with time, so the center of the crankpin has periodic motion relative to the connecting rod bearing center. The X, Y, Z coordinate system, whose origin is fixed at the undeformed bearing center, is used as a reference coordinate system of the analysis.

The problem is to find the pressure and temperature fields in the lubricant film, in order to find the most important characteristic in fonctionnement, which is the minimum film thickness.

**Reynolds equation in transient state:** The Reynolds equation is obtained from the Navier-Stokes equations, in the case of the dynamic mode, the additional data is the variation of the load in module and direction. (Fig. 1).

The crankshaft speeds are written:

\[
\begin{align*}
U_a &= \dot{e}\sin\theta - e(\dot{\varphi} + \psi)\cos\theta + R_d\omega_a \equiv R_a\omega_a \\
V_a &= \dot{e}\cos\theta + e(\dot{\varphi} + \psi)\sin\theta + R_d\omega_a \frac{dh}{dx}
\end{align*}
\]

For the bearing: \( U_a = R_{c\theta} \).

By making the approximation \( R_a \approx R \approx (C/R) \ll 1 \), the Reynolds modified equation is written as:

\[
\frac{\partial}{\partial x} \left( h^3 \frac{\partial}{\partial x} p \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial}{\partial z} p \right) = \frac{12 \mu C \left( \frac{\phi - \psi}{\varphi} \right) \varepsilon \sin\theta + \varepsilon \cos\theta}{(R/L)^2} \frac{\partial}{\partial \theta}
\]

the dimensionless Reynolds equation is written:

\[
\frac{\partial}{\partial \theta} \left( \frac{H^3 \frac{\partial}{\partial \theta} P}{\theta} \right) + \left( \frac{R}{L} \right)^2 \frac{\partial}{\partial \theta} = \frac{12 \mu LD}{F (C/R)^2} \left[ \left( \frac{\phi - \psi}{\varphi} \right) \varepsilon \sin\theta + \varepsilon \cos\theta \right]
\]

\[\psi: \text{Load angular velocity}\]
\[\phi: \text{Angular Velocity of the centers line relatively at load.}\]
\[\varepsilon: \text{Crushing velocity (}\varepsilon = \dot{e}/C\text{)}\]
\[\omega_a, \omega_c: \text{Angular velocities of the crankshaft and bearing.}\]

**Mobility method:** The second member of the Eq. (2) fact of appearing the two unknown factors of the problem: \( \varepsilon \) and \( \phi \). The traditional solution is to give two values arbitrary to \( \varepsilon \) and to \( \phi \) and to use an iterative method on these two speeds until the hydrodynamic load \( W \) calculated equal and is opposed to the load applied \( F \).

Speeds of the crankshaft center inside the bearing are determined by writing the equality between the hydrodynamic force in film and the whole of the forces applied to the journal:

\[
F + \int \rho \cdot dS = 0
\]

Relatively for the axis system lied to the centers line, the Eq. (3) became:

\[
\begin{align*}
F_{C} &= +F \cos \phi = - \int_{-L/2 \theta_c}^{+L/2 \theta_c} p(\theta, z) \cos \theta \ Rd \theta \ dz = -W_{C} \\
F_{\phi} &= +F \sin \phi = - \int_{-L/2 \theta_c}^{+L/2 \theta_c} p(\theta, z) \sin \theta \ Rd \theta \ dz = -W_{\phi}
\end{align*}
\]

\( W_c \) and \( W_\phi \) are the components of the hydrodynamic load according to the direction of eccentricity and its perpendicular.
Numerical calculations being very significant, we prefer to use the mobility method of Booker (1984) which allows a fast and precise resolution of the problem.

$$
\varepsilon = \frac{F(C/R)^2}{L D} M_e \text{ et } \varepsilon = \frac{F(C/R)^2}{L D} M_{\phi}
$$

(5)

The vector of mobility has as components:

$$
M_e = + M \cos \alpha \quad M_{\phi} = - M \sin \alpha
$$

(6)

$\alpha$ is the angle between the mobility vector and the eccentricity direction.

The Reynolds equation modified is put in the form, while posing $P = \overline{P} M$:

$$
\frac{\partial}{\partial Z} \left( \frac{R^2}{L} \frac{\partial P}{\partial \theta} \right) + \frac{\partial}{\partial \theta} \left( \frac{R^2}{L} \frac{\partial P}{\partial Z} \right) = 12 \cos (\theta + \alpha)
$$

(7)

The second member of the Eq. (7) utilizes only one unknown factor, the direction of mobility $\alpha$; the solution of this equation became simplified. However the determination of $\alpha$, for a given eccentricity, is only function of the angle of chock $\phi$, requires to use a numerical method of interpolation (the relation enters $\alpha$ and $\phi$ is a priori unknown). Moreover the module of the vector mobility $M$ equal to 1 and is reactualized with the computed value with the step of previous time. The boundary conditions associated with the Eq. (7) are those of Reynolds, by taking atmospheric pressure like reference, they are written:

$$
\begin{align*}
\overline{P} (Z = \pm 1/2) &= 0 \quad \text{que le soit } \theta \\
\overline{P} (\theta = 0) &= \overline{P} (\theta = 2\pi) \quad \text{que le soit } Z \\
\overline{P} &= 0 \quad \text{sur la frontière du film} \\
\frac{\partial \overline{P}}{\partial \theta} &= 0 \quad \text{sur la zone de cavitation}
\end{align*}
$$

(8)

The components of the load without dimension are written:

$$
\begin{align*}
\overline{W}_e &= \int_{1/2 \delta_e}^{+\frac{1}{2} \delta_e} p(\theta, Z) \cos \theta \, d\theta \, dZ \\
\overline{W}_\phi &= \int_{1/2 \delta_e}^{+\frac{1}{2} \delta_e} p(\theta, Z) \sin \theta \, d\theta \, dZ
\end{align*}
$$

(9)

The module of the load is given by:

$$
\overline{W} = \sqrt{\overline{W}_e^2 + \overline{W}_\phi^2}
$$

(10)

What makes it possible to calculate the module of the vector mobility and the angle of chock:

$$
M = \frac{2}{W} \quad \text{et} \quad \phi = \text{Arg} \left( - \frac{\overline{W}_{\phi}}{\overline{W}_e} \right)
$$

(11)

The dimensioned hydrodynamic load is equal to the load applied $F$ and is written:

$$
\begin{align*}
\overline{W}_e &= \frac{F}{2} M \overline{W}_e \\
\overline{W}_\phi &= \frac{F}{2} M \overline{W}_\phi
\end{align*}
$$

(12)

Since the eccentricity varies in time, therefore it is necessary to choose an interval of time between two successive points:

$$
\Delta t = \frac{\Delta \theta_2}{60 \omega_c} \quad \text{et} \\
\left\{ \begin{array}{l}
\omega_c \text{ (tr/min): Crankshaft speed} \\
\Delta \theta_2 \text{ (°): The step between two successive points}
\end{array} \right.
$$

The minimal film thickness: The minimal film thickness of lubricating, without thermal and elastic deformation, is expressed by:

$$
h_{\min} = C (1 + s \cos \theta)
$$

(13)

to which we must add the deformations due to the fields of pressures and the thermal deformations or dilations.

$$
h_{\min f} = h_{\min} + \delta h_e + \delta h_d
$$

(14)

where: $\delta h_e$ is the elastic strain
$\delta h_d = \delta h_e - \delta h_s$ is the differential dilatation between the crankpin and the bearing.

The energy equation: The energy equation permitted to calculate the temperature field in the fluid, it translates the energy conservation and permitted to study the thermal transfers in the journal. According to Boncompain (1984) the energy equation is written:
\[ \rho C_p \left( \frac{u}{R} \frac{\partial T}{\partial \theta} + v \frac{\partial T}{\partial \phi} + w \frac{\partial T}{\partial z} \right) = \]
\[ K \left( \frac{\partial^2 T}{\partial \theta^2} + \mu \left( \frac{\partial u}{\partial \phi} \right)^2 + \left( \frac{\partial w}{\partial \phi} \right)^2 \right) \]
(15)

Where the first term of the first member represents the convection, the first term of the second member conduction and the second term of the second member, viscous dissipation.

**Heat conduction equation:** Our study takes into account the transfer of heat by conduction in the bearing. In order to determine the thermal deformations of the elements of the journal, the temperatures in the solid elements must be known, the equation of heat (16) is:

\[ \frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r_c} \frac{\partial T}{\partial r_c} + \frac{1}{r_c^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} = 0 \]
(16)

Where \( T \) is the temperature in any point of the bearing according to \( r_c, \theta \) and \( z \).

**Calculation of final film thickness:**
- Owing to the fact that the crankpin is massive and in rotation, owing to the fact that metals are of good thermal drivers relative to other materials, owing to the fact that the temperature does not have an effect localised like the pressure, but cumulative until thermal stability, we estimate that a calculation of simple dilation at an average temperature of the crankpin is sufficient.
- Contrary to the non revolving bearings, the crank pins are constantly in rotation, it is not the same surface which is always under constraint, therefore heat is not localised in the same portion of surface, but distributed according to its structure.

We estimate that the ideal and finally are that after the established mode and thus thermal stability, the radial clearance under operation is not compromised and is assured.

**Global heating effect:** The determination of the distribution of the temperature in lubricating film like in the solids in contact is done by the resolution of the equation of energy, the equation of Reynolds generalized and also the equation of conduction of heat in the crankpin and the bearing. The resolution of this three-dimensional problem was considered only very recently.

**DISSIPATED POWER**

The energy dissipated in the contact is significant and the temperatures in the fluid and contiguous materials with film are raised, it results from it a fall from viscosity and thus a reduction from the bearing pressure of the journal. A simple analysis consists in carrying out a total heat balance and thus determining an average temperature value and viscosity of the lubricant. The average temperature value will be calculated starting from the power dissipated during the cycle, average viscosity being obtained starting from a law of variation of viscosity according to the temperature, generally we neglect the effect of the pressure on viscosity in journal bearings.

The formula of Mac Coul and Walther was retained by the ASTM. It is expressed in the form:

\[ \log(\log(v + \alpha)) = A \log(T) + B \]
(17)

Where \( v \) is the kinematic viscosity in centistoke (\( \text{mm}^2/\text{s} \)), \( T \) is the absolute temperature and \( A, B \) are the specific constants of the lubricant, the parameter \( \alpha \) depends on viscosity \( v \).

\[ P = P_1 + P_2 + P_3 \]
(18)

Where:

\[ P_1 = e\phi F \sin \phi \]
(19)
represents the power dissipated by load rotation.

\[ P_2 = eF \cos \phi \]
(20)
represents the power dissipated by the film crushing.

\[ P_3 = C_a \omega_a + C_c \omega_c \]
(21)
represents the power dissipated by the fluid shear.

**Temperature elevation:** If it is admitted that the power dissipated in film is evacuated by the lubricant, the rise in the temperature can be written:

\[ \Delta T = \frac{P_{\text{moy}}}{Q_{\text{moy}} \rho C_p} \]
(22)

With \( C_p \) specific heat of the lubricant, \( \rho \) density of the lubricant at the supply temperature and \( Q_{\text{moy}} \) the medium flow.

Fréne (1982) and Fréne et al. (1990) propose to approach the maximum value of the temperature by the following empirical relation:
\[ T_{\text{max}} = T_a + 2 \Delta T \]  

(23)

The minimal film thickness will be finally the minimal thickness of the fluid film at which we add displacements due to the pressures (mechanics) and displacements due to the dilatation.

**MECHANICAL DEFORMATIONS**

The connecting rod material is assumed to be isotropic, the stress-strain relationship can be written as:

\[
\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = [D] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \]  

(24)

Where

\[
[D] = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} 1-v & v & 0 & 0 & 0 \\ v & 1-v & 0 & 0 & 0 \\ v & v & 1-v & 0 & 0 \\ 0 & 0 & 0 & 1-2v & 0 \\ 0 & 0 & 0 & 0 & 1-2v \end{bmatrix} \]

(25)

Stress analysis for the bearing structure is accomplished by the finite elements software CASTEM 2000, with a pressure of 1Mpa, the meshing used is hexaedral of 8 nodes.

\[ \delta h_p = M_4(0, \theta) \times p(\theta, z) \]  

(26)

Where \( M_4(\theta, z) \) is the compliance matrix, obtained for a unit pressure of 1Mpa, displacement in each point is obtained by multiplication of the coefficient of the matrix by the corresponding pressure of the node considered, in this way we do not have recourse to each iteration on the pressure to calculate by finite elements displacement (Bouyer and Fillon, 2003).

The calculation of the mechanical deformations is applied only to the bearing, the crankpin relative with the bearing is bulkier and the deformations are negligible.

**Dilatation effect:** Both of the crankpin and the bearing are dilated.

For the crankpin and the bearing:

\[ \delta h_{a,c} = R_a \sigma a_c(T_{a,c} - T_0) \]  

(27)

\( R_a, R_c \) are respectively the crankpin and the bearing radius. \( T_a, T_c \) are respectively the crankpin and bearing average temperatures.

The two effects combined bring to a differential expansion and must be added to the film thickness.

\[ \delta h_d = \delta h_c - \delta h_a \]  

(28)

\[ h_{\text{min}} = h_{\text{min}} + \delta h_p + \delta h_d \]  

(29)

**Resolution procedure:** To determine the pressure field in lubricating film, finite difference method is applied for the resolution of the modified Reynolds Eq. (8). The associated linear system cannot be solved directly, because of use of the boundary condition of Reynolds relations (9), we thus applied the iterative method of Gauss Seidel with sur-relaxation coefficient, the calculation of deformations was done according to laws of elasticity and is solved by the known software CASTEM 2000.

Displacements are given for a unit pressure of 1Mpa, to have the real displacement of a node it is necessary to multiply the matrix compliance obtained, by the pressure in this node (Byung and Kyung, 2001), (Bouyer and Fillon, 2003).

The reference pressure is the atmospheric pressure, the temperature is determinate by the heat flow in the journal, due to heat transfer in the bearing and the viscous dissipation in lubricant. The temperatures in film and bearing interface are equal (thin film), the thermal transfer in the crankpin is neglected. And we consider for this last, only dilatation between an average and reference temperatures.

The pressures in the sections of entry and of exit of film are equal to the supply pressure, that of the edges to the atmospheric pressure.

For displacements, the bearing being embedded in the big end, radial displacements on the level of the external radius of the bearing are taken as null.

![Fig. 2: Load diagram](image-url)
Simulation results: We have made our calculation in the connecting rod bearing of Ruston and Hornsby 6veb-x MkIII4 Stroke Diesel engine, as the bearing has a full circumferential oil groove, the calculation is performed on a single land of the bearing. The load diagram at 600 rpm is in Fig. 2. Bearing dimensions, material properties and operating conditions are listed in Table 1. Boundary conditions are Reynolds conditions. Figure 3 shows the three dimensional pressure distribution for some crankshaft angles, the minimal film thickness, the maximal film pressure with and without deformation effect, the dissipater power, outlet flow rate and the rubbing torque are represented respectively in Fig. 4-8.
these characteristics the minimal film thickness is the most important and must be assured, because it characterizes the working clearance.

CONCLUSION

A thermo-elastohydrodynamic lubrication analysis of connecting rod bearing is proposed which includes thermal dilatation and elastic deformation of the bearing surface. Simulation results show that the thermal distortion has remarkable effects on the bearing performance such as the minimum film thickness, maximum film pressure and oil flow rate. It is concluded that the thermo-elastohydrodynamic lubrication analysis is very recommended to predict the performance of connecting rod bearings in internal combustion engines.

Our results within sight of the results obtained by other researchers are concordant, the few differences obtained are due to the boundary conditions and the methods of resolutions.

Appendix:
Thermal deformation of the bearing

According to Laroze (1991), the equation of heat (16) can be written,

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dt}{dr} \right) = 0$$

By integration, we obtain:

$$T = T_1 - T_2 \log r + \frac{T_3 \log r_1 - T_1 \log r_2}{\log \frac{r_1}{r_2}}$$

where $r_1$ and $r_2$ are respectively the internal and external radius, $T_1$ and $T_2$ are respectively the temperatures of the internal and external walls of the bearing.

By considering the assumption of plane deformation and according to the equation of Navier we have:

$$\frac{d}{dr} \left( \frac{du}{r} + \frac{u}{r} \right) = \frac{1+v}{1-v} \frac{dT}{dr}$$

Where $u(r)$ represents radial displacement. By integration successive, we have:

$$u(r) = \frac{1+v}{1-v} \alpha A \left[ \frac{1}{2} \log r + \frac{1}{2} \left( B + C - \frac{A}{2} \right) + \frac{D}{r} \right]$$
With:

\[ A = T_1 - T_2, \quad B = T_2 \log \frac{t_1}{t_2} - T_1 \log \frac{t_2}{t_1}, \]
\[ C = (1 - 2v) \frac{T_2}{t_2^2} - \frac{T_1}{t_1^2} - (1 - 2v) \frac{A}{2} - 2(1 - v)T_0 \]
\[ D = \frac{1}{2} \left( \frac{t_1}{t_2^2} \right) \left( T_2 - T_1 \right) \]

For the crankpin:

\[ u(a) = R_a \cdot \alpha(T - T_0) \]

\( \bar{T} \) Average temperature of the crankpin
\( T_0 \) Temperature of reference to which dimensions were taken

**NOMENCLATURE**

- \( C \): Radial clearance (Rc-Ra)
- \( C_a, C_r \): Friction torques on the crankpin and bearing
- \( C_P \): Specific heat of the lubricant
- \( D \): Average diameter of the journal
- \( e \): Eccentricity of the journal
- \( F \): Dynamic load applied to the journal
- \( h \): Film thickness \( H \): Dimensionless film thickness \( \left( h/H \right) \)
- \( h_{min} \): Minimal film thickness
- \( L \): Length of the bearing
- \( L_2 \): Crank radius
- \( L_3 \): Rod length
- \( M \): Mobility factor
- \( m \): Rod mass
- \( M_x, M_y \): Components of the mobility vector
- \( O_a \): Crankpin Center
- \( O_c \): Bearing Center
- \( OX, Y_0 \): Axis System fixed to the rod
- \( OXYZ \): Axis System related to the centers locus
- \( P \): Hydrodynamic pressure in film
- \( \bar{P} \): Dimensionless Pressure
- \( \varphi_\infty \): Dissipated power
- \( Q \): Axial flow
- \( Q \): Dimensionless Axial flow
- \( R \): Journal average radius
- \( R_a \): Crankpin radius.
- \( R_c \): Bearing radius.
- \( t \): Time
- \( T \): Temperature
- \( T_s \): Supply temperature
- \( W \): Hydrodynamic load supported by the journal
- \( x \): Coordinate in the (circumferential direction)
- \( y \): Coordinate in the film thickness direction.
- \( z \): Coordinate in the axial direction
- \( \alpha \): Mobility direction.
- \( \Delta \tau \): Time step.
- \( \Delta T \): Temperature rise
- \( \epsilon \): Relative eccentricity
- \( \theta \): Circumferential coordinate in the Oxyz system
- \( \theta_\psi, \theta_\phi \): Circumferential co-ordinates of the entry and the exit of lubricating film
- \( \mu \): Lubricant dynamic viscosity.
- \( \rho \): Lubricant density.
- \( \iota \): Angle between the load \( F \) and axis \( OX \)
- \( \bar{\omega} \): Average angular velocity of the crankshaft and the bearing.
- \( \omega_\psi, \omega_\phi \): Angular velocities of the crankshaft and bearing brought back to \( O, X_0 Y_0 \)
- \( \psi \): Load angular velocity
- \( \phi \): Angular velocity of calage angle.

**REFERENCES**


