On the Performance of Distributed Power Control Algorithm over Wireless CDMA

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Abstract: Most Power Control Algorithms require that the link gain between an access port and a mobile terminal should be known. This means a centralized measurement mechanism to track the movement of mobile terminals should be employed which in turn will result into heavy signaling between access ports and mobile terminals. Hence the practicability of designing such systems is not trivial. In this study, focus was made on Distributed Power Control (DPC) algorithm which avoids the centralized power control schemes. Performance of DPC Algorithm and its convergence was done through simulation. The results show what may be expected when DPC power control is implemented in a practical system.

Key words: Power control, fading channels, CDMA, wireless networks

INTRODUCTION

Code Division Multiple Access (CDMA) is interference limited Network. CDMA Network capacity depends significantly on the near-far-effect where a mobile terminal is instructed by the base station to either increase or reduce its transmitter power. The transmitter power affects the link signal quality and the interference environment in a wireless system. However adjusting the transmitter power to improve the link performance is not a trivial problem. If a terminal with a low SIR increases its transmitter power, the SIR is momentarily increased. The increase in transmitter power will on the hand increase the interference in the other links in the system, causing these terminals to increase their powers which results into power competition. If a mobile terminal’s transmitter power is lowered, this will decrease interference to the other links, but could jeopardize its own link. In CDMA systems, many terminals will communicate with the same access port through a common frequency channel (Glisi, 2003). Many researchers have worked on different perspectives of power control algorithms during the recent decades (Rosberg and Zander, 1998; Bambos, 1998; Hanly, 1999 for reviews on power control). Power control in cellular radio systems, especially, has drawn much attention since (Zander’s 1992) work on centralized and distributed SIR balancing. SIR balancing was further investigated by Grandhi et al. (1993). Foschini and Miljancic (1993) considered a more general and realistic model, in which a positive receiver noise and a respective target SIR were taken into account. Foschini’s and Miljancic’s distributed algorithm (FMA) was shown to converge either synchronously or asynchronously to a fixed point of a feasible system. The convergence rate of power control is especially important when propagation and traffic conditions are changing rapidly. Jung et al. (2006) worked on prioritized data services under power constraints. It is expected that Next Generation Wireless Networks will be dominated by bursty traffic than today’s voice-dominated traffic. With bursty traffic, slow algorithms will perhaps not even be able to converge before the data burst ends. To track these changes, the power control algorithm must converge quickly. For instance, in a W-CDMA system, the interference situation can change drastically from frame to frame due to changes in the traffic load. In this study we introduced the effect of fading channels on Distributed power control.

SYSTEM MODEL

Assume that there are Q transmitters assigned to the channel c, where transmitter j uses a transmission power p_j. By using the following vector notation to describe all transmission powers of the transmitters (terminals).

\[ P = (p_1, p_2, p_3, ..., p_Q) \]

(1)

In the uplink case, the value p_i means the transmission power of terminal j. However in the downlink, it denotes the transmission power dedicated to terminal j by the access port to which terminal j is connected.

The expression for the SIR in the receiver i on the channel can be derived:

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\[ \Gamma_i = \frac{g_u p_i}{\sum_{j=i}^{Q} g_j p_j + n_i} \]  

where:
- \( g_u \) = Link gain between receiver i and transmitter i.
- \( g_j \) = Link gain between receiver i and transmitter j.
- \( p_i \) = Transmission power of terminal i.
- \( P_j \) = Transmitter power of terminal j.
- \( n_i \) = Noise power at receiver i.

**Proposition:** A transmitter (terminal) is said to be supported by an Access Point (AP) if it has the SIR satisfying:

\[ \Gamma_i \geq \gamma_0 \]  

where, \( \gamma_0 \) is a target SIR threshold.

Substituting this value in the Eq. 2, we get:

\[ p_i \geq \gamma_0 \left( \sum_{i=1}^{Q} \frac{g_u}{g_j} p_j + \frac{n_i}{g_u} \right) \]  

Equation 3 shows the minimal power that a Mobile Terminal (MT) i should use to achieve the target SIR, assuming the other transmitters' powers are fixed.

Define the Q×Q (non-negative) normalized link gain matrix \( H_\omega = (h_\omega) \) such that:

\[ h_i = \gamma_0 \frac{g_u}{g_u} \text{ for } i \neq j \text{ and } h_{ii} = 0 \text{ for } i = j \]  

Let the normalized noise vector \( \eta = (\eta_i) \) such that:

\[ \eta_i = \gamma_0 \frac{n_i}{g_u} \]  

Rewriting (3)

\[ p_i \geq \sum_{j=1}^{Q} (h_{ij} p_j + \eta_i) \]  

With the matrix notation, the Q linear inequalities (\( \Gamma_i \geq \gamma_0 \) for all i) can be described by:

\[ (I-H) P \geq \eta \]  

**Definition:** The target SIR \( \gamma_0 \) is said to be achievable if there exists a non-negative power vector \( P \) such that \( \Gamma_i \geq \gamma_0 \) for all i.

**Proposition 1:** The target SIR \( \gamma_0 \) is achievable if the largest eigenvalue of the matrix \( H_\omega \) denoted by \( \rho(H) \) is less than or equal to one. The case of \( \rho(H) \) will make the \( \gamma_0 \) achievable only when the receiver noise is zero.

**DISTRIBUTED POWER CONTROL AND ITERATIONS**

From Eq. 2 it can be concluded that link gain matrix G should be known. This assumption requires that a centralized measurement mechanism should be employed which in turn will result into very heavy signaling between access ports and terminals. Hence the practicability of designing such a system is not trivial. In this study, we directed our focus into how to avoid such a centralized control by using the Distributed Power Control (DPC) algorithms. DPC algorithms were first suggested by (Foschini and Miljanic, 1993).

In this study it is assumed that the receiver noise is not negligible and that there exists a unique and non-negative power vector \( P^* \) that solves Eq. 7. In other words, \( \rho(H) < 1 \) so that the matrix \((I-H) = \eta\) is nonsingular and:

\[ P^* = (I-H)^{-1} \eta \geq 0 \]  

Every element in the matrix H is hardly available in practical systems; hence methods such as Gaussian elimination method for solving systems of linear equations cannot be used here. A general iterative method used for power control algorithms are derived from numerical linear algebra. A general iterative method proposed to solve to solve (8) is given by:

\[ P^{(n+1)} = M^{-1} N P^n + M^{-1} \eta \]  

Where, M and N are matrices of appropriate sizes such that:

\[ P^* = M^{-1} N P^* + M^{-1} \eta \]  

The vector \( P^{(n)} \) represents the power level at iteration \( n \). When M and N are appropriately selected, the iterative method in (9) can converge, that is,

\[ \lim_{n \to \infty} P^{(n)} = P^* = (I-H)^{-1} \eta \]  

Let \( M = I \) and \( N = H \), a power control algorithm can be constructed as:

\[ P^{(n+1)} = HP^{(n)} + \eta, \ n = 0, 1, 2, \ldots \]
Hence for each transmitter $i$, the iterative power $P_i^{(n)}$ becomes:

$$P_i^{(n+1)} = \frac{\gamma_i}{g_i} \left( \sum_{j=1}^{n} g_j P_j^{(n)} + n_i \right) = \frac{\gamma_i}{g_i} P_j^{(n)}, \quad n = 0, 1, 2, \ldots$$  \hspace{1cm} (13)

Where, $\gamma_i$ and $P_i^{(b)}$ denote the received SIR and transmission power of transmitter $i$ at iteration $n$, respectively.

**Convergence of the iterative method:** In the general iterative method in Eq. 9, let $\alpha_0, \alpha_1$ be the eigen values of the iteration matrix, $M^{-1}$ $N$ and define $\rho(M^{-1} N) = \max_1 \{ |\alpha_k| \}$. If we define the vector error by

$$e^{(b)} = p^{(b)} - p_0$$  \hspace{1cm} (14)

From (9), the error vector $e^{(b)}$ can be expressed as:

$$e^{(b)} = M^{-1} N e^{(b-1)} = (M^{-1} N)^b e^{(0)}$$  \hspace{1cm} (15)

According to proof Zander *et al.* (2001), in order for (15) to converge to zero vector, $\rho(M^{-1} N) < 1$ should hold. From (15) it can be proved that the power vector converges to a fixed point with a geometric rate.

Hence for DPC

$$\rho(M^{-1} N) = \rho(H)$$  \hspace{1cm} (16)

As stated in proposition 1 that $\rho(H) < 1$ when the target SIR is achievable and receiver noise is positive. Therefore DPC will converge to $P^*$ whenever the given target is achievable.

**Convergence speed of iterative method:** Convergence speed of power control is important characteristic by which we can determine the practical applicability of a given power control algorithm. It has always been assumed that the link gain matrix is variable during the power control process. However, in practical systems the values of the gain matrix and the size of the matrix are changing continuously due to mobile movement and the propagation condition change. A good power control algorithm should quickly converge to the state where the system supports as many users as possible. Hence the smaller $\rho(M^{-1} N)$ is, the faster the convergence.

**SIMULATION AND METHODOLOGY**

The simulation setup is described in this section. We simulated the distributed power control (DPC) algorithm. The path loss model used for the simulations is COST231. Present assumptions and default parameter values are stated.

**Path loss model:** We considered path loss and shadowing in our path model. Fading affects the signal strength measurements and transmit power values. The path loss was modeled using the COST231-Hata model (Mogensen *et al.*, 1991). The signal from the BS to the MT is assumed to decay at the rate of 4th power of the distance. The signal received by a MT from all other BSs except the one that is serving the MT is treated as interference. Considering only path loss, the interference power from each interfering BS $j$ to a MT $i$ is:

$$S_j = \frac{P_0 c}{d_j^{15}}$$  \hspace{1cm} (17)

Where, $d_j$ is the distance between the BS $j$ and the MT $i$. The constant $c$ corresponds to the intercept in the path loss model and is assumed to be 28.5 dB when distance is in meters (Mogensen *et al.*, 1991). The slow shadow fading is modeled by independent log-normal variables. To account for the spatial correlation of the shadows, we assume the model proposed by Gudmundson (1991) where log-normal shadowing was modeled as a Gaussian white noise process that is filtered by a first-order low-pass filter:

$$\Psi_{i+1|j} = \zeta \Psi_{i|j} + (1 - \zeta) \tilde{\Psi}_i$$  \hspace{1cm} (18)

Where, $\Psi_{i|j}$ is the mean-squared envelope expressed in decibels, that is experienced at location $i$, $\zeta$ is a zero-mean Gaussian random variable with the standard deviation of 8 dB and $\zeta$ is a parameter that controls the spatial correlation of the shadows. After every given time interval $T$ in seconds, the spatial correlation factor $\zeta$ for a mobile that is traveling with velocity $v$ is calculated as:

$$\zeta = \frac{v T}{\sigma_D}$$  \hspace{1cm} (19)

Where, $\zeta$ represents a shadow correlation between two points separated by a spatial distance of $D_m$. In present simulation $\zeta$ is set to 0.82 for a distance of 100 m, based on the experiments by Gudmundson (1991). Taking into account the shadowing, the interference power received from an interfering BS $j$ by a MT $i$ at location $l$ is:

$$S_j = \frac{P_0 c}{d_j^{15}} 10^\frac{\gamma_j}{10}$$  \hspace{1cm} (13)
Fig. 1: Generation of MTs in various BSs

Simulation parameter assumptions: To simulate a very large cellular network, (Lin and Mak, 1994) recommend a wraparound topology. This approach eliminates the boundary effects in an unwrapped topology. Hence we simulated our network using a wrapped mesh topology with 81 hexagonal cells. Each cell is surrounded by two rings of BSs so that a significant fraction of interference is captured. We make the following assumptions in our simulations:

- The mobile terminals move based on a two-dimensional random walk model, that is, the mobiles can travel in any direction in a plane with an equal probability. The speed of a mobile is chosen randomly below the maximum speed. We set the maximum speed to 120 km h⁻¹, unless otherwise stated. Mobile terminals (MT) are generated randomly and uniformly across the cells and can appear anywhere with an equal probability (Fig-1).

- The target SIR = 6 dB, number of iterations = 20, the default diameter of a cell is 1 km and all the BSs are assumed to use the same transmission power of 15 W. The spread bandwidth is 3.24 MHz and the thermal noise is set to -105 dBm, derived from (WEE, 2000).

RESULTS AND DISCUSSION

Figure 2 and 3 demonstrate the scenario where two mobile terminals behave under the influence of DPC algorithm. The scenario shows the power required by each terminal to reach a point of convergence. When an appropriate target SIR is attained, the terminals will converge. In Fig. 1, the SIR was set to a different value.
Fig. 5: Convergence of Distributed Power Control (DPC), SIR = 6 dB

other than the intended target SIR of 6 dB. As can be observed no proper convergence was attained by the system. In Fig. 2, the SIR value was set to 6 dB which is the target SIR and this has resulted into convergence.

Figure 4 and 5 show the number of iterations against the normalized Euclidean error for SIR values of 1 dB and 6 dB, respectively. It is observed that, the Euclidean error is better for an SIR of 1 dB than 6 dB which is our target SIR.

CONCLUSIONS

In this research Distributed Power Control (DPC) algorithm has been investigated. CDMA systems are power and interference limited; hence, controlling the transmission power of Mobile Terminals in a cell is crucial to enhance the overall capacity of the network. The advantage of DPC algorithm is that the transmission power of MTs is not centrally controlled. This saves a lot of capacity in terms of massive signaling.

REFERENCES


