Determination of Gravity Anomalies over the Arabian Sea

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Abstract: Gravity anomalies derived from satellite altimetry are progressively taking the place of ship-borne gravity for many marine geo-scientific investigations this paper compares the marine gravity anomalies derived from multi-mission satellite altimetry with gravity anomalies implied by the EGM96 global geopotential model. The results show some significant differences among these gravity data sources. Ocean satellite altimetry implied free-air gravity anomalies have had the shortest wavelength removed during the processing to generate the optimal solution between multiple radar altimeter missions. Compute gravity anomalies by LSC using along-track, differenced geoidal heights and height slopes. Gravity Anomalies over the Arabian Sea (latitudes: 0-25°N and longitudes: 35-70°E) is computed by using along track deflections of vertical and grid along track deflections of vertical using Shepard’s method of gridding procedure and finally the gravity anomalies were computed using the inverse Vening-Meinesz formula we used altimeter data from Topex/Poseidon, Jason1, Geosat, ERS1 and ERS2 missions.

Key words: Satellite altimetry, mean sea surface, deflection of the vertical, gravity anomalies

INTRODUCTION

The main goal of satellite altimetry is the study and observation of the processes and properties of the marine environment, so that when utilizing altimetry data the monitoring of phenomena like the mean sea level variations and changes, the ice transfer, the wind speed, the wave height and the water temperature would be feasible. It is possible, through various techniques, to derive information about the marine gravity field and the ocean tides as well (Emadi et al., 2003).

Altimetric measurements have a vital importance for geodesy, since we are able to provide a direct measurement of the main estimation quantity of geodesy i.e., geoid heights. Since the altimetric observations, the Sea Surface Heights (SSHs), correspond to the separation of the sea surface from the reference ellipsoid is very close to geoid undulations. The difference between Mean sea surface height and geoid is called Sea Surface Topography (SST). The deflections of the vertical are calculated directly by differentiating the sea surface height observations in the along track direction, or by differentiating the geoid undulations in the regular grid and then the free-air gravity anomalies, the difference between the magnitude of the actual gravity (Wₜ) on the geoid and the magnitude of the normal gravity (γₜ) on the ellipsoid’ were computed using the inverse Vening-Meinesz formula.

2-D of deflections of vertical components and gravity anomaly with satellite altimetry data: Deflection of vertical is defined as the spatial angle between the normal gravity vector on the reference ellipsoid and the actual gravity vector on the geoid (θ).

It can also be interpreted as the maximum slope of the geoid with respect to the reference ellipsoid at the point of interest.

\[
\xi = \frac{1}{r \cos \phi} \frac{\partial N}{\partial \lambda} \\
\eta = \frac{1}{r \cos \phi} \frac{\partial N}{\partial \lambda} 
\]

By using gradients of the sea surface height many of the long wavelength altimetry error sources are limited. An obvious advantage is that the task of having to perform a crossover reduction can be avoided. The deflections of the vertical are calculated directly by differentiating the sea surface height observations in the along track direction and then computing the deflections

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of the vertical \((\xi, \eta)\) from equation (1) at the crossover location where ascending and descending ground tracks intersect from either the same or different satellites a much more stable determination of the slopes can be obtained.

**Vertical deflections from along-track slopes:** Taking the first horizontal derivatives of the altimeter-sensed sea surface heights along-track yields the negative deflections of the vertical at the geoid. These are interpolated onto a regular grid and can be converted to gravity anomalies using the inverse Vening-Meinesz formula.

The derivative of the geoid height \(N\) with respect to time \(t\) along the ascending profile is (Smith and Sandwell, 1994):

\[
\dot{N}_s = \frac{\partial N}{\partial t} = \frac{\partial N}{\partial \varphi} \dot{\varphi}_s + \frac{\partial N}{\partial \lambda} \dot{\lambda}_s
\]  

and along the descending profile is:

\[
\dot{N}_d = \frac{\partial N}{\partial t} = \frac{\partial N}{\partial \varphi} \dot{\varphi}_d + \frac{\partial N}{\partial \lambda} \dot{\lambda}_d
\]  

Where \(\varphi, \lambda\) are the geodetic latitude and geodetic longitude of data points, respectively. At the crossover point the following relationships are accurate to better than 0.1%.

\[
\begin{align*}
\dot{\varphi}_s &= -\dot{\varphi}_d \\
\dot{\lambda}_s &= \dot{\lambda}_d
\end{align*}
\]  

The geoid gradient (deflection of the vertical) is obtained by solving (1) using (4). Then we can write:

\[
\frac{\partial N}{\partial \varphi} = \frac{1}{2\varphi} (\dot{N}_s + \dot{N}_d) \quad \text{(5)}
\]

\[
\frac{\partial N}{\partial \lambda} = \frac{1}{2|\lambda|} (N_s - N_d) \quad \text{(6)}
\]

When two or more satellites with different orbital inclinations are available, the situation is slightly more complex but more stable.

**Gridding the deflection of vertical components:** The computed the deflections of vertical components were discrete values, which were randomly distributed over Arabian sea and that the distances between them were not uniform. As required by FFT techniques that the input data should be uniformly spaced on a regular grid (Dadzie, 2005).

We used Shepard’s method of gridding procedure to interpolate values of deflection of vertical components at locations where no data existed, using a grid spacing of 2.5° arc-minute in both longitude and latitude directions. the computational steps as follow:

Knowing the coordinates of the discrete crossover point positions \((\varphi_s, \lambda_s)\) in a spherical coordinate system \((\varphi, \lambda)\) and the corresponding values of the components of the deflection of vertical, \(f = f(\varphi, \lambda_i)\) (\(i = 1, 2, ..., N\)) the differences in longitude \((\Delta\lambda_i)\) and latitude \((\Delta\varphi_i)\), the coordinates of \((\varphi, \lambda)\) at each of the discrete crossover points \((\varphi_s, \lambda_s)\) within a grid cell and the coordinates \((\varphi, \lambda_i)\) of the grid node (i.e., the centre point of the grid cell where \(Z\) value is to be interpolated) are computed:

\[
\begin{align*}
\Delta\varphi &= \varphi - \varphi_s \\
\Delta\lambda &= \lambda - \lambda_s
\end{align*}
\]  

Where \(r_i\) is the distance between the grid node, termed the computation point \(P\) and the \(i\)th discrete crossover point within the cell, termed the running point \(Q_i\), is computed by:

\[
r_i = 2R\sin\frac{\psi_i}{2}
\]

where \(R\) is the mean radius of the Earth, \(\psi_i\), is the spherical distance between \(P\) and \(Q_i\) and

\[
\sin^2\frac{\psi_i}{2} = \sin^2\frac{\Delta\varphi_i}{2} + \sin^2\frac{\Delta\lambda_i\cos\varphi_s\cos\varphi_i}{2}
\]

the weight function can be calculated as (Jiang, 2001):

\[
\phi(\varphi, \lambda) = \begin{cases} 
\frac{1}{r} & 0 < r < \frac{S}{3} \\
\frac{27}{4S} (\frac{r}{S} - 1)^2 & \frac{S}{3} < r < S \\
0 & r > S
\end{cases}
\]

where \(r\) is the search radius and \(S\) is the spherical cap, which was set at 2°. The interpolator was finally given by:

\[
z = F(\varphi, \lambda) = \sum_{i=1}^{N} \frac{f_i}{\phi^*(r_i)} 
\]

where \(f_i\) is the value at the grid point.

878
where $N$ is the number of data points used in fitting the interpolated value at the grid node and $\mu$ is the smoothing factor, which was set at 2 for this study.

**Computation gravity anomaly using inverse vening-meinesz formula:** The concepts of determining marine gravity anomalies from satellite radar altimetry are as follows. The altimeter essentially measures the distance between the satellite and the instantaneous sea surface along the nadir using pulse-limited radar at a series of footprints along the sub-satellite tracks (Fu and Cazenave, 2001).

The Inverse Vening Meinesz formula used in this study to compute gravity anomalies using altimeter-derived components of deflection of vertical (Eqs. 1-5) as input data and is given as (Cheng et al., 2001; Hwang, 1998):

$$
\Delta g = -\frac{\gamma}{4\pi R} \int_{\sigma} \left[ \frac{3 \csc \psi - \cos \psi \csc \frac{\psi}{2} \tan \frac{\psi}{2}}{2} \right] \frac{\partial N}{\partial \psi} \, d\sigma 
$$

where the component of deflection of vertical along the azimuth $\alpha$ is related to the geoid slope ($\frac{\partial N}{\partial \psi}$)

$$
\frac{\partial N}{\partial \psi} = R(x \cos \alpha + \eta \sin \alpha) 
$$

where $R$ is the mean radius of the earth $x$ and $\eta$ are the altimeter-derived north-south and east-west components of the deflection of the vertical, respectively.

$$
\sin \alpha = -\frac{\cos \phi \sin (\lambda_p - \lambda)}{\sin \psi} 
$$

$$
\alpha = \frac{\cos \phi \sin \varphi - \sin \phi \cos \phi \cos (\lambda_p - \lambda)}{\sin \psi} 
$$

substituting Eq. (13) into Eq. (12) yields

$$
\Delta g(\phi, \lambda_p) = -\frac{\gamma}{4\pi} \int_{\varphi} \left( 3 \csc \psi - \cos \psi \csc \frac{\psi}{2} \tan \frac{\psi}{2} \right) (x \cos \alpha + \eta \sin \alpha) \, d\sigma 
$$

considering the Eq. (14) and (15) Eq. (17) is written as:

$$
\Delta g(\phi, \lambda_p) = -\frac{\gamma}{4\pi} \int_{\varphi} \left( xIV_x + \eta IV_y \right) \, d\sigma 
$$

where

$$
IV_x = \cos \alpha (3 \csc \psi - \cos \psi \csc \frac{\psi}{2} \tan \frac{\psi}{2})
$$

$$
= \left( \frac{\cos \phi \sin \phi - \sin \phi \cos \phi \cos (\lambda_p - \lambda)}{\sin \psi} \right) \left( 3 \csc \psi - \cos \psi \csc \frac{\psi}{2} \tan \frac{\psi}{2} \right) 
$$

$$
= \left( \frac{\cos \phi \sin (\lambda_p - \lambda)}{\sin \psi} \right) \left( 3 \csc \psi - \cos \psi \csc \frac{\psi}{2} \tan \frac{\psi}{2} \right) 
$$

and

$$
\sin (\psi / 2) = \left[ \sin^2 \frac{1}{2}(\phi - \varphi) + \sin^2 \frac{1}{2}(\lambda_p - \lambda) \cos \phi \cos \phi \right]^{1/2} 
$$

so the 1D convolution of Eq. (17) is expressed as:
\[
\Delta g(\phi, \lambda) = \frac{\gamma}{4\pi} \int \left[ \left( \xi(\phi, \lambda) \cdot \sin \phi \right) \cdot \chi \left( \phi, \lambda \right) \right] \cdot \text{d} \phi
\]

\[
= \frac{\gamma}{4\pi^2} \int \left[ \left( \xi(\phi, \lambda) \cdot \cos \phi \right) \cdot \chi \left( \phi, \lambda \right) \right] \cdot \text{d} \phi
\]

where and represent the 2D-FFT operator and its inverse. With Eq. (28) the gravity anomalies at all gridded points are computed simultaneously.

RESULTS AND ANALYSES

The differences between the altimeter-derived gravity anomalies (Fig. 1 and 2) and the EGM96-derived gravity anomalies are equal to the residual gravity anomalies computed utilizing the inverse Vening Meinisz formula and they represent the short-wavelength component of the gravity anomaly signal. Table 1 and 2 show the descriptive statistics of the differences between the altimeter-derived
Fig. 1: 2.5'×2.5' Map of altimeter-derived gravity anomalies

Table 3: Statistics of the deflection of vertical over the Arabian Sea via 1D-FIT (unit, mgal)

<table>
<thead>
<tr>
<th>Mission</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
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<tr>
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<td>-4.12514</td>
<td>34.7438</td>
<td>34.4930</td>
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</tbody>
</table>

Table 4: Statistics of the deflection of vertical over the Arabian Sea via 2D-FIT (unit, mgal)

<table>
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<th>Mission</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
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<td>-4.12276</td>
<td>34.5154</td>
<td>34.2683</td>
</tr>
</tbody>
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CONCLUSIONS

This study describes the procedure and accuracy for marine gravity anomalies over the Arabian Sea from multisatellite altimetry. For the present research, it could be concluded that the influence of the Dynamic Ocean Topography on the deflection of vertical should be taken into account in order to improve the accuracy of the determination of gravity anomalies. The gridded residual vertical deflections over unobserved area can be supplied by the corresponding model value when the inverse Vening-Meinesz formula is used to derive the marine gravity anomalies.

REFERENCES


