A Search Algorithm for Determination of Economic Order Quantity in a Two-Level Supply Chain System with Transportation Cost

Mohammadali Pirayesh Neghab and Rasoul Haji
Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran

Abstract: This study considers a two-level supply chain system consisting of one warehouse and a number of identical retailers. In this system, we incorporate transportation costs into inventory replenishment decisions. The transportation cost contains a fixed cost and a variable cost. We assume that the demand rate at each retailer is known and the demand is confined to a single item. First, we derive the total cost which is the sum of the holding and ordering cost at the warehouse and retailers as well as the transportation cost from the warehouse to retailers. Then, we propose a search algorithm to find the economic order quantities for the warehouse and retailers which minimize the total cost.

Key words: Economic order quantity, inventory control, transportation, supply chain management

INTRODUCTION

The goal of most research efforts related to the supply chain management is to present mechanisms to reduce operational costs. The most important operational costs in a supply chain are the inventory cost and the transportation cost.

In the replenishment process, other than the inventory cost, the transportation cost is a major cost factor that affects the optimal shipment size.

Some articles in supply chain consider the transportation cost as a part of the ordering cost and assume it is independent of the shipment size (Hill, 1997; Goyal and Nebebe, 2000; Hoque and Goyal, 2000). Schuster and Bassok (1997), Qu et al. (1999), Nozick and Tornquist (2001) and Aghezzaf et al. (2006) studied the integrated inventory-transportation systems. In their works, the transportation cost is affected by the routing decisions and does not depend on the shipment size.

In many practical cases, the transportation cost is affected by the shipment size and vice versa. So, it is important to determine the economic order quantity which minimizes the overall logistics costs.

Bregman et al. (1990) proposed a heuristic method for the control of inventory in a multi-level environment with transportation cost under deterministic and dynamic demand and finite horizon. Ganeshan (1999) introduced a three-level supply chain consisting of a number of identical retailers, one central warehouse and a number of identical suppliers. In his model, the objective function consists of the ordering, the holding and the transportation costs. He considered the transportation cost as a function of the order quantity but ignored the capacity of the vehicle. Swenseth and Godfrey (2002) studied the effect of the transportation cost discounts on ordering decision when over declaring a shipment is possible. Huang et al. (2005) considered a two-level supply chain system with transportation capacity constraint. They applied the Zero Inventory Ordering (ZIO) policy in which the replenishment is made at equally spaced time intervals and orders are placed only when inventory levels are zero. Ertogrul et al. (2007) considered a vendor-buyer supply chain model and incorporated the transportation cost. In their study, the transportation is made by one type of vehicle whose cost is a function of the shipment size; this function has an all-unit-discount structure. Our model differs from the one proposed by Ertogrul et al. (2007) in the sense that we assume there are three types of vehicles which are defined as small, medium and large. Each type has its own fixed cost, variable cost and the capacity size.

THE MODEL

In this study, we consider a two-level supply chain consisting of one warehouse and a number of identical retailers (Fig. 1). We assume that the demand rate at each retailer is known and the demand is confined to a single item. Shortage is allowed neither at the retailers nor at the warehouse. The transportation time for an order to arrive at a retailer from the warehouse is assumed to be constant. The warehouse orders to an external supplier.
In this model, we suppose that there are three types of vehicles and delivery of each order from warehouse to a retailer is made by a single vehicle without splitting. It is a common transportation scheme in most practical cases. We define these types as small (S), medium (M) and large (L). Each type has its own fixed cost, variable cost and the capacity size (Table 1).

It is assumed that $F_1 < F_2 < F_3$, $v_1 > v_2 > v_3$, $q_1 < q_2 < q_3$, $F_3 = F_1 + q_3 (v_3 - v_2)$ and $F_2 = F_1 + q_2 (v_2 - v_1)$. These equations are supposed to avoid any over declaration. Hence, the transportation cost varies according to the order quantity as shown in Fig. 2.

**FORMULATION OF THE TOTAL COST**

In this section, we intend to derive the total cost. The total cost is the sum of the holding and ordering costs at the warehouse and retailers as well as the transportation cost from the warehouse to retailers.

The notations used in the formulation are as follows:

\[
\begin{align*}
D_r & = \text{Demand rate at a retailer} \\
A_r & = \text{Ordering cost for a retailer} \\
A_w & = \text{Ordering cost for the warehouse} \\
h_r & = \text{Rate of holding cost at a retailer} \\
h_w & = \text{Rate of holding cost at the warehouse} \\
Q_r & = \text{Order quantity at a retailer} \\
Q_w & = \text{Order quantity at the warehouse} \\
m & = \text{Number of retailers}
\end{align*}
\]

We have assumed that the demand rate at the retailers and the transportation time to the retailers are constant and shortage is not allowed at the retailers. Hence, the inventory level at the retailers is a simple EOQ model.

It is assumed that there is no lot-splitting at the warehouse. Furthermore, shortage is not allowed at the warehouse so the order quantity of the warehouse includes an integer multiple (n) of the order quantity of each retailer. Since there are m identical retailers therefore the order quantity of the warehouse is $Q_w = mQ_r$. For optimal solution the arrival of an order to the warehouse corresponds to the delivery of an order to each retailer. Thus, the maximum inventory level at the warehouse is $Q_w = mQ_r$.

The total cost is the sum of the holding and ordering costs at the retailers and the warehouse plus the transportation cost from the warehouse to retailers. Thus, the total cost can be written as:

\[
C_T(Q_r, Q_w) = \frac{D_r A_r + h_r (Q_r - mQ_r)}{Q_r} \\
+ m \left( \frac{D_r A_w}{Q_w} + \frac{h_w Q_w}{Q_r} + \frac{D F}{Q_r} + D v_r \right), \quad i = 1, 2, 3
\] (1)
To minimize the above cost we must consider the following constraints:

\[ q_{i-1} < Q_i \leq q_i \]  
(1.1)

\[ Q_n - mQ_i \]  
(1.2)

\[ n \text{ is a positive int and } q_0 = 0 \]  
(1.3)

Index \( i \) in (1) denotes the vehicle types, 1, 2 and 3, respectively for S, M and L. \( D_n \) is the demand rate at the warehouse which is sum of the demand rates at the retailers, \( D_n = mD_i \).

Substituting \( mnQ_i \) for \( Q_n \) and \( mD_i \) for \( D_n \) in (1) then our mathematical problem can be defined as:

\[ \text{Min } C_T(n, Q) = \frac{D_i A_{\alpha}}{nQ_i} + \frac{h \cdot mQ_i (n - 1)}{2} \]  
(2)

\[ + m \left( \frac{D_i A_{\alpha}}{Q_i} + \frac{h Q_i}{2} + \frac{D_i F}{Q_i} + D_i v_i \right), \quad i = 1, 2, 3 \]

subject to:

\[ q_{i-1} < Q_i \leq q_i \]  
(2.1)

\[ n \text{ is a positive int and } q_0 = 0 \]  
(2.2)

**SEARCH ALGORITHM TO FIND THE OPTIMAL SOLUTION**

The total cost function (2) has the piece-wise convex property for a given value of \( n \). This property is originated by the transportation scheme supposed in the model (Fig. 3).

Figure 2 shows that the transportation cost has an incremental discount structure. Hence, for a given value of \( n \) the method of obtaining the optimal value of \( Q_i \) is the same as the one described by Hadley and Whitin (1963) for incremental quantity discount model.

We develop a search algorithm to obtain the optimal value of \( n \) and \( Q_i \). As mentioned earlier, we apply the incremental quantity discount method for a given value of \( n \). To create our search algorithm we need a lower bound and an upper bound for \( n \). Since \( n \) is a positive integer thus 1 can be a lower bound for \( n \). The following proposition generates an upper bound for \( n \).

**Proposition:** The upper bound of \( n \) is:

\[ n_u = \left[ \frac{A_{\alpha} (h_q - h_n)}{mh_n (A_{\alpha} + F)} \right] \text{ if } \left[ \frac{A_{\alpha} (h_q - h_n)}{mh_n (A_{\alpha} + F)} \right] = 0 \text{ then } n_u = 1 \]

(\( \left[ X \right] \) represents the largest integer less than or equal to \( X \)).

**Proof:** In the first interval of \( Q_n \), the total cost function is:

\[ C_T(n, Q) = \frac{D_i A_{\alpha}}{nQ_i} + \frac{h \cdot mQ_i (n - 1)}{2} \]  
(3)

\[ + m \left( \frac{D_i A_{\alpha}}{Q_i} + \frac{h Q_i}{2} + \frac{D_i F}{Q_i} + D_i v_i \right) \]

If we set the derivatives of \( C_T \) with respect to \( Q \) and \( n \) equal to zero we obtain:

\[ Q^* = \frac{D_i A_{\alpha}}{m \left( \frac{m h_A - h_n}{n} \right)} \]  
(4)

\[ n^* = \frac{1}{m} \frac{D_i A_{\alpha}}{m h_A} \]  
(5)

Substituting \( Q^* \) in (3) we have:

\[ C_T(n) = \sqrt{2D_i m c_f (m A_{\alpha} + m F_i) (h_n (n - 1) + h_r)} + D_i v_i \]  
(6)

The value of \( n \) which optimizes \( C_T(n) \) is obtained as:

\[ n^* = \frac{A_{\alpha} (h_q - h_n)}{m h_n (A_{\alpha} + F)} \]  
(7)

From Eq. 5, it is clear that \( n \) and \( Q_n \) have an inverse relation. The value of \( Q_n \) obtained from Eq. 4 is a lower bound on \( Q_n \), because there is no gain to decrease \( Q_n \) less than \( Q^* \). Hence, the \( n^* \) in Eq. 7 would be an upper bound on \( n \).

In our search algorithm we need the incremental quantity discount method. The steps of this method are as follows (Hadley and Whitin, 1963):

**Step 1:** Compute \( Q_n \), the value of \( Q \) which minimizes \( C_T(n, Q) \) for \( i = 1, 2, 3 \). From Eq. 2 we see that:

\[ Q_n = \frac{D_i (\frac{m h_A}{n} + m F_i)}{m h_A (n - 1) + h_r} \]  
(8)
Step 2: For those \( Q_{n} \) which are \( q_{0}(1) < Q_{n} < q_{i} \) determine \( C_{T_{L}}(n, Q_{n}) \).

Step 3: The \( Q_{n} \) corresponding to the minimum of those costs (in step 2) is the optimal value of \( Q_{n} \).

In summary, the search algorithm to obtain the optimal values of \( n \) and \( Q_{n} \) is as follows:

Algorithm:

Step 1:

Set \( n = \sqrt{\frac{A_{w}(h_{w} - h_{r})}{m h_{w}(A_{n} + 0)}} \) if \( \sqrt{\frac{A_{w}(h_{w} - h_{r})}{m h_{w}(A_{n} + 0)}} = 0 \) than \( n = 1 \)

Step 2: For \( n = 1, 2, \ldots, n \), find the corresponding optimal value of \( Q_{n} \) as follows:

Step 2.1. Compute \( Q_{n} \) using Eq. 8.

Step 2.2. For those \( Q_{n} \) which are \( q_{0}(1) < Q_{n} < q_{i} \) determine \( C_{T_{L}}(n, Q_{n}) \).

Step 2.3. The \( Q_{n} \) corresponding to the minimum of those costs (in step 2.2) is the optimal value of \( Q_{n} \).

Step 3: For \( n = 1, 2, \ldots, n \), and the corresponding optimal value of \( Q_{n} \), calculate the total cost.

Step 4: The solution which has the minimum total cost among the solutions in step 3 is the overall optimal solution.

NUMERICAL EXAMPLE

To clarify the steps of the search algorithm, we solve a numerical example. The problem data is as follows:

\[
\begin{align*}
A_{w} &= 300 \, \text{€} \\
A_{n} &= 25 \, \text{€} \\
h_{w} &= 2 \, \text{€/unit}^{-1} \text{year}^{-1} \\
h_{r} &= 10 \, \text{€/unit}^{-1} \text{year}^{-1} \\
D_{n} &= 1500 \, \text{unit} \text{year}^{-1} \\
m &= 3
\end{align*}
\]

Table 2 gives the relevant data on the transportation.

Solution Algorithm:

Step 1:

\[
n_{n} = \sqrt{\frac{300 \times (10 - 2)}{3 \times 2 \times (25 + 10)}} = 3
\]

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Capacity (unit)</th>
<th>Fixed cost (€)</th>
<th>Variable cost (€/unit(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>100</td>
<td>10</td>
<td>0.30</td>
</tr>
<tr>
<td>M</td>
<td>200</td>
<td>20</td>
<td>0.20</td>
</tr>
<tr>
<td>L</td>
<td>275</td>
<td>30</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Step 2:

For \( n = 1 \):

Step 2.1: Using Eq. 8 \( Q_{n} = 201.2, Q_{n} = 208.6 \) and \( Q_{n} = 215.6 \).

Step 2.2: \( 200 < Q_{n} < 275 \) and \( C_{T_{L}}(1, 215.6) = 7144.2 \).

Step 2.3: For \( n = 1 \) the optimal value of \( Q_{n} \) is 215.6.

For \( n = 2 \):

Step 2.1: Using Eq. 8 \( Q_{n} = 145.8, Q_{n} = 154.1 \) and \( Q_{n} = 162.0 \).

Step 2.2: \( 100 < Q_{n} < 200 \) and \( C_{T_{L}}(2, 154.1) = 6448.0 \).

Step 2.3: For \( n = 2 \) the optimal value of \( Q_{n} \) is 154.1.

For \( n = 3 \):

Step 2.1: Using Eq. 8 \( Q_{n} = 121.0, Q_{n} = 129.6 \) and \( Q_{n} = 137.6 \).

Step 2.2: \( 100 < Q_{n} < 200 \) and \( C_{T_{L}}(3, 129.6) = 6341.5 \).

Step 2.3: For \( n = 3 \) the optimal value of \( Q_{n} \) is 129.6.

Step 3:

For \( n = 1 \) and \( Q_{n} = 215.6 \):

\( C_{T_{L}}(1, 215.6) = 7144.2 \).

For \( n = 2 \) and \( Q_{n} = 154.1 \):

\( C_{T_{L}}(2, 154.1) = 6448.0 \).

For \( n = 3 \) and \( Q_{n} = 129.6 \):

\( C_{T_{L}}(3, 129.6) = 6341.5 \).

Step 4:

The optimal solution is \( n = 3, Q_{n} = 129.6 \) and \( C_{T_{L}}(3, 129.6) = 6341.5 \).

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

In this study, we considered a two-level supply chain system consisting of one warehouse and a number of identical retailers. Unlike the common practice which determines the economic order quantity according to inventory costs only, in this model we incorporated transportation costs into inventory replenishment decisions. We derived the total cost which is the sum of the holding and ordering cost at the warehouse and retailers as well as the transportation cost from the
warehouse to retailers. The total cost function is a piece-wise convex function. Based on this property, we proposed a search algorithm to obtain the optimal solution. We provided a numerical example to show that one can apply easily the steps of the search algorithm.

For future research one can expand this model by including the multi-item lot-sizing problem. This model can be more practical if the number of vehicles is a decision variable.

REFERENCES


