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Scheduling Three Stage Flowshop Processes with
No Intermediate Storage Using Novel Matrix Approach

Amir Shafeeq, M.I. Abdul Mutalib, K.A. Amminudin and Ayyaz Muhammad
Department of Chemical Engineering, Universiti Teknologi Petronas, 31750 Tronoh,
Perak Darul Ridzuan, Malaysia

Abstract: Scheduling optimisation normally aimed at minimizing makespan, leading to overall optimisation of
the production cost. This study focuses on determining the production sequence with minimum makespan for
scheduling of a flowshop process using no intermediate storage (NIS) transfer policy. Unlike previous methods
which uses mathematics considerably and lacks interactivity, the method introduces a simple approach based
on matrix formulation to determine the makespan for all possible batch production sequences thus allowing
simple screening to be done for selecting the optimal sequence.

Key words: Batch process, scheduling, matrix, no intermediate storage

INTRODUCTION

The chemical industry has become more attracted towards batch processes due to their flexibility and
suitability for the production of relatively small volume and high variety products (specialty chemicals,
pharmaceuticals and agrochemicals), which offer advantages in present economic and business situations.
The manufacturing of such products generally involves multi-stage unit operations with tight specifications.
The significant development in flexible production systems has made it possible to produce wide variety of products
using flowshop processes where more than one product is produced in a pre-specified sequence using same
equipments in successive campaigns. More recent works have considered the more complicated cases of jobshop
processes where each product has its own production sequence and make use of processing units in different
combinations. As a result, several products may be produced concurrently (Voudouris and Grossmann, 1992;
Örçün et al., 2001).

The competition among different products in a flowshop process for shared and limited production
resources has caused complexity in scheduling problems. Much of the difficulty arises due to a large set of diverse
constraints and multiple objectives that might be conflicting at times. Significant amount of works on
mathematical programming approach has been applied to batch process scheduling by formulating the problem as
mixed-integer linear or nonlinear problem. However, many industries are still hesitant to apply the scheduling
techniques and mostly prefer to do so manually by their plant operators though the mathematical methods were
made available through computer applications.

The problem with scheduling design of batch processes could be divided into different categories depending on the way the intermediate products are handled in between the unit operations. Such problems are normally referred to as applying different transfer policies. The scheduling problem will become more complicated when feed uncertainty and product market demand pattern are taken into account. On the transfer policy, cases that are usually considered consist of (i) Zero Wait (ZW), (ii) Unlimited Intermediate Storages (UIS), (iii) Finite Intermediate Storage (FIS) and (iv) No Intermediate Storage (NIS) Transfer Policy (Birewar and Grossmann, 1989; Das et al., 1990; Grau et al., 1996; Biegler et al., 1997). When different transfer policy is selected for a batch process operation, it has an impact on the optimum production sequence which, in turn affects the optimum makespan of the batch process. Generally, for a given batch process recipe, the problem refers to the determination of the makespan for corresponding optimum production sequence (Jung et al., 1994; Balasubramanian and Grossmann, 2002; Ryu and Pistikopoulos, 2007).

Contrary to the mathematical programming approach, works related to the use of heuristic approach for
determining optimal batch process or production sequence were generally based on the use of Gantt charts
which is simple but can be extremely tedious particularly for the case of large scale problems. Comparisons have to
be made on huge number of different possible batch production sequences in order to find the optimal

Corresponding Author: Amir Shafeeq, Department of Chemical Engineering, Universiti Teknologi Petronas, 31750 Tronoh,
Perak Darul Ridzuan, Malaysia Tel: +60124069384 Fax: +6053654090
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solution. Nevertheless, it keeps the designer fully in control in terms of the selection of the batch production sequence during design process.

In this study, a simple matrix approach is proposed, which could calculate quickly the makespan of a specified batch production sequence from a given batch process recipes based on the No Intermediate Storage (NIS) transfer policy (Suhami and Mah, 1981; Ku and Karimi, 1988, 1990, 1992; Kim et al., 1996). The generic matrix formulation could also be used for determining the makespan for all other possible production sequences that could be derived from the same batch process recipes. A list of possible production sequences together with their respective makespan can be generated from which screening can then be done to select a few for short listing as candidates for the optimal solution(s). The proposed matrix approach is also capable of handling relatively large number of products and with computer programming, the task of finding the optimal solution(s) could be achieved within a short span of time.

**Formulation of batch scheduling problem using Gantt chart method:** The basis for formulating solution strategy for any problem depends on having pertinent information and understanding the nature of the problem to be solved. In the case of batch scheduling design, these information consists of (i) the number of products to be produced, (ii) the number of process stages, (iii) the batch process recipe for each product and (iv) the interstage transfer policy used e.g., NIS in the present study.

The batch scheduling problem consist of three main stages i.e., mixing (S₁), reaction (S₂) and separation (S₃), with negligible clean up and transfer times is used for the purpose of elaborating the matrix approach.

**Example 1: Makespan for two products i.e., AB:** The processing time for two products (A and B) using three stages (S₁, S₂ and S₃) are shown in Table 1. The makespan of the process for products A and B is determined using Gantt chart method, which is 21 h shown in Fig. 1.

It is obvious that there are three possible paths to determine the makespan. Firstly, the makespan is calculated based on taking the sum of AS₁, BS₁, BS₂, BS₃ and the waiting period of the intermediate product in stage BS₁, i.e., 2 h as represented by the shaded area in Fig. 1. Secondly, it can also be calculated by taking the sum of AS₁, AS₂, BS₂, BS₃, the idle time after AS₁, i.e., 1 h (Fig. 1) and the waiting period in BS₁, i.e., 2 h (Fig. 1). Finally, it could also be calculated by taking the sum of AS₁, AS₂, AS₃, and BS₃. It is observed that result of makespan calculation is same from all the possible paths i.e., 21 h. Apparently, it seems to be a lot easier to use the last path, which does not require calculation of any idle or waiting time. However, this may not be so easy to choose when dealing with number of products greater than two as will be illustrated in the forthcoming examples.

**Example 2: Makespan for three products i.e., ABC:** In this example, the makespan calculation is performed for three products i.e., A, B and C. The respective processing time for the batch process recipes producing three products in the sequence of product A, followed by product B and lastly product C, are shown in Table 2. It appears from the Fig. 2 that possible number of paths to calculate makespan is now eight instead of three as earlier observed in example 1. The calculated makespan for the specified production sequence in this case is 27 h.

The first path to calculate makespan is by taking sum of AS₁, AS₂, AS₃, BS₂, BS₃, idle time between BS₁ and CS₁, and CS₃. The second path is by taking sum of AS₁, AS₂, AS₃, CS₁ and CS₃. The third path is by taking sum of AS₁, AS₂, idle time between AS₁ and BS₁, BS₂, waiting time in BS₂, BS₃ and CS₃. The fourth path is by taking sum of AS₁, AS₂, idle time between AS₁ and BS₁, BS₂, BS₃, waiting time in BS₂, BS₃, idle time between BS₁ and CS₁, and CS₃. The fifth path is by taking sum of AS₁, BS₁, BS₂, waiting time in BS₂,
The matrix approach is designed on the basis of this observation to calculate the makespan for any number of products.

**THE PROPOSED MATRIX APPROACH**

The past application of matrix has managed to simplify considerably the amount of calculation required for solving set of mathematical equations developed to represent a specific system. The proposed matrix formulation in the current work is also capable of simplifying the calculation required to determine the makespan for a specified batch production sequence as opposed to the Gantt chart method. The ability to quickly do so enables the matrix approach to be used to calculate the makespan for all possible production sequences derived from a given batch process recipes. A simple screening procedure could then be employed to select the few best sequences as potential solutions and the decision for the best option is left to the designer. The proposed matrix formulation does not involve any complex mathematical theorem but limited to only simple mathematical formulae combined with some logical understandings derived from the Gantt chart method. Also, it could be easily programmed on computer thus making the execution of the optimization procedure significantly faster and easier.

**The procedure for the matrix approach:** The proposed matrix approach was derived from observations made when applying the Gantt chart method to determine makespan of a specified batch production sequence. The idea is to replace the traditional Gantt chart method with an easier one that could calculate the makespan much quickly. The guideline developed for the matrix approach to calculate the makespan is provided below using the case of a batch process producing four products using three processing stages.

**Step 1:** Firstly, arrange the product recipes according to the arrangement shown below in the form of Matrix $M_i$ where $A, B, C$ and $D$ represent the products in rows $i$ where $i = 1, 2, 3, 4$ while $S_1, S_2$ and $S_3$ represent the corresponding stages in column $j$ where $j = 1, 2, 3$. In this respect the scheduling would be based on a sequence where product $A$ is produced first, followed by product $B$, then product $C$ and lastly product $D$.

$$
\begin{array}{ccc}
1 & 2 & 3 \\
1 & AS_1 & AS_2 & AS_3 \\
2 & BS_1 & BS_2 & BS_3 \\
3 & CS_1 & CS_2 & CS_3 \\
4 & DS_1 & DS_2 & DS_3 \\
\end{array}
$$
Step 2: On the basis of the observations made from above examples for the selection of common path to calculate makespan, select the first element in the first row, the entire elements in the second column and the third element in the bottom most row of the matrix i.e., in the above case the elements are AS₁, AS₂, BS₁, CS₂, DS₁ and DS₂.

\[
\begin{array}{ccc}
1 & 2 & 3 \\
1 & AS₁ & AS₂ & AS₃ \\
2 & BS₁ & BS₂ & BS₃ \\
3 & CS₁ & CS₂ & CS₃ \\
4 & DS₁ & DS₂ & DS₃ \\
\end{array}
\]

Step 3: It is also very much obvious from above examples that two parameters which are in addition to processing times in the stages, must also be included in makespan calculation namely (i) the idle time that exists between stages and it represents the delay prior to starting up the next stage and (ii) the waiting time required within the stage and it represents the waiting time for the intermediate products until the availability of the next stage. The calculation of idle time and waiting times can be done by introducing slack variables in between each of the second column elements in the matrix. The value of each slack variable will be based on summation of idle and waiting times calculated in between the second stage of all the products. In the present case, there will be three slack variables i.e., \(V₁\) located in between elements AS₁ and BS₂, \(V₂\) located in between elements BS₁ and CS₂ and \(V₃\) located in between elements CS₁ and DS₁. Note that the number of slack variables will be one less, than the number of products. The respective idle times and waiting times are represented by letters d and w, respectively as illustrated below.

\[
\begin{array}{ccc}
1 & 2 & 3 \\
1 & AS₁ & AS₂ & AS₃ \\
2 & BS₁ & BS₂ & BS₃ \\
3 & CS₁ & CS₂ & CS₃ \\
4 & DS₁ & DS₂ & DS₃ \\
\end{array}
\]

First, the calculation of the slack variable is made based on the value of the matrix elements located diagonally between the first two rows i.e., BS₁ and AS₂ and BS₂ and AS₃.

For the matrix shown above the formula for calculating the slack variable \(Vᵢ\) between AS₂ and BS₁ is:

\[V₁ = d₁ + w₁ \text{ where } d₁ = BS₁ - (AS₂ + w₁) \text{ and } w₁ = AS₂ - (BS₁ + d₁)\]

The value of \(w₁\) is assumed to be zero at the beginning of the calculation. The same procedure is then repeated between the second and third row, followed by the third and fourth row of the matrix element in order to determine the second and third slack variable i.e., \(V₂\) in between BS₂ and CS₂ and \(V₃\) in between CS₁ and DS₁ as follows;

The formulae adopted to perform the calculation for the two slack variables are as given below.

\[V₂ = d₂ + w₂ \text{ where } d₂ = CS₂ - (BS₂ + w₂) \text{ and } w₂ = BS₂ - (CS₂ + d₂)\]

\[V₃ = d₃ + w₃ \text{ where } d₃ = DS₁ - (CS₁ + w₃) \text{ and } w₃ = CS₁ - (DS₁ + d₃)\]

However, should the value of either or both \(d\) and \(w\) for the two respective stages appear to be negative, a zero value is taken instead.

Step 4: The makespan for the batch process is calculated using the formula;

\[\text{Makespan} = AS₂ + BS₂ + BS₁ + CS₂ + DS₁ + DS₂ + V₁ + V₂ + V₃\]

The generalized mathematical expression for the matrix approach: From the procedure developed above, generalized mathematical expressions could be developed for the matrix approach particularly for the calculation of the slack variables and the makespan. The developed mathematical equations are applicable to \(n\) number of products and are shown below;

\[
\begin{align*}
  dᵢ &= Mᵢᵢᵢ - (Mᵢᵢᵢ + wᵢᵢ) & i = (1, \ldots, n-1) \\
  wᵦ &= Mᵦᵦ - (Mᵦᵦ + dᵦ) & k = (1, \ldots, n-1) \\
  Vₖ &= dₖ + wₖ & (iii)
\end{align*}
\]

Note that \(Mᵢᵢᵢ\) represent the value of the matrix element situated in row \(i\) and column \(j\) starting from \(1,1\). The respective values for the idle time, \(d\) and the waiting time, \(w\) calculated from equation (i) and (ii), are substituted into equation (iii) in order to calculate the value of the respective slack variable, \(V\). The makespan is then determined using equation (iv) below.

\[\text{Makespan} = M₁₁ + \sum_{i=1}^{n} Mᵢᵢᵢ + Mᵦᵦ + \sum_{i=1}^{n-1} Vᵦ \] (iv)
Application of matrix approach: The effectiveness of the application of the suggested matrix approach for scheduling flowshop process is illustrated with an example problem. The example problem has been taken from Ku and Karimi (1992) for which, they have developed an MILP formulation to solve for the optimum production sequence offering minimum makespan (Edgar et al., 2001). Using the matrix formulation, the makespan of the optimum batch sequence as suggested by Ku and Karimi (1992) is recalculated.

Example 4: Makespan of four products using Matrix approach

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.5</td>
<td>4.3</td>
<td>8.7</td>
</tr>
<tr>
<td>2</td>
<td>4.0</td>
<td>5.5</td>
<td>3.5</td>
</tr>
<tr>
<td>3</td>
<td>3.5</td>
<td>7.5</td>
<td>6.0</td>
</tr>
<tr>
<td>4</td>
<td>12.0</td>
<td>3.5</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Using the matrix formulation, the slack variables $V_i$, $V_j$ and $V_k$ are determined using equations (i), (ii) and (iii).

$V_i = d_i + w_i = 3.2$

where, $d_i = 4.0 - (4.3 + 0) = -0.3 = 0$, $w_i = 8.7 - (5.5 + 0) = 3.2$

$V_j = d_j + w_j = 0$

where, $d_j = 3.5 - (5.5 + 3.2) = -5.2 = 0$, $w_j = 3.5 - (7.5 + 0) = -4 = 0$

$V_k = d_k + w_k = 4.5$

where, $d_k = 12.0 - 7.5 = 4.5$, $w_k = 6.0 - (3.5 + 4.5) = -2 = 0$

Subsequently, the makespan for the batch production sequence is calculated using Eq. iv.

Makespan = 3.5 + 4.3 + 5.5 + 7.5 + 3.5 + 8.0 + 3.2 + 0 + 4.5 - 40 h

The makespan is found to be the same as those obtained by Ku and Karimi (1992) which demonstrate the validity of the matrix approach. Again it should be noted that the makespan calculated is based on the production sequence producing the products in the order of product A, followed by product B, then product C and lastly product D.

**Determination of optimum batch production sequence:** In optimizing the batch scheduling for a given batch process, it is important that the optimum sequence or few best sequences are firstly determined from all possible batch production sequences that could be derived. The number of possible batch production sequences could be determined using the following permutation rule:

$P(n) = n!$ where:

$P(n) = $ No. of possible batch production sequences

$n = $ No. of products

For example, the number of possible batch production sequences for producing four products i.e., A, B, C and D is $P(4) = 4! = 24$. Based on the permutation result, the various possible sequences are then developed and these are shown in Table 4. Given the matrix approach ability to quickly calculate the makespan for a specified production sequence, the task of calculating the makespan for all the possible sequences could be done easily by repeating the procedure. Table 4 shows the makespan calculated using matrix approach for all the possible production sequences producing the specified four products with batch process recipes as stated in example 4. It is obvious from Table 4 that the production sequence ACDB has the lowest makespan i.e., 34.8 h, which is the same as determined by Ku and Karimi (1992) using MILP formulation. This proves that the developed matrix approach is able to calculate the makespan for a specified batch process with NIS transfer policy. The makespan calculation in Table 4 has been performed using matrix approach which was programmed on a Microsoft Visual C++ (Version 6.0)™.

**CONCLUSIONS**

The design of batch process scheduling appears to be rather complex in view of the various parameters involved during optimization. Often intermediate products are allowed to stay within the current process unit while waiting for the availability of the next process unit. This creates another dimension to the design problem of the batch process scheduling. Mathematical programming methods based on MILP and MINLP were used widely in the past in view of its ability to overcome the complex optimization problem while designing batch process
scheduling. The matrix approach introduced in the present work uses simple formulation derived from the logic obtained through the use of Gantt chart for determining makespan. Computer programming helps to execute the required formulation swiftly allowing the makespan calculation to be done for the various potential solutions and at the same time, screened for the best few. Given the options and having considered all the constraints (including subjective ones), the designer could then rationalize the solutions and select the best. Future work is planned for developing the approach further to account for other transfer policies such as the allocation of intermediate storage between process units. It is also envisaged that the matrix approach could be extended to incorporate uncertainties in product demand and supply.

REFERENCES


