Intelligent Error Covariance Matrix Resetting for Maneuver Target Tracking

M.H. Bahari, A. Karsaz and M.B. Naghibi-S
Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

Abstract: In this study, an intelligent approach is applied to reset the error covariance matrix of Kalman filter (KF) for high maneuver target tracking. In practice, the standard KF is used for non-maneuvering target tracking applications, which is optimal in the Minimum Mean Square Error (MMSE) sense. Furthermore, it has fast convergence rate. However, after some iterations the steps of the KF become very small. Because of small steps in KF, the accuracy of target tracking may be severely degraded in presence of maneuver. This drawback can be overcome by resetting the error covariance matrix of the KF. Since the information of earlier updates will be partially lost by resetting the error covariance matrix, system should reset it just when the target maneuvers and KF steps are not large enough to track the target accurately. Moreover, resetting factor should be proportional to the maneuver. Therefore, we present an intelligent approach based on target maneuver detection to determine proper instants for resetting the error covariance matrix. In addition, the new scheme is able to determine the optimal value of resetting factor in each iteration effectively. Simulation results illustrate that the tracking ability of the proposed scheme is more than conventional approaches, especially for high maneuvering target tracking applications.

Key words: Fuzzy logic, Kalman filter, high maneuver target, maneuver detector, tracking problem

INTRODUCTION

Track radars provide continuous range, bearing and elevation data on one or more specific targets. These radars are equipped with a radar signal processor and plot extractor. Radar signal processor reduces the noise interference and decreases the effect of clutter. Plot extractor extracts the plot position data and sends them to the radar data processor with azimuth and range sequences. The radar data processor applies tracking algorithm to the received data and gives a track (Edde, 1993). This algorithm should be able to track high maneuvering targets with reasonable accuracy.

The KF has been used extensively in target tracking problems. However, while the target maneuvers the quality of the position and velocity estimation could be degraded considerably (Lee et al., 2005). To solve this problem, some techniques have recently been issued to modify the conventional KF, such as the augmented state Kalman estimator (AUSKE). The AUSKE solves the problems by including the input parameters as a part of an augmented state (Haessig and Friedland, 1998; Moeckerjee and Reifler, 1999; Mehrotra and Mahapatra, 1997). The AUSKE suffers from complexity of computational effort and numerical problems when state dimensions are large. Hsieh and Chen (1999, 2000) suggested an optimal two-stage Kalman estimator (OTSKE) for a general case to reduce the computational complexity of the AUSKE. However, OTSKE suffers from two major drawbacks. These drawbacks stem from assuming constant acceleration and input term assumed to be observable from the measurement equation (Hsieh and Chen, 1999; Qiu et al., 2005). Wang and Varshney (1993) proposed a tracking algorithm which was developed based on constant acceleration (jerk is zero) for low maneuvering targets. Therefore, the performance of the estimation is reduced when target moves with non-constant or high acceleration.

Goodwin and Sin (1984) proposed an adaptive control of time varying systems. They used a finite data window for error covariance matrix in least square algorithm and reset that matrix periodically. In this scheme, resetting times and resetting factor are two important parameters. Any changes in these two parameters will significantly influence the tracking accuracy. In Goodwin and Sin (1984) these two parameters are determined off-line and are constant during the simulation. We use a soft computing approach to determine these two parameters on-line and adaptively regarding the target maneuver.

The equivalent-noise approaches assume that the filter correction can take a simple form of increasing the KF gain equivalent to placing more weights on the measurements (Cardillo et al., 1999; Li and Jilkov, 2002;
Li and Bar-Shalom, 1994). In the other words, the equivalent-noise approaches assume that the maneuver effect can be modeled by a white or colored noise process (Li and Jilkov, 2002). Of course, the statistics of these weights for maneuver compensation in general, are not known. This fundamental assumption converts the problem of maneuvering target tracking to a state estimation problem in the presence of nonstationary process noise with unknown statistics (Li and Jilkov, 2002). This conversion is the most important drawback of this fundamental assumption. The proposed algorithm in this paper intends to overcome this major drawback by determining the matrix covariance resetting level value.

There are some literature about using fuzzy logic in maneuvering target tracking with intelligent adaptation and capability to add human knowledge to the system (Duh and Lin, 2004; Lee et al., 2004).

Continuing these efforts, in this research we find some logical rules for resetting error covariance matrix and add them to the Goodwin and Sin method with the use of fuzzy logic.

MANEUVERING TARGET TRACKING FORMULATION

Some researches in change-point detection have been explored in Gustafsson (1996) and Malladi and Speyer (1999). It is assumed that the target moves in a plane, which is the two-dimensional case, such as a helicopter with fixed elevation. The state equation for the nonmaneuvering model is given by 1.

\[
\begin{align*}
X(n+1) &= F(n)X(n) + G(n)w(n) \\
Z(n) &= H(n)X(n) + v(n)
\end{align*}
\]

(1)

Where:

- **X(.)** = State vector
- **x(.)** = White system driving uncertainty
- **X(0)** = Initial condition which may be uncertain
- **Z(.)** = Observation vector
- **v(.)** = White observation uncertainty

\[
\begin{align*}
E\{v(n)v^T(n)\} &= R(n) \\
E\{w(n)w^T(n)\} &= Q(n) \\
E\{x(0)x^T(0)\} &= \psi E\{x(0)x(0)\} - 0 \\
E\{w(0)\} &= 0, E\{x(0)\} = 0 \\
E\{w(0)v^T(0)\} &= 0 \\
X(n) &= \begin{bmatrix} x(n) & v_x(n) & y(n) & v_y(n) \end{bmatrix}^T
\end{align*}
\]

where, \(R(n)\), \(Q(n)\) and \(\psi\) denote the measurement, process and initial state covariance matrices, respectively. The expression for \(G(n)\), \(F(n)\) and \(H(n)\) as functions of the update time \(T\) (\(T\) is the time interval between two consecutive measurements) are:

\[
\begin{align*}
F &= \begin{bmatrix} 1 & T/2 & 0 \[0 & 1 & 0 \[0 & 0 & 1 \end{bmatrix}, G &= \begin{bmatrix} T^2/2 & 0 & 0 & 0 \[0 & T & 0 \[0 & 0 & T \end{bmatrix}, H &= \begin{bmatrix} 1 & 0 \end{bmatrix}
\end{align*}
\]

The standard KF which is an efficient and unbiased filter (Schwepp, 1973) is summarized in the following relations.

\[
\begin{align*}
\hat{x}(n+1|n+1) &= F(n)\hat{x}(n|n) + G(n)w(n) \\
\Sigma(n+1|n+1) &= \Sigma(n+1|n) + G(n)H(n)\Sigma(n|n)H(n)^T + Q(n) \\
\Sigma(n+1|n) &= F(n)\Sigma(n|n)F(n)^T + G(n)Q(n)G(n)^T \\
\Sigma(0|0) &= 0, \hat{x}(0|0) = 0
\end{align*}
\]

(2)

\(K(n)\) is the Kalman gain and notation \(\hat{x}(n+1|n)\) denotes the prediction at the \(n+1\)th sample point given the measurement up to and including the \(n\)th whilst \(\hat{x}(n|n)\) denotes the estimation at the \(n\)th sample point given the measurement up to and including the \(n\)th. \(\Sigma(n|n)\) is the error covariance matrix and \(\Sigma(n+1|n)\) is the error covariance matrix of the one-step prediction.

The maneuvering model treats the acceleration as an additive term:

\[
\begin{align*}
X(n+1) &= F(n)X(n) + C(n)U(n) + G(n)w(n) \\
\Sigma(n+1|n) &= F(n)\Sigma(n|n)F(n)^T + G(n)Q(n)G(n)^T + C(n)R(n)C(n)^T
\end{align*}
\]

(3)

Where:

- **U(n) = [u_x(n) u_y(n)]**
- **U(n) = Target acceleration which is modeled as an unknown variable**

ERROR COVARIANCE MATRIX resetting

Kalman filter is known to provide extremely rapid initial convergence rate and optimal tracking in nonmaneuvering problem. However, the algorithm was developed with some assumptions. The most important one is constant speed of target movement. To be more precise, while the target maneuvers, the quality of the position and velocity estimation could be decreased significantly. Therefore, using KF is suitable until target starts to maneuver.

As we know, in this algorithm the error covariance matrix \(\Sigma(n|n)\) becomes small after a few iterations
(Goodwin and Sin, 1984). Consequently, when the target begins to maneuver with high acceleration, tracker which uses KF would not be functionally accurate. This motivates a related scheme in which \( \Sigma(n|n) \) is reset at various times. In other words, old data is discarded to keep the algorithm alive. The main idea of resetting \( \Sigma(n|n) \) is to retain the fast initial convergence of KF and track a target maneuver immediately.

The trace norm is usually used as a matrix measure. Therefore, after some iterations the error covariance matrix norm (Trace(\( \Sigma(n|n) \))) becomes smaller than specified value called Trace Limit (TL), the following resetting procedure is performed:

\[
\text{Trace}(\Sigma(n|n)) < \text{TL} \Rightarrow \Sigma(n|n) = K_v \Sigma(n-1|n-1)
\]  

(4)

where, \( K_v \) is resetting factor.

The main drawbacks of the conventional resetting scheme are:

- The TL of the algorithm is determined off-line and remains constant during the simulation. This drawback may cause a delay in resetting \( \Sigma(n|n) \) when the target maneuvers. Furthermore, it can lead to unwanted resetting when the target does not maneuver.
- The Resetting Factor (\( K_v \)) is determined off-line and remains constant during the simulation. This drawback may produce a small Kalman gain when the target maneuvers (incomplete compensation) or a large Kalman gain when the target does not maneuver.

To overcome these drawbacks the Intelligent Error Covariance Matrix Resetting Algorithm is suggested in this study.

**INTELLIGENT ERROR COVARIANCE MATRIX RESETTING**

Target maneuver value plays a critical role in determining the resetting factor and the TL value in each step. Since target maneuver is unknown for the tracker, its estimation would be essential to determine these two parameters.

**Fuzzy maneuver detector**: Radar output signal has no explicit mathematical relationship with target maneuver. However, there exists a complex nonlinear mapping between them. Finding effective input elements is essential to map the input vector to target acceleration vector. In this research, two features are used as inputs of fuzzy acceleration estimator system.

**Absolute value of difference between last target course (\( \psi \)) and observation target course (\( \xi \))**: Figure 1 is target movement geometry in Cartesian coordinates. \( \Delta \theta \) is one of the most useful elements to detect the target maneuver (Bahari et al., 2007).

When \( |\Delta \theta| \) is low, then the target with high probability is moving around its last direction and when \( |\Delta \theta| \) High, then the target with high probability is moving toward sensor's observation. This fact was used as a fuzzy rule in fuzzy acceleration estimator.

\[
|\Delta \theta|, \psi \text{ and } \xi 
\]

are calculated with the use of following equations.

\[
\Delta \theta = \psi - \xi
\]

(5)

Where:

\[
\psi = \text{Last target course} \quad \xi = \text{Observation target course}
\]

\[
\psi = \text{Angle}(\text{H}(n-1|n) - HX(n-1|n))
\]

\[
\xi = \text{Angle}(Z(n|n) - HX(n-1|n))
\]

(6)

**Absolute value of measurement residue (R)**: The objective here is to develop a maneuver detection algorithm, which detects the acceleration and jerk of a maneuvering target. Similar idea of quickest detection and change detection algorithm only for constant acceleration has been investigated in the Wang and Varshney (1993). The standard KF is an efficient and un biased filter with the Measurement residual as follows:

\[
Z(n+1) = Z(n+1) - \hat{Z}(n+1|n) = Z(n+1) - H(n+1)\hat{X}(n+1|n)
\]

(7)

The measurement residue for nonmaneuvering target is a stochastic zero mean white process i.e.,
where, $\Re(.)$ denote the measurement covariance matrix.

Therefore, for nonmaneuvering targets, the mean of this sequence ($\bar{z}(n+1)$) is zero. But, for maneuvering target case, this sequence is no longer zero and contains more information. In fact, acceleration term leads to a bias in measurement residue. The amount of this bias, supply some information about the existence of target acceleration. This fact was used as another fuzzy rule in fuzzy acceleration estimator system for target maneuver detection and estimation.

**Intelligent error covariance matrix resetting:** Block diagram of proposed system is shown in Fig. 2. In Fig. 2 block 1, calculates $\Delta \theta$ and R. Block 2 is a fuzzy controller. The fuzzy system has two inputs and one output. The input variables of fuzzy system are $|\Delta \theta|$ and R. Inputs and output fuzzy sets all have three Gaussian membership functions with the following membership grade $u_i(x_i)$.

$$u_i(x_i) = \exp \left[ -\frac{1}{2} \left( \frac{x_i - \mu_i}{\sigma_i} \right)^2 \right]$$

(8)

where, $\mu_i(x_i)$ and $\sigma_i$ are the center value and the standard deviation of Gaussian membership function for $i^{th}$ input variable of $i^{th}$ fuzzy rule, respectively. The output of the fuzzy logic controller determines the estimated acceleration value of the target ($a_t$) based on $\Delta \theta$ and R inputs. Fuzzy inference rules support mentioned information.

Block 3 is a simple low pass filter. The main purpose of using low pass filter is that output of block 2 ($a_t$) is a noisy signal. To vivify, $a_t$ is target acceleration signal added with high frequency noise.

After passing this noisy signal through a low pass filter, real value of target acceleration will be achieved. Block 4, which determines $K_c$ is another fuzzy system.

This fuzzy system has two inputs and one output. Inputs are the target acceleration (output of Block 3) and Trace[$\Sigma(n,n)$]. Output of this fuzzy controller is optimum value for $K_c$ in each iteration.

Obviously, error covariance matrix should be reset when the target starts to maneuver and tracker steps are large enough ($\Sigma(n-1)$ is small) to track the target. In such a situation, fuzzy system in block 4 determines a proper value for $K_c$ proportional to the target acceleration. While the target does not maneuver or $\Sigma(n,n)$ is large enough to track the target accurately, output of block 4 remains zero. Inputs and output fuzzy sets all have two Gaussian membership functions.

Block 5 is error covariance matrix resetting centre. Decision about resetting $\Sigma(n,n)$ is made in this block of the system. Input of this block is just $K_c$. In block five if $K_c$ is a nonzero value then system reset $\Sigma(n,n)$ using relation 4 (with the use of determined $K_c$ in block 4). At the other pole, if resetting factor is equal to zero then system updates the error covariance matrix using (2) as the conventional KF equations.

**RESULTS**

Here, simulation was carried out to illustrate the efficiency of the proposed scheme. Simple KF, method of Wang and Varshney (1993), conventional resetting scheme and proposed method are compared in the different case studies.

In experiments reported in this section, the following assumptions and parameter values are used. In these simulations, the sampling time is $T = 0.015$ (sec). Covariance elements generated for R and $\theta$ axis are both Gaussian random variables. In addition, the measurement noise vector in Cartesian coordinates is related to the measurement noise vector in polar coordinates by the following equation (Bahari et al., 2007).

$$
\begin{bmatrix}
\delta_x \\
\delta_y
\end{bmatrix} =
\begin{bmatrix}
\cos^2(\theta(0)) & R(0)\sin(\theta(0)) \\
\sin^2(\theta(0)) & R(0)\cos(\theta(0))
\end{bmatrix}
\begin{bmatrix}
\delta_x \\
\delta_y
\end{bmatrix}
$$

(9)

Where:

- $\delta_x = 200$
- $\delta_y = 1$
- $R(0)$ and $\theta(0)$ denote the target initial range and azimuth, respectively.

It is important to mention that in our simulations the initial state (including initial position and initial speed) of the target is not known for the trackers.

**First case study:** The initial position of the target is given by $[x(0), y(0)] = [1532.1(m), -1285.6(m)]$ with an initial speed of $[v_x(0), v_y(0)] = [38.3(m/sec), -32.1(m/sec)]$. Target moves with constant acceleration $[a_x(0),$
Fig. 3: Trajectory of the maneuvering target in Cartesian coordinates and the tracking result of the proposed method and method of Wang in the first case study.

Fig. 4: Speed of the maneuvering target and the estimation result of the proposed method and method of Wang in the first case study. $u(0) = [0.5(\text{m sec}^{-2}),0.5(\text{m sec}^{-2})]$ until $t = 75(\text{sec})$ (sample time = 5000). Then, it starts to maneuver with acceleration value $[u_x(5001), u_y(5001)] = [-4(\text{m sec}^{-2}),-4(\text{m sec}^{-2})]$. This acceleration continues until $t = 105(\text{sec})$ (Sample time = 7000). At $t = 105(\text{sec})$ moment, target starts to another maneuver with acceleration value $[u_x(7001), u_y(7001)] = [8(\text{m sec}^{-2}),8(\text{m sec}^{-2})]$. Target moves with this acceleration up to end of this simulation at $t = 150(\text{sec})$.

Figure 3 shows, target trajectory estimation by proposed method and method of Wang in this Case Study. As can be seen in this figure, proposed scheme tracks target more accurately in comparison with the other method. The Fig. 4 shows method of Wang tracks maneuvering target speed with lower accuracy in comparison with the proposed scheme.

In order to compare proposed scheme with method of Wang and simple KF (with no error covariance matrix resetting), a Mont Carlo (Robert and Casella, 1999) simulation of 50 runs was performed. The standard deviation (STD) of estimation error of range, azimuth, course and speed of all three methods in this case study is compared in Table 1.

Table 1: Estimation error in the first case study

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Method of Wang</th>
<th>Simple KF</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>397.93</td>
<td>435.25</td>
<td>20.64</td>
</tr>
<tr>
<td>Azimuth</td>
<td>5.12</td>
<td>5.71</td>
<td>0.52</td>
</tr>
<tr>
<td>Course</td>
<td>52.10</td>
<td>53.53</td>
<td>10.12</td>
</tr>
<tr>
<td>Speed</td>
<td>83.26</td>
<td>112.42</td>
<td>16.91</td>
</tr>
</tbody>
</table>

Fig. 5: Trajectory of the maneuvering target in Cartesian coordinates and tracking result of the proposed method, conventional resetting scheme and the method of Wang in the second case study.

Second case study: The initial position of the target is given by $[x(0), y(0)] = [-100(\text{m}),-173.2(\text{m})]$ with an initial speed of $[v_x(0), v_y(0)] = [-50(\text{m sec}^{-1}), -86.6(\text{m sec}^{-1})]$. During this case study which longs 45 (sec) target moves with the following jerk value $[J_x, J_y] = 4\text{Cos}(t/8)(\text{m sec}^{-1}), 4\text{Cos}(t/8)(\text{m sec}^{-1})]$. Tracking result of three methods Method of Wang and Varshney (1993) Conventional Resetting Scheme and proposed Method is compared in this case study. In this simulation resetting factor ($k_r$) and Trace Limit of conventional resetting scheme are assumed to be 1.2 and 100, respectively. Note that proper values for these two parameters were found using try and error method.

Figure 5 shows the accuracy of proposed method in tracking target, which moves with jerk. As can be seen in Fig. 5, very quick maneuvers are detected by proposed
maneuver detector system and error covariance matrix is reset when target maneuvers. Therefore, an accurate estimation result is obtained. Figure 6 and 7 show, target range and azimuth estimation by three methods. In fact, proposed scheme is able to track target speed and course much more accurately, as far as Fig. 8 and 9 suggest, in comparison with the other methods when target maneuvers quickly.

Table 2 highlights ability of proposed method to estimate target trajectory, range, azimuth, course and
speed in contrast with method of (Wang and Vershney, 1993) conventional resetting scheme. Mont Carlo simulation of 50 runs was performed and STD of estimation error on different parameters in the second case study obtained.

CONCLUSION

In order to track a maneuvering target, a new algorithm has been introduced in this paper, which uses an intelligent technique to reset the error covariance matrix of the KF. The proposed intelligent method is equipped with a fuzzy acceleration estimator system working based on some information about target maneuver dynamics. Proposed algorithm uses estimated acceleration in an innovative architecture in order to add two crucial ability to the conventional error covariance matrix resetting scheme. First, finding proper instances to reset the error covariance matrix. Second, determining optimized value for the resetting factor in each of iterations. Therefore, KF is used when target velocity is constant and when the target maneuvers system reset the error covariance matrix intelligently in order to have an accurate estimation.

Proposed method has been compared with the method of Wang, simple Kalman filter and the conventional resetting scheme in two different case studies of target movement including a high maneuvering target (in the first case study) and jerking target (in the second case study). Target range, azimuth, course, speed and trajectory estimation accuracy were considered in the simulations. Results indicate that the proposed method is significantly superior to three other methods in all target parameters estimation accuracy.

REFERENCES


