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Developing a New ELECTRE Method with Interval Data in Multiple Attribute Decision Making Problems

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Abstract: This study aims at providing a new and unique method to rank the alternatives with interval data in multi attribute decision making problems. Indeed, it is better to use interval data for problems which don't follow deterministic and exact data due to their nature (e.g., time, distance, temperature) or can not be presented easily as deterministic and specific numbers. So, in this study, we seek to develop the solution method for such kind of decision making problem (especially, when it is impossible to present data as fuzzy and thereby the use of fuzzy decision making method.

Key words: ELECTRE method, interval data, multi-criteria decision making, multiple attribute decision making, decision maker

INTRODUCTION

In the present world, most of the problems given to the managers and even our everyday life problems have various dimensions which are formulized with multiple variables. In other words, the final decision cannot be made with optimization of just one variable. So, it is natural that solving of these problems becomes more complicated especially when the variables are in contradiction and increasing the desirability of one variable may require decreasing the desirability of other one. So Multi-Criteria Decision Making (MCDM) and especially Multi Attribute Decision Making (MADM) have been developed for solving such kind of problems. A Multi Attribute Decision Making (MADM) problem can be summarized in the form of a matrix in which the rows are different alternatives and the columns are the criteria specifying the properties of the alternatives (Hwang and Masud, 1981). Also the matrix cells show the position of the row alternative in relation to the correspondent column criterion. Until now, we just presented the problem. Now to prioritize the alternatives, a decision technique is required that specifies an alternative of higher rank by interchanging and trading off of the different criteria (Hwang and Yoon, 1995).

Due to the nature of present data, the ELECTRE method must be developed and applied in this method. Since in the real world we are confronted with data having interval nature like temperature, energy-related problems and etc, the decision methods are required to be developed and matched to these kinds of data. The current method is presented to develop the ELECTRE method.

BACKGROUND

As a ranking method, ELECTRE has its own uses in energy planning and has been used in several researches. For instance, this method has been used in determining and defining an accessible penetration level for renewable energy resources in an isolated decentralized system for generating electricity (Papadopoulos and Karagiannidis, 2008). Also, it has been used in another research to assess an action plan for diffusing the renewable energy technologies. This method helps the energy department to choose the most suitable innovative technology (Beccali *et al*., 2003).

Almeida (2007) has used the ELECTRE method to choose the contractor for outsourcing the activities. In this study, the value and the cost of the service quality are considered and each criterion is evaluated against a utility function. Roy and Figueira (2002) proposed a method based on ELECTRE method and Simo's approach to determine the criteria. Combined together, ELECTRE and similar branch and bound techniques can be used to determine the non-dominated answers in a multiple objective mixed integer linear problem (Lourenco and Costa, 2003).

ELECTRE method can be used in facility layout problem. For instance in the research of Aiello *et al.* (2006) the Pareto-optimal solutions are determined by employing a multi-objective constrained genetic algorithm and the subsequent selection of the optimal solution is carried out by means of ELECTRE method which is one of the multi criteria decision methods. The transportation costs and the distance between the departments are some of the criteria used in the analysis.

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Group decision is one of the issues that deserves considerable attention. When multiple decision makers are present, the final decision will be the result of interaction of different ideas interaction, since each decision maker has his own information and values systems. ELECTRE method enhancement can help a decision group to achieve a consensus on a set of possible alternatives (Leyva-Lopez and Fernandez-Gonzalez, 2003).

In all the above mentioned problems, ELECTRE method is used as a tool for ranking and determining the best alternative. So, after reviewing literature and conducting researches, it seems necessary to do a comprehensive study on the development of this method as a fundamental study. In fact, this is one of the objectives pursued in the present study which has been little noticed so far. ELECTRE method is one of the effective methods for multi attribute decision making with qualitative and quantitative features. So, the development of this method for increasing the capability of such decision making and satisfying the exact requests of the decision maker is a very important task. Besides, in many cases we face with data of interval nature such as temperature, time, date and energy-related computations. In the real world, many problems conditions are impossible to be described using absolute numerical data. Indeed, fuzzy, interval and bounded data can often express the information well. In some situations, we can not use absolute and exact data because of the data collecting conditions. So, we must use interval data. For instance, for locating gas stations and first aid stations on roads or locating a fire station or hospital in a city, an exact location can not be usually candidate and these locations have to be considered as interval. One of the most important researches conducted in this area, in which an algorithmic method is presented to develop TOPSIS for the data having interval nature (Jahanshahloo ***et al.***, 2006).

In the above research, the existing steps in TOPSIS method (technique for order-preference by similarity to ideal solution) have been changed and a new structure has been developed for options ranking. Finally, a numerical example showing the necessity of developing the method has been solved. However, since many problems such as locating and energy problems are analyzed using ELECTRE methods and are expressed by interval data, we try to meet such needs in the present study. Since classic and fuzzy methods cannot provide exact and suitable answers to many problems, we present a new algorithmic method for ELECTRE method with interval data for ranking.

THE PRINCIPLES OF DECISION METHODS

Generally, Multi-Criteria Decision making (MCDM) models have been proposed for complicated decision having multiple optimization assessment criteria. Some uses of MCDM in engineering include flexible manufacturing systems (Cambron and Evans, 1991), layout design (Putrus, 1990), integrated manufacturing systems (Boucher and Macstravic, 1991) and the evaluation of technology investment decisions (Wanga and Triantaphylloub, 2008; Wabalickis, 1988.). These models are generally classified into two categories (Hwang and Masud, 1981; Hwang and Yoon, 1995):

- Multi Objective Decision (MODM)
- Multi Attribute Decision Making (MADM)

Here, the main emphasis is on multi attribute decision making. The MADM models are choosers and are used to choose the most suitable alternative among m criteria and as we know, MODM models are used for designing. Any MADM problem has multiple attributes that the decision maker has to exactly specify them in the problem. The number of the criteria depends on the nature of the problem. In Multi-Criteria Decision making, multiple criteria which are sometimes in contradiction are considered and this usually happens in daily life. For instance in the personal life factors such as job selection, job prestige, work place, salary and wages, progress opportunities, work conditions and etc, are considered as criteria and could be of high importance for a person. In organizational problems, when strategy selection of an organization is considered, the criteria such as obtained income in a period of time, stock value of an organization, market share, organization image in society (key money) and etc can be important. In governmental problems, the transportation sector has to design the transportation system in a way that the travel time, delays, transportation cost and etc have become minimized. Multi attribute decision making is normally formularized like the following matrix:

$$\begin{array}{c|cccc} & x_1 & x_2 & \cdots & x_n \\ \hline A_1 & r_{11} & r_{12} & \cdots & r_{1n} \\ A_2 & . & . & . & . \\ . & . & . & . & . \\ . & . & .. & . & . \\ A_m & r_{m1} & r_{m2} & \cdots & r_{mn} \end{array}$$

where, A_i indicates the ith alternative and r_{ij} indicates the value of jth criteria for ith alternative.

MADM models can be analyzed using different information processing techniques based on the criteria provided by the decision maker. To this aim, the MADM data are classified into two general categories (Hwang and Masud, 1981; Hwang and Yoon, 1995):

- **Compensatory models:** In these models, interchange among criteria is allowed. This indicates indicating that changing of a criterion is compensated by an opposite change (in converse direction) in the other attribute(s). Compensatory models include simple weight mean, TOPSIS (technique for order-preference by similarity to ideal solution), ELECTRE (elimination et choice translating reality), linear assignment, AHP and etc
- **Non-compensatory models:** In these models, interchange among criteria is not allowed. This indicates that the weakness of an criterion is not compensated by the strength of another one, but each criterion is separately considered as the assessment basis of competing alternatives. The important advantage of these models is their simplicity that is compatible with the behavior and information limitation of the decision maker. Non-compensatory methods include dominance method, lexicography, elimination, maximin, minimin, conjunctive-satisfying-method and disjunctive-satisfying-method

Compensatory models are themselves classified into three main subgroups:

- **Scoring sub-model:** This model aims at finding a desirability function for each alternative and choosing the alternative with highest desirability. The related methods include simple additive weighting method (SAW), interactive simple average weighting method and hierarchical additive weighting method
- **Compromising subgroup:** In this subgroup, the option nearest to the ideal option is chosen. The methods belongs to this subgroup include TOPSIS, MRS (marginal rate of substitution of attributes), MDS (multidimensional-scaling-with ideal-point) and LINMAP (linear programming for multidimensional analysis of preferences)
- **Concordance subgroup:** In these models, the output is a set of ranks such that the necessary concordance will be provided in the most suitable way. This subgroup includes ELECTRE methods and linear assignment (Hwang and Masud, 1981; Hwang and Yoon, 1995)

THE REASON OF ELECTRE METHOD SELECTION BY CONSIDERING INTERVAL DATA

This study aims at analyzing a problem with a suitable MADM model. Having this fact in mind that in compensatory models the interchange among the criteria is permitted, the criteria are not viewed separately but are analyzed as a whole set and the interactions and relations among the criteria are taken into account. So, when there is such kind of relations among the criteria, using the compensatory models is recommended. Besides, in real world we face with a lot of problems which are of compensatory models kind.

In scoring-sub model, there is always the need for finding a desirability function and this in turn requires the complete recognition of the problem and high experience. This is a weakness of this subgroup. So, the problem is how to find the multi attribute desirability function. Also, the compromising subgroup selects the option nearest to the optimized answer. This may not be free of errors and not be usable practically in the problem. In the present study, we chose the ELECTRE method of the concordance subgroup, because we need to find an order and in fact, rank the existing alternatives (Hwang and Masud, 1981; Hwang and Yoon, 1995).

ELECTRE METHOD

ELECTRE is a multi attribute decision making method for ranking multiple alternatives based on some criteria. This method is a very effective assessment solution that equips the decision activities with quantitative and qualitative features (Huang and Chen, 2005). As mentioned before, ELECTRE is one of the most important compensatory methods (Hwang and Masud, 1981).

In this method, the output is a set of ranks such that the necessary concordance will be provided in the most appropriate form. ELECTRE uses a new concept known as outranking. For example $A_k \rightarrow A_l$ indicates that although k and l options do not have any priority to each other mathematically, the decision maker accepts that A_k is better than A_l (Hwang and Masud, 1981). In this method, all the alternatives are assessed using the outranking comparisons and the non-effective alternatives are omitted. Pair comparisons performed based on agreement rank of weights (W_j) and difference rank from weighting assessment values (V_{ij}) and are tested simultaneously for alternatives assessment. All these steps are planned according to a concordant and a discordant set that is known as concordance analysis.

ELECTRE METHOD WITH INTERVAL DATA

In the present study, we have tried to develop the ELECTRE model by using the interval data. As mentioned earlier, in many problems the output data are not exact values and fluctuate or due to the nature of the problem can be expressed as interval. We can mention time, date and temperature are some examples.

Decision matrix based on the interval data is shown below. In this matrix, A_i s are analyzed alternatives and C_j s are selection criteria:

	C_1	C_2	....	C_n
A_1	$[r_{11}^L, r_{11}^U]$	$[r_{12}^L, r_{12}^U]$	....	$[r_{1n}^L, r_{1n}^U]$
A_2	$[r_{21}^L, r_{21}^U]$	$[r_{22}^L, r_{22}^U]$	....	$[r_{2n}^L, r_{2n}^U]$
A_m	$[r_{m1}^L, r_{m1}^U]$	$[r_{m2}^L, r_{m2}^U]$	....	$[r_{mn}^L, r_{mn}^U]$

$$W = \{w_1, w_2, ..., w_n\}$$

where, W_j is the weight of criterion C_j.

The developed ELECTRE algorithm is described as follows.

Step 1

Computing dimensionless decision making matrix: Here, we need a new method of unscaling, because interval data are present. In this method, the upper and lower limits of the numbers are unscaled separately. In this step, the decision matrix is transformed into a normalized matrix by using the following relationships:

$$n_{ij}^L = \frac{r_{ij}^L}{\sqrt{\sum_{i=1}^{m}(r_{ij}^L)^2 + \sum_{i=1}^{m} r_{ij}^U}} \qquad \begin{matrix} i = 1,...,n \\ j = 1,...,m \end{matrix} \qquad (1)$$

$$n_{ij}^U = \frac{r_{ij}^U}{\sqrt{\sum_{i=1}^{m}(r_{ij}^L)^2 + \sum_{i=1}^{m} r_{ij}^U}} \qquad \begin{matrix} i = 1,...,n \\ j = 1,...,m \end{matrix} \qquad (2)$$

Step 2

Creating a weighted dimensionless matrix (V): Here, a weighted dimensionless matrix is calculated by using a known vector W.

$$W = \{w_1, w_2, ..., w_n\} \approx \qquad \text{Assumed from DM}$$

$$V_{ij}^L = w_i * n_{ij}^L \qquad \begin{matrix} i = 1,...,n \\ j = 1,...,m \end{matrix} \qquad (3)$$

$$V_{ij}^U = w_i * n_{ij}^U \qquad \begin{matrix} i = 1,...,n \\ j = 1,...,m \end{matrix} \qquad (4)$$

$$V = \begin{vmatrix} (V_{11}^L, V_{11}^U), & ...... & (V_{1j}^L, V_{1j}^U), & ...... & (V_{1n}^L, V_{1n}^U) \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ (V_{m1}^L, V_{m1}^U) & ...... & (V_{mj}^L, V_{mj}^U) & ...... & (V_{mn}^L, V_{mn}^U) \end{vmatrix} \qquad (5)$$

Such that n_{ij}^L and n_{ij}^U are the data of ND matrix that their criterion scores are dimensionless and comparable. W_{n*n} is a diagonal matrix obtained from weighting criteria.

Step 3: Specifying the concordance and discordance sets for each pair of alternatives $l \neq k$; k,l = 1,2,3,...,n. In this step, the existing attributes set $J = \{j|j = 1,2,...,n\}$ is divided by distinguished concordant ($S_{k,l}$) and discordant ($D_{k,l}$) subsets.

The concordant set ($S_{k,l}$) will include A_k and A_l with criteria such that A_k will be preferred to A_l. On the other hand, due to interval nature of data, if two concordance and discordance sets are presented in ELECTRE classic algorithm, there will be some difficulties in data distribution. In fact, in this case it is not clear which of the upper or lower limits has to be used for formation of concordant and discordant sets. So, the best possible method is dividing each concordance and discordance sets by upper and lower limit sets. In this procedure, the concordance set includes two sets $S_{k,l}^L$ and $S_{k,l}^U$ and the discordance set includes two sets $D_{k,l}^L$ and $D_{k,l}^U$. In other words, at first the lower limits of the decision matrix is analyzed like a classic ELECTRE problem and two concordance and discordance sets are formed. After that, the same procedure is repeated for upper limits. (r_{ij} is assumed to have increasing desirability). So, concordance set for upper and lower limits is as follows:

$$S_{k,l}^L = \{i \in I \mid r_{ki}^L \geq r_{li}^L, \quad j \in J \mid r_{ki}^L \leq r_{li}^L\} \qquad (6)$$

$$S_{k,l}^U = \{i \in I \mid r_{ki}^U \geq r_{li}^U, \quad j \in J \mid r_{ki}^U \leq r_{li}^U\} \qquad (7)$$

where, I and J indicate the positive (profit) and the negative (cost) sets, respectively.

In order to find the upper and lower limits of the discordance sets, the following formulas are used:

$$D_{k,l}^L = \{i \in I \mid r_{ki}^L \prec r_{li}^L, \quad j \in J \mid r_{ki}^L \succ r_{li}^L\} \qquad (8)$$

$$D_{k,l}^U = \{i \in I \mid r_{ki}^U \prec r_{li}^U, \quad j \in J \mid r_{ki}^U \succ r_{li}^U\} \qquad (9)$$

Step 4

Computing concordance matrix: The concordance criterion $I_{k,l}$ is equal to the average of $I_{k,l}^L$ and $I_{k,l}^U$ values. Since, here the concordance set includes two subsets $S_{k,l}^U$ and $S_{k,l}^U$, these two matrices are separately determined and finally the average of two values is calculated in order to form the concordance matrix. The concordance matrix $I^L_{k,l}$ is the weight sum (W_j) of criteria forming the $S^L_{k,l}$ set. So the concordance criterion ($I^L_{k,l}$) between A_k and A_l is as follows:

$$I^L_{k,l} = \sum_{j \in S^L_{k,l}} w_j; \qquad \sum_{j=1}^{n} w_j = 1 \qquad \forall j \in S^L_{k,l} \tag{10}$$

And the concordance matrix $I^U_{k,l}$ is the weight sum (W_j) of criteria forming the $S^U_{k,l}$ set. The concordance criterion ($I^U_{k,l}$) between A_k and A_l is as follows:

$$I^U_{k,l} = \sum_{j \in S^U_{k,l}} w_j; \qquad \sum_{j=1}^{n} w_j = 1 \qquad \forall j \in S^U_{k,l} \tag{11}$$

Finally, the concordance matrix $I_{k,l}$ must be created:

$$I_{k,l} = \frac{\sum_{j \in S^L_{k,l}} w_j + \sum_{j \in S^U_{k,l}} w_j}{2} \tag{12}$$

The concordance criterion $I_{k,l}$ reflects the relative importance of A_k in comparison with A_l such that $0 \le I_{k,l} \le 1$. Higher value of $I_{k,l}$ means that the priority of A_k over A_l is more concordant. So, we have to find only one matrix and that is why the mentioned values are averaged.

Hence, the sequential values of $I_{k,l}$ criteria ($k,l = 1,2,...,m$, $k \neq 1$) form the asymmetric concordance matrix (I) as follows:

$$I = \begin{vmatrix} - & I_{1,2} & I_{1,3} & .... & I_{1,m} \\ I_{2,1} & - & I_{2,3} & .... & I_{2,m} \\ . & . & - & .... & I_{2,m} \\ . & . & . & .... & . \\ . & . & . & - & . \\ I_{m1} & I_{m2} & .... & I_{m(m-1)} & - \end{vmatrix} \tag{13}$$

Step 5

Computing discordance matrix: In contrast to $I_{k,l}$ criterion, the discordance criterion shows how much A_k assessment is worse than A_l. Because of the presence of interval data, two values $D^L_{k,l}$ and $D^U_{k,l}$ are defined like the concordance matrix; it means that this criterion for each discordance sets $D^L_{k,l}$ is calculated by using the V matrix elements (weighted scores) for the correspondent discordant set and finally the average value of $NI^L_{k,l}$ and $NI^U_{k,l}$ is used for computing discordance matrix $NI_{k,l}$ as follows:

$$NI^L_{k,l} = \frac{\max_{j \in D^L_{k,l}} |V_{kj} - V_{lj}|}{\max_{j \in J} |V_{kj} - V_{lj}|} \tag{14}$$

$$NI^U_{k,l} = \frac{\max_{j \in D^U_{k,l}} |V_{kj} - V_{lj}|}{\max_{j \in J} |V_{kj} - V_{lj}|} \tag{15}$$

o, matrix $NI_{k,l}$ is computed as follows:

$$NI_{k,l} = \frac{NI^L_{k,l} + NI^U_{k,l}}{2} \tag{16}$$

Hence, the discordance matrix for each pair comparison of the alternatives will be as follows:

$$NI = \begin{vmatrix} - & NI_{1,2} & NI_{1,3} & .... & NI_{1,m} \\ NI_{2,1} & - & NI_{2,3} & .... & NI_{2,m} \\ . & . & - & .... & NI_{2,m} \\ . & . & . & .... & . \\ . & . & . & - & . \\ NI_{m1} & NI_{m2} & .... & NI_{m(m-1)} & - \end{vmatrix} \tag{17}$$

It is clear that the information present in I and NI matrixes are considerably different and yet complement each other. Matrix I shows the weights w_j of concordant criteria and the asymmetrical NI matrix indicates the biggest relative difference from $V_{ij} = n_{ij}.w_j$ for discordant criteria.

Step 6

Specifying the effective concordant matrix: $I_{k,l}$ values of concordance matrix have to be assessed in relation to a threshold value so, that the priority chance of A_k over A_l can be better judged. This chance will increase if $I_{k,l}$ exceeds a minimum threshold ($\bar{I}$):

$$I_{kl} \ge \bar{I} \tag{18}$$

For instance, $\bar{I}$ (optional) can be calculated by averaging concordance criteria as follows:

$$\bar{I} = \sum_{k=1}^{m} \sum_{l=1}^{m} I_{k,l} / m(m-1) \tag{19}$$

Then according to the $\bar{I}$, a Boolean F matrix (having 0s and 1s) is created such that:

$$f_{k,l} = 1 \rightarrow I_{k,l} \ge \bar{I} \tag{20}$$

$$f_{k,l} = 0 \rightarrow I_{k,l} \prec \bar{I} \tag{21}$$

Then, each element of matrix F (the effective concordant matrix) will show an effective alternative and will be dominant alternative.

Step 7

Specifying the effective discordant matrix: The $Ni_{k,l}$ elements of discordant matrix also have to be assessed against a threshold value as in step 6. For instance, the threshold value $\bar{NI}$ can be calculated as follows:

$$\overline{NI} = \sum_{k=1}^{m}\sum_{l=1}^{m} NI_{k,l} / m(m-1) \tag{22}$$

Then a Boolean G matrix (known as the effective discordant matrix) is created such that:

$$g_{k,l} = 1 \rightarrow NI_{k,l} \leq \overline{NI} \tag{23}$$

$$g_{k,l} = 0 \rightarrow NI_{k,l} \succ \overline{NI} \tag{24}$$

Elements of matrix G having the value of 1 show the dominance relations among the alternatives.

Step 8

Specifying the global and effective matrix: The common elements (h_{kl}) form the global matrix (h) for decision through peer to peer multiplication of matrices F and G.

$$h_{k,l} = f_{k,l} \cdot g_{k,l} \tag{25}$$

Step 9

Neglecting non-effective alternatives: The global matrix h shows the relative priority order of the alternatives. It means that $h_{kl} = 1$ indicates that A_k is preferred to A_l from the viewpoint of concordance and discordance criteria, but A_k may be still dominated by other alternatives. Therefore, In order to have A_k as an effective alternative when using the ELECTRE method, the following conditions must be met:

$$h_{kl} = 1 \rightarrow \quad \text{so, that for at least one I} \quad I = 1,2,...,m \tag{26}$$

$$h_{kl} = 0 \rightarrow \quad \text{for I} \qquad i = 1,2,...,m; \quad i \neq k; i \neq 1 \tag{27}$$

Meeting both of these two conditions at the same time may be rarely occurs. But the effective elements can simply be distinguished in matrix h in a way that each column in h having at least one 1 element can be omitted, since this column is dominated by one or more rows.

One way for determining ranking is using graphs. In such graphs, all the alternatives are displayed using nodes. $f_{kl} = 1$ shows the path (arc) between two nodes k and l and $f_{kl} = 0$ shows that there is no path between k and l. In this graph the alternative having the most output is selected as the best option. Also the options can be sorted according to their output numbers.

NUMERICAL EXAMPLE

Here, the problem is solved based on data provided in (Jahanshahloo *et al*., 2006) which are interval data of 15 bank branches (A_1, A_2,...A_{15}) in Iran. The method used in this example for calculating numerical values by the developed ELECTRE method is presented below.

Step 1: Dimensionless decision matrix computation (using the formulas (1) and (2)). The interval decision matrix and interval normalized decision matrix are shown in Table 1 and 2, respectively.

Step 2: Creating a weighted dimensionless matrix (V) (using the formulas (3 and 4)). The interval weighted normalized decision matrix is presented in Table 3.

Step 3: Specifying the concordance and discordance sets for each pair of alternatives (using the formulas (6) and (7) for concordance set and formulas (8) and (9) for discordance sets). The upper and lower limits of concordance set and discordance sets are presented in Table 4 and 5, respectively.

Step 4: Computing concordance matrix: (using formulas (10-12)). The concordance matrix is as following matrix:

$$I = \begin{vmatrix}
- & 0.375 & 0.375 & 0.5 & 0.375 & 0.1875 & 0.25 & 0.5 & 0.3125 & 0.5 & 0.375 & 0.3125 & 0.5 & 0.25 & 0.4375 \\
0.125 & - & 0.1875 & 0.375 & 0.3125 & 0.1875 & 0.3125 & 0.5 & 0.3125 & 0.375 & 0.375 & 0.25 & 0.3125 & 0.1875 & 0.375 \\
0.125 & 0.3125 & - & 0.5 & 0.375 & 0.25 & 0.25 & 0.5 & 0.375 & 0.375 & 0.375 & 0.375 & 0.5 & 0.3125 & 0.4375 \\
0 & 0.125 & 0 & - & 0.125 & 0.0.0625 & 0.0.0625 & 0.125 & 0 & 0 & 0.125 & 0.125 & 0.125 & 0 & 0.125 \\
0.125 & 0.1875 & 0.125 & 0.375 & - & 0.125 & 0.125 & 0.375 & 0.1875 & 0.25 & 0.25 & 0.1875 & 0.25 & 0.125 & 0.375 \\
0.3125 & 0.3125 & 0.25 & 0.4375 & 0.375 & - & 0.375 & 0.375 & 0.25 & 0.375 & 0.375 & 0.375 & 0.5 & 0.25 & 0.375 \\
0.25 & 0.1875 & 0.25 & 0.4375 & 0.375 & 0.125 & - & 0.375 & 0.3125 & 0.375 & 0.4375 & 0.375 & 0.5 & 0.25 & 0.375 \\
0 & 0 & 0 & 0.3125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.125 & 0.0625 & 0.1875 \\
0.1875 & 0.1875 & 0.125 & 0.5 & 0.3125 & 0.25 & 0.1875 & 0.375 & - & 0.5 & 0.5 & 0.25 & 0.4375 & 0.1875 & 0.3125 \\
0 & 0.125 & 0.125 & 0.5 & 0.25 & 0.125 & 0.125 & 0.375 & 0 & - & 0.25 & 0.1875 & 0.25 & 0 & 0.375 \\
0.125 & 0.125 & 0.125 & 0.375 & 0.25 & 0.125 & 0.0625 & 0.375 & 0 & 0.25 & - & 0.125 & 0.25 & 0 & 0.375 \\
0.1875 & 0.25 & 0.125 & 0.375 & 0.3125 & 0.125 & 0.125 & 0.375 & 0.25 & 0.3125 & 0.375 & - & 0.375 & 0.125 & 0.375 \\
0 & 0.1875 & 0 & 0.375 & 0.25 & 0 & 0 & 0.375 & 0.0625 & 0.25 & 0.25 & 0.125 & - & 0 & 0.25 \\
0.25 & 0.3125 & 0.1875 & 0.5 & 0.375 & 0.25 & 0.25 & 0.4375 & 0.3125 & 0.5 & 0.5 & 0.375 & 0.5 & - & 0.375 \\
0.0625 & 0.125 & 0.0625 & 0.375 & 0.125 & 0.125 & 0.125 & 0.375 & 0.1875 & 0.125 & 0.125 & 0.125 & 0.25 & 0.125 & -
\end{vmatrix} \tag{28}$$

Table 1: The interval decision matrix

	C_1		C_2		C_3		C_4	
	r^U_{1j}	r^L_{1j}	r^U_{2j}	r^L_{2j}	r^U_{3j}	r^L_{3j}	r^U_{4j}	r^L_{4j}
A_1	961.37	500.37	3126798	2696995	38254	26364	6957.33	965.97
A_2	1775.50	873.70	1061260	1027546	50308	3791	3174.00	2285.03
A_3	196.39	95.93	1213541	1145235	26846	22964	510.93	207.98
A_4	1752.66	848.07	395241	390902	1213	492	92.30	63.32
A_5	120.47	58.69	165818	144906	18061	18053	370.81	176.58
A_6	955.61	464.39	416416	408163	48643	40539	5882.53	4654.71
A_7	342.89	155.29	410427	335070	44933	33797	2506.67	560.26
A_8	3629.54	1752.31	768593	700842	1519	1437	86.86	58.89
A_9	495.78	244.34	696338	641680	24108	11418	2283.08	1070.81
A_{10}	1417.11	730.27	481943	453170	2955	2719	559.85	375.07
A_{11}	931.01	454.75	342598	309670	2617	2016	1468.45	936.62
A_{12}	630.01	303.58	317186	286149	27070	14918	4335.24	1203.79
A_{13}	1345.58	658.81	347848	321435	8045	6616	399.80	200.36
A_{14}	860.79	144.68	835839	618105	40457	24428	4555.42	2781.24
A_{15}	292.15	144.68	12028	119948	1749	1494	471.22	282.73

Table 2: The interval normalized decision matrix

	C_1		C_2		C_3		C_4	
	n^U_{1j}	n^L_{1j}	n^U_{2j}	n^L_{2j}	n^U_{3j}	n^L_{3j}	n^U_{4j}	n^L_{4j}
A_1	0.1645	0.0856	0.6001	0.5176	0.2865	0.1974	0.5086	0.0706
A_2	0.3038	0.1495	0.2037	0.1972	0.3768	0.0283	0.2320	0.1670
A_3	0.0336	0.0164	0.2329	0.2198	0.2010	0.1720	0.0373	0.0152
A_4	0.2999	0.1451	0.0758	0.0750	0.0090	0.0036	0.0067	0.0046
A_5	0.0206	0.0100	0.0318	0.0278	0.1352	0.1352	0.0271	0.0129
A_6	0.1635	0.0794	0.0799	0.0783	0.3643	0.3036	0.4300	0.3403
A_7	0.0586	0.0265	0.0787	0.0643	0.3365	0.2531	0.1832	0.0409
A_8	0.6211	0.2999	0.1475	0.1345	0.0113	0.0107	0.0063	0.0043
A_9	0.0848	0.0418	0.1336	0.1231	0.1805	0.0855	0.1669	0.0782
A_{10}	0.2425	0.1249	0.0925	0.0869	0.0221	0.0203	0.0409	0..0274
A_{11}	0.1078	0.0519	0.0657	0.0594	0.0196	0.0151	0.1073	0.0684
A_{12}	0.1078	0.0519	0.0608	0.0549	0.2027	0.1117	0.3169	0.0880
A_{13}	0.2303	0.1127	0.0667	0.0616	0.0602	0.0495	0.0292	0.0146
A_{14}	0.1473	0.0719	0.1604	0.1186	0.3030	0.1829	0.3330	0.2033
A_{15}	0.0500	0.0247	0.0230	0.0230	0.0131	0.0111	0.0344	0.0206

Table 3: The interval weighted normalized decision matrix

	C_1		C_2		C_3		C_4	
	υ^U_{1j}	υ^L_{1j}	υ^U_{2j}	υ^L_{2j}	υ^U_{3j}	υ^L_{3j}	υ^U_{4j}	υ^L_{4j}
A_1	0.0205	0.0107	0.07505	0.06471	0.0358	0.0246	0.0635	0.0088
A_2	0.0379	0.0186	0.0254	0.0246	0.0471	0.0035	0.0209	0.0208
A_3	0.0042	0.0020	0.0291	0.0274	0.0251	0.0215	0.0046	0.0019
A_4	0.0374	0.0181	0.0094	0.0093	0.0011	0.0004	0.0008	0.0005
A_5	0.0025	0.0012	0.0039	0.0034	0.0169	0.0169	0.0033	0.0016
A_6	0.0204	0.0099	0.0099	0.0097	0.0455	0.0379	0.0537	0.0425
A_7	0.0073	0.0033	0.0098	0.0080	0.0420	0.0316	0.0229	0.0051
A_8	0.07766	0.0374	0.0184	0.0168	0.0014	0.0013	0.0007	0.0005
A_9	0.0303	0.0156	0.0115	0.0108	0.0225	0.0025	0.0208	0.0097
A_{10}	0.0303	0.0156	0.0115	0.0108	0.0027	0.0025	0.0051	0.0034
A_{11}	0.0199	0.0097	0.0082	0.0074	0.0024	0.0018	0.0134	0.0085
A_{12}	0.0134	0.0064	0.0076	0.0068	0.0253	0.0139	0.0396	0.0110
A_{13}	0.0287	0.0140	0.0083	0.0077	0.0075	0.0061	0.0036	0.0018
A_{14}	0.0184	0.0089	0.0200	0.0148	0.0378	0.0228	0.0416	0.0254
A_{15}	0.0062	0.0030	0.0028	0.0028	0.0016	0.0013	0.0043	0.0025

Step 5: Computing discordance matrix (using formulas (14) and (15)). The discordance matrix shown below is a big matrix. So, we assign one matrix to the lower limit and another one to the upper limit (as shown in matrices (29) and (30), respectively).

Table 4: The concordance sets for 15 existing alternatives (upper and lower limits)

$C^{U}_{1,5} = \{2,3,4\}$	$C^{L}_{1,5} = \{2,3,4\}$	$C^{U}_{1,4} = \{1,2,3,4\}$	$C^{L}_{1,4} = \{1,2,3,4\}$	$C^{U}_{1,3} = \{2,3,4\}$	$C^{L}_{1,3} = \{2,3,4\}$	$C^{U}_{1,2} = \{1,2,4\}$	$C^{L}_{1,2} = \{1,2,3\}$
$C^{U}_{1,9} = \{2,3,4\}$	$C^{L}_{1,9} = \{2,3\}$	$C^{U}_{1,8} = \{1,2,3,4\}$	$C^{L}_{1,8} = \{1,2,3,4\}$	$C^{U}_{1,7} = \{2,4\}$	$C^{L}_{1,7} = \{2,4\}$	$C^{U}_{1,6} = \{2,4\}$	$C^{L}_{1,6} = \{2\}$
$C^{U}_{1,13} = \{1,2,3,4\}$	$C^{L}_{1,13} = \{1,2,3,4\}$	$C^{U}_{1,12} = \{2,3,4\}$	$C^{L}_{1,12} = \{2,3\}$	$C^{U}_{1,11} = \{2,3,4\}$	$C^{L}_{1,11} = \{2,3,4\}$	$C^{U}_{1,10} = \{1,2,3,4\}$	$C^{L}_{1,10} = \{1,2,3,4\}$
$C^{U}_{2,3} = \{3,4\}$	$C^{L}_{2,3} = \{4\}$	$C^{U}_{2,1} = \{3\}$	$C^{L}_{2,1} = \{4\}$	$C^{U}_{1,15} = \{1,2,3,4\}$	$C^{L}_{1,15} = \{2,3,4\}$	$C^{U}_{1,14} = \{2,4\}$	$C^{L}_{1,14} = \{2,3\}$
$C^{U}_{2,7} = \{2,3,4\}$	$C^{L}_{2,7} = \{2,4\}$	$C^{U}_{2,6} = \{2,3\}$	$C^{L}_{2,6} = \{2\}$	$C^{U}_{2,5} = \{2,3,4\}$	$C^{L}_{2,5} = \{2,4\}$	$C^{U}_{2,4} = \{2,3,4\}$	$C^{L}_{2,4} = \{2,3,4\}$
$C^{U}_{2,11} = \{2,3,4\}$	$C^{L}_{2,11} = \{2,3,4\}$	$C^{U}_{2,10} = \{2,3,4\}$	$C^{L}_{2,10} = \{2,3,4\}$	$C^{U}_{2,9} = \{2,3,4\}$	$C^{L}_{2,9} = \{2,4\}$	$C^{U}_{2,8} = \{1,2,3,4\}$	$C^{L}_{2,8} = \{1,2,3,4\}$
$C^{U}_{2,15} = \{2,3,4\}$	$C^{L}_{2,15} = \{2,3,4\}$	$C^{U}_{2,14} = \{2,3\}$	$C^{L}_{2,14} = \{2\}$	$C^{U}_{2,13} = \{2,3,4\}$	$C^{L}_{2,13} = \{2,4\}$	$C^{U}_{2,12} = \{2,3\}$	$C^{L}_{2,12} = \{2,4\}$
$C^{U}_{3,5} = \{2,3,4\}$	$C^{L}_{3,5} = \{2,3,4\}$	$C^{U}_{3,4} = \{1,2,3,4\}$	$C^{L}_{3,4} = \{1,2,3,4\}$	$C^{U}_{3,2} = \{1,2\}$	$C^{L}_{3,2} = \{1,2,3\}$	$C^{U}_{3,1} = \{1\}$	$C^{L}_{3,1} = \{1\}$
$C^{U}_{3,9} = \{1,2,3\}$	$C^{L}_{3,9} = \{1,2,3\}$	$C^{U}_{3,8} = \{1,2,3,4\}$	$C^{L}_{3,8} = \{1,2,3,4\}$	$C^{U}_{3,7} = \{1,2\}$	$C^{L}_{3,7} = \{1,2\}$	$C^{U}_{3,6} = \{1,2\}$	$C^{L}_{3,6} = \{1,2\}$
$C^{U}_{3,13} = \{1,2,3,4\}$	$C^{L}_{3,13} = \{1,2,3,4\}$	$C^{U}_{3,12} = \{1,2\}$	$C^{L}_{3,12} = \{1,2,3,4\}$	$C^{U}_{3,11} = \{1,2,3\}$	$C^{L}_{3,11} = \{1,2,3\}$	$C^{U}_{3,10} = \{1,2,3\}$	$C^{L}_{3,10} = \{1,2,3\}$
$C^{U}_{4,2} = \{1\}$	$C^{L}_{4,2} = \{1\}$	$C^{U}_{4,1} = \{\}$	$C^{L}_{4,1} = \{\}$	$C^{U}_{3,15} = \{1,2,3,4\}$	$C^{L}_{3,15} = \{1,2,3\}$	$C^{U}_{3,14} = \{1,2\}$	$C^{L}_{3,14} = \{1,2,3\}$
$C^{U}_{4,7} = \{\}$	$C^{L}_{4,7} = \{2\}$	$C^{U}_{4,6} = \{2\}$	$C^{L}_{4,6} = \{\}$	$C^{U}_{4,5} = \{1,3,4\}$	$C^{L}_{4,5} = \{2\}$	$C^{U}_{4,3} = \{\}$	$C^{L}_{4,3} = \{\}$
$C^{U}_{4,11} = \{2\}$	$C^{L}_{4,11} = \{2\}$	$C^{U}_{4,10} = \{\}$	$C^{L}_{4,10} = \{\}$	$C^{U}_{4,9} = \{\}$	$C^{L}_{4,9} = \{\}$	$C^{U}_{4,8} = \{1,4\}$	$C^{L}_{4,8} = \{1,4\}$
$C^{U}_{4,15} = \{2\}$	$C^{L}_{4,15} = \{2\}$	$C^{U}_{4,14} = \{\}$	$C^{L}_{4,14} = \{\}$	$C^{U}_{4,13} = \{2\}$	$C^{L}_{4,13} = \{2\}$	$C^{U}_{4,12} = \{2\}$	$C^{L}_{4,12} = \{2\}$
$C^{U}_{5,4} = \{1,3,4\}$	$C^{L}_{5,4} = \{2\}$	$C^{U}_{5,3} = \{1\}$	$C^{L}_{5,3} = \{1\}$	$C^{U}_{5,2} = \{1\}$	$C^{L}_{5,2} = \{1,3\}$	$C^{U}_{5,1} = \{1\}$	$C^{L}_{5,1} = \{1\}$
$C^{U}_{5,9} = \{1\}$	$C^{L}_{5,9} = \{1,3\}$	$C^{U}_{5,8} = \{1,3,4\}$	$C^{L}_{5,8} = \{1,3,4\}$	$C^{U}_{5,7} = \{1\}$	$C^{L}_{5,7} = \{1\}$	$C^{U}_{5,6} = \{1\}$	$C^{L}_{5,6} = \{1\}$
$C^{U}_{5,13} = \{1,3\}$	$C^{L}_{5,13} = \{1,3\}$	$C^{U}_{5,12} = \{1\}$	$C^{L}_{5,12} = \{1,3\}$	$C^{U}_{5,11} = \{1,3\}$	$C^{L}_{5,11} = \{1,3\}$	$C^{U}_{5,10} = \{1,3\}$	$C^{L}_{5,10} = \{1,3\}$
$C^{U}_{6,2} = \{1,4\}$	$C^{L}_{6,2} = \{1,3,4\}$	$C^{U}_{6,1} = \{1,3\}$	$C^{L}_{6,1} = \{1,3,4\}$	$C^{U}_{5,15} = \{1,2,3\}$	$C^{L}_{5,15} = \{1,2,3\}$	$C^{U}_{5,14} = \{1\}$	$C^{L}_{5,14} = \{1\}$
$C^{U}_{6,7} = \{2,3,4\}$	$C^{L}_{6,7} = \{2,3,4\}$	$C^{U}_{6,5} = \{2,3,4\}$	$C^{L}_{6,5} = \{2,3,4\}$	$C^{U}_{6,4} = \{1,3,4\}$	$C^{L}_{6,4} = \{1,2,3,4\}$	$C^{U}_{6,3} = \{3,4\}$	$C^{L}_{6,3} = \{3,4\}$
$C^{U}_{6,11} = \{2,3,4\}$	$C^{L}_{6,11} = \{2,3,4\}$	$C^{U}_{6,10} = \{1,3,4\}$	$C^{L}_{6,10} = \{1,3,4\}$	$C^{U}_{6,9} = \{3,4\}$	$C^{L}_{6,9} = \{3,4\}$	$C^{U}_{6,8} = \{1,3,4\}$	$C^{L}_{6,8} = \{1,3,4\}$
$C^{U}_{6,15} = \{2,3,4\}$	$C^{L}_{6,15} = \{2,3,4\}$	$C^{U}_{6,14} = \{3,4\}$	$C^{L}_{6,14} = \{3,4\}$	$C^{U}_{6,13} = \{1,2,3,4\}$	$C^{L}_{6,13} = \{1,2,3,4\}$	$C^{U}_{6,12} = \{2,3,4\}$	$C^{L}_{6,12} = \{2,3,4\}$
$C^{U}_{7,4} = \{1,2,3,4\}$	$C^{L}_{7,4} = \{1,3,4\}$	$C^{U}_{7,3} = \{3,4\}$	$C^{L}_{7,3} = \{3,4\}$	$C^{U}_{7,2} = \{1\}$	$C^{L}_{7,2} = \{1,3\}$	$C_{7,1} = \{1,3\}$	$C^{L}_{7,1} = \{1,3\}$
$C^{U}_{7,9} = \{1,3,4\}$	$C^{L}_{7,9} = \{1,3\}$	$C^{U}_{7,8} = \{1,3,4\}$	$C^{L}_{7,8} = \{1,3,4\}$	$C^{U}_{7,6} = \{1\}$	$C^{L}_{7,6} = \{1\}$	$C^{U}_{7,5} = \{2,3,4\}$	$C^{L}_{7,5} = \{2,3,4\}$
$C^{U}_{7,13} = \{1,2,3,4\}$	$C^{L}_{7,13} = \{1,2,3,4\}$	$C^{U}_{7,12} = \{1,2,3\}$	$C^{L}_{7,12} = \{1,2,3\}$	$C^{U}_{7,11} = \{1,2,3,4\}$	$C^{L}_{7,11} = \{1,2,3\}$	$C^{U}_{7,10} = \{1,3,4\}$	$C^{L}_{7,10} = \{1,3,4\}$
$C^{U}_{8,2} = \{\}$	$C^{L}_{8,2} = \{\}$	$C^{U}_{8,1} = \{\}$	$C^{L}_{8,1} = \{\}$	$C^{U}_{7,15} = \{2,3,4\}$	$C^{L}_{7,15} = \{2,3,4\}$	$C^{U}_{7,14} = \{1,3\}$	$C^{L}_{7,14} = \{1,3\}$
$C^{U}_{8,6} = \{2\}$	$C^{L}_{8,6} = \{2\}$	$C^{U}_{8,5} = \{2\}$	$C^{L}_{8,5} = \{2\}$	$C^{U}_{8,4} = \{2,3\}$	$C^{L}_{8,4} = \{2,3,4\}$	$C^{U}_{8,3} = \{\}$	$C^{L}_{8,3} = \{\}$
$C^{U}_{8,11} = \{2\}$	$C^{L}_{8,11} = \{2\}$	$C^{U}_{8,10} = \{2\}$	$C^{L}_{8,10} = \{2\}$	$C^{U}_{8,9} = \{2\}$	$C^{L}_{8,9} = \{2\}$	$C^{U}_{8,7} = \{2\}$	$C^{L}_{8,7} = \{2\}$
$C^{U}_{8,15} = \{2\}$	$C^{L}_{8,15} = \{2,3\}$	$D^{U}_{8,14} = \{\}$	$C^{L}_{8,14} = \{2\}$	$C^{U}_{8,13} = \{2\}$	$C^{L}_{8,13} = \{2\}$	$C^{U}_{8,12} = \{2\}$	$C^{L}_{8,12} = \{2\}$
$C^{U}_{9,4} = \{1,2,3,4\}$	$C^{L}_{9,4} = \{1,2,3,4\}$	$C^{U}_{9,3} = \{4\}$	$C^{L}_{9,3} = \{4\}$	$C^{U}_{9,2} = \{1\}$	$C^{L}_{9,2} = \{1,3\}$	$C^{U}_{9,1} = \{1\}$	$C^{L}_{9,1} = \{1,4\}$
$C^{U}_{9,8} = \{1,3,4\}$	$C^{L}_{9,8} = \{1,3,4\}$	$C^{U}_{9,7} = \{2\}$	$C^{L}_{9,7} = \{2,4\}$	$C^{U}_{9,6} = \{1,2\}$	$C^{L}_{9,6} = \{1,2\}$	$C^{U}_{9,5} = \{2,3,4\}$	$C^{L}_{9,5} = \{2,4\}$
$C^{U}_{9,13} = \{1,2,3,4\}$	$C^{L}_{9,13} = \{2,3,4\}$	$C^{U}_{9,12} = \{1,2\}$	$C^{L}_{9,12} = \{1,2\}$	$C^{U}_{9,11} = \{1,2,3,4\}$	$C^{L}_{9,11} = \{1,2,3,4\}$	$C^{U}_{9,10} = \{1,2,3,4\}$	$C^{L}_{9,10} = \{1,2,3,4\}$
$C^{U}_{10,2} = \{1\}$	$C^{L}_{10,2} = \{1\}$	$C^{U}_{10,1} = \{\}$	$C^{L}_{10,1} = \{\}$	$C^{U}_{9,15} = \{3,4\}$	$C^{L}_{9,15} = \{2,3,4\}$	$C^{U}_{9,14} = \{1\}$	$C^{L}_{9,14} = \{1,2\}$
$C^{U}_{10,6} = \{2\}$	$C^{L}_{10,6} = \{2\}$	$C^{U}_{10,5} = \{2,4\}$	$C^{L}_{10,5} = \{2,4\}$	$C^{U}_{10,4} = \{1,2,3,4\}$	$C^{L}_{10,4} = \{1,2,3,4\}$	$C^{U}_{10,3} = \{4\}$	$C^{L}_{10,3} = \{4\}$
$C^{U}_{10,11} = \{2,3\}$	$C^{L}_{10,11} = \{2,3\}$	$C^{U}_{10,9} = \{\}$	$C^{L}_{10,9} = \{\}$	$C^{U}_{10,8} = \{1,3,4\}$	$C^{L}_{10,8} = \{1,3,4\}$	$C^{U}_{10,7} = \{2\}$	$C^{L}_{10,7} = \{2\}$
$C^{U}_{10,15} = \{2,3,4\}$	$C^{L}_{10,15} = \{2,3,4\}$	$C^{U}_{10,14} = \{\}$	$C^{L}_{10,14} = \{\}$	$C^{U}_{10,13} = \{2,4\}$	$C^{L}_{10,13} = \{2,4\}$	$C^{U}_{10,12} = \{2,3\}$	$C^{L}_{10,12} = \{2\}$
$C^{U}_{11,4} = \{1,3,4\}$	$C^{L}_{11,4} = \{1,3,4\}$	$C^{U}_{11,3} = \{4\}$	$C^{L}_{11,3} = \{4\}$	$C^{U}_{11,2} = \{1\}$	$C^{L}_{11,2} = \{1\}$	$C^{U}_{11,1} = \{1\}$	$C^{L}_{11,1} = \{1\}$
$C^{U}_{11,8} = \{1,3,4\}$	$C^{L}_{11,8} = \{1,3,4\}$	$C^{U}_{11,7} = \{\}$	$C^{L}_{11,7} = \{4\}$	$C^{U}_{11,6} = \{1\}$	$C^{L}_{11,6} = \{1\}$	$C^{U}_{11,5} = \{2,4\}$	$C^{L}_{11,5} = \{2,4\}$
$C^{U}_{11,13} = \{1,4\}$	$C^{L}_{11,13} = \{1,4\}$	$C^{U}_{11,12} = \{2\}$	$C^{L}_{11,12} = \{2\}$	$C^{U}_{11,10} = \{1,4\}$	$C^{L}_{11,10} = \{1,4\}$	$C^{U}_{11,9} = \{\}$	$C^{L}_{11,9} = \{\}$
$C^{U}_{12,2} = \{1,4\}$	$C^{L}_{12,2} = \{1,3\}$	$C^{U}_{12,1} = \{1\}$	$C^{L}_{12,1} = \{1,4\}$	$C^{U}_{11,15} = \{2,3,4\}$	$C^{L}_{11,15} = \{2,3,4\}$	$C^{U}_{11,14} = \{\}$	$C^{L}_{11,14} = \{\}$
$C^{U}_{12,6} = \{1\}$	$C^{L}_{12,6} = \{1\}$	$C^{U}_{12,5} = \{2,3,4\}$	$C^{L}_{12,5} = \{2,4\}$	$C^{U}_{12,4} = \{1,3,4\}$	$C^{L}_{12,4} = \{1,3,4\}$	$C^{U}_{12,3} = \{3,4\}$	$C^{L}_{12,3} = \{\}$
$C^{U}_{12,10} = \{1,4\}$	$C^{L}_{12,10} = \{1,3,4\}$	$C^{U}_{12,9} = \{3,4\}$	$C^{L}_{12,9} = \{3,4\}$	$C^{U}_{12,8} = \{1,3,4\}$	$C^{L}_{12,8} = \{1,3,4\}$	$C^{U}_{12,7} = \{4\}$	$C^{L}_{12,7} = \{4\}$
$C^{U}_{12,15} = \{2,3,4\}$	$C^{L}_{12,15} = \{2,3,4\}$	$C^{U}_{12,14} = \{1\}$	$C^{L}_{12,14} = \{1\}$	$C^{U}_{12,13} = \{1,3,4\}$	$C^{L}_{12,13} = \{1,3,4\}$	$C^{U}_{12,11} = \{1,3,4\}$	$C^{L}_{12,11} = \{1,3,4\}$
$C^{U}_{13,4} = \{1,3,4\}$	$C^{L}_{13,4} = \{1,3,4\}$	$C^{U}_{13,3} = \{\}$	$C^{L}_{13,3} = \{\}$	$C^{U}_{13,2} = \{1\}$	$C^{L}_{13,2} = \{1,3\}$	$C^{U}_{13,1} = \{\}$	$C^{L}_{13,1} = \{\}$
$C^{U}_{13,8} = \{1,3,4\}$	$C^{L}_{13,8} = \{1,3,4\}$	$C^{U}_{13,7} = \{\}$	$C^{L}_{13,7} = \{\}$	$C^{U}_{13,6} = \{\}$	$C^{L}_{13,6} = \{\}$	$C^{U}_{13,5} = \{2,4\}$	$C^{L}_{13,5} = \{2,4\}$
$C^{U}_{13,12} = \{2\}$	$C^{L}_{13,12} = \{2\}$	$C^{U}_{13,11} = \{2,3\}$	$C^{L}_{13,11} = \{2,3\}$	$C^{U}_{13,10} = \{1,3\}$	$C^{L}_{13,10} = \{1,3\}$	$C^{U}_{13,9} = \{\}$	$C^{L}_{13,9} = \{1\}$
$C^{U}_{14,2} = \{1,4\}$	$C^{L}_{14,2} = \{1,3,4\}$	$C^{U}_{14,1} = \{1,3\}$	$C^{L}_{14,1} = \{1,4\}$	$C^{U}_{13,15} = \{2,3\}$	$C^{L}_{13,15} = \{2,3\}$	$C^{U}_{13,14} = \{\}$	$C^{L}_{13,14} = \{\}$
$C^{U}_{14,6} = \{1,2\}$	$C^{L}_{14,6} = \{1,2\}$	$C^{U}_{14,5} = \{2,3,4\}$	$C^{L}_{14,5} = \{2,3,4\}$	$C^{U}_{14,4} = \{1,2,3,4\}$	$C^{L}_{14,4} = \{1,2,3,4\}$	$C^{U}_{14,3} = \{3,4\}$	$C^{L}_{14,3} = \{4\}$
$C^{U}_{14,10} = \{1,2,3,4\}$	$C^{L}_{14,10} = \{1,2,3,4\}$	$C^{U}_{14,9} = \{2,3,4\}$	$C^{L}_{14,9} = \{3,4\}$	$C^{U}_{14,8} = \{1,2,3,4\}$	$C^{L}_{14,8} = \{1,3,4\}$	$C^{U}_{14,7} = \{2,4\}$	$C^{L}_{14,7} = \{2,4\}$
$C^{U}_{14,15} = \{2,3,4\}$	$C^{L}_{14,15} = \{2,3,4\}$	$C^{U}_{14,13} = \{1,2,3,4\}$	$C^{L}_{14,13} = \{1,2,3,4\}$	$C^{U}_{14,12} = \{2,3,4\}$	$C^{L}_{14,12} = \{2,3,4\}$	$C^{U}_{14,11} = \{1,2,3,4\}$	$C^{L}_{14,11} = \{1,2,3,4\}$
$C^{U}_{15,4} = \{1,3,4\}$	$C^{L}_{15,4} = \{1,3,4\}$	$C^{U}_{15,3} = \{\}$	$C^{L}_{15,3} = \{4\}$	$C^{U}_{15,2} = \{1\}$	$C^{L}_{15,2} = \{1\}$	$C^{U}_{15,1} = \{\}$	$C^{L}_{15,1} = \{1\}$
$C^{U}_{15,8} = \{1,3,4\}$	$C^{L}_{15,8} = \{1,3,4\}$	$C^{U}_{15,7} = \{1\}$	$D^{L}_{15,7} = \{1\}$	$C^{U}_{15,6} = \{1\}$	$C^{L}_{15,6} = \{1\}$	$C^{U}_{15,5} = \{4\}$	$C^{L}_{15,5} = \{4\}$
$C^{U}_{15,12} = \{1\}$	$C^{L}_{15,12} = \{1\}$	$C^{U}_{15,11} = \{1\}$	$C^{L}_{15,11} = \{1\}$	$C^{U}_{15,10} = \{1\}$	$C^{L}_{15,10} = \{1\}$	$C^{U}_{15,9} = \{1,2\}$	$C^{L}_{15,9} = \{1\}$
				$C^{U}_{15,14} = \{1\}$	$C^{L}_{15,14} = \{1\}$	$C^{U}_{15,13} = \{1,4\}$	$C^{L}_{15,13} = \{1,4\}$

Table 5: The discordance sets for 15 existing alternatives (upper and lower limits)

$D_{1,5}^{U}=\{1\}$	$D_{1,5}^{L}=\{1\}$	$D_{1,4}^{U}=\{\}$	$D_{1,4}^{L}=\{\}$	$D_{1,3}^{U}=\{1\}$	$D_{1,3}^{L}=\{1\}$	$D_{1,2}^{U}=\{3\}$	$D_{1,2}^{L}=\{4\}$
$D_{1,9}^{U}=\{1\}$	$D_{1,9}^{L}=\{1,4\}$	$D_{1,8}^{U}=\{\}$	$D_{1,8}^{L}=\{\}$	$D_{1,7}^{U}=\{1,3\}$	$D_{1,7}^{L}=\{1,3\}$	$D_{1,6}^{U}=\{1,3\}$	$D_{1,6}^{L}=\{1,3,4\}$
$D_{1,13}^{U}=\{\}$	$D_{1,13}^{L}=\{\}$	$D_{1,12}^{U}=\{1\}$	$D_{1,12}^{L}=\{1,4\}$	$D_{1,11}^{U}=\{1\}$	$D_{1,11}^{L}=\{1\}$	$D_{1,10}^{U}=\{\}$	$D_{1,10}^{L}=\{\}$
$D_{2,3}^{U}=(1,2)$	$D_{2,3}^{L}=\{1,2,3\}$	$D_{2,1}^{U}=\{1,2,4\}$	$D_{2,1}^{L}=\{1,2,3\}$	$D_{1,15}^{U}=\{\}$	$D_{1,15}^{L}=\{1\}$	$D_{1,14}^{U}=\{1,3\}$	$D_{1,14}^{L}=(1,4)$
$D_{2,7}^{U}=\{1\}$	$D_{2,7}^{L}=\{1,3\}$	$D_{2,6}^{U}=\{1,4\}$	$D_{2,6}^{L}=\{1,3,4\}$	$D_{2,5}^{U}=\{1\}$	$D_{2,5}^{L}=\{1,3\}$	$D_{2,4}^{U}=\{1\}$	$D_{2,4}^{L}=\{1\}$
$D_{2,11}^{U}=\{1\}$	$D_{2,11}^{L}=\{1\}$	$D_{2,10}^{U}=\{1\}$	$D_{2,10}^{L}=\{1\}$	$D_{2,9}^{U}=\{1\}$	$D_{2,9}^{L}=\{1,3\}$	$D_{2,8}^{U}=\{\}$	$D_{2,8}^{L}=\{\}$
$D_{2,15}^{U}=\{1\}$	$D_{2,15}^{L}=\{1\}$	$D_{2,14}^{U}=\{1,4\}$	$D_{2,14}^{L}=\{1,3,4\}$	$D_{2,13}^{U}=\{1\}$	$D_{2,13}^{L}=\{1,3\}$	$D_{2,12}^{U}=\{1,4\}$	$D_{2,12}^{L}=\{1,3\}$
$D_{3,5}^{U}=\{1\}$	$D_{3,5}^{L}=(1)$	$D_{3,4}^{U}=()$	$D_{3,4}^{L}=\{\}$	$D_{3,2}^{U}=(3,4)$	$D_{3,2}^{L}=\{4\}$	$D_{3,1}^{U}=\{2,3,4\}$	$D_{3,1}^{L}=(2,3,4)$
$D_{3,9}^{U}=(4)$	$D_{3,9}^{L}=\{4\}$	$D_{3,8}^{U}=\{\}$	$D_{3,8}^{L}=\{\}$	$D_{3,7}^{U}=(3,4)$	$D_{3,7}^{L}=\{3,4\}$	$D_{3,6}^{U}=\{3,4\}$	$D_{3,6}^{L}=(3,4)$
$D_{3,13}^{U}=()$	$D_{3,13}^{L}=\{\}$	$D_{3,12}^{U}=(3,4)$	$D_{3,12}^{L}=()$	$D_{3,11}^{U}=(4)$	$D_{3,11}^{L}=\{4\}$	$D_{3,10}^{U}=\{4\}$	$D_{3,10}^{L}=\{4\}$
$D_{4,2}^{U}=\{2,3,4\}$	$D_{4,2}^{L}=\{2,3,4\}$	$D_{4,1}^{U}=\{1,2,3,4\}$	$D_{4,1}^{L}=(1,2,3,4)$	$D_{3,15}^{U}=()$	$D_{3,15}^{L}=\{4\}$	$D_{3,14}^{U}=\{3,4\}$	$D_{3,14}^{L}=\{4\}$
$D_{4,7}^{U}=(1,2,3,4)$	$D_{4,7}^{L}=\{1,3,4\}$	$D_{4,6}^{U}=\{1,3,4\}$	$D_{4,6}^{L}=\{1,2,3,4\}$	$D_{4,5}^{U}=(1,3,4)$	$D_{4,5}^{L}=\{1,3,4\}$	$D_{4,3}^{U}=\{1,2,3,4\}$	$D_{4,3}^{L}=(1,2,3,4)$
$D_{4,11}^{U}=\{1,3,4\}$	$D_{4,11}^{L}=\{1,3,4\}$	$D_{4,10}^{U}=(1,2,3,4)$	$D_{4,10}^{L}=\{1,2,3,4\}$	$D_{4,9}^{U}=(1,2,3,4)$	$D_{4,9}^{L}=\{1,2,3,4\}$	$D_{4,8}^{U}=\{2,3\}$	$D_{4,8}^{L}=\{2,3,4\}$
$D_{4,15}^{U}=(1,3,4)$	$D_{4,15}^{L}=\{1,3,4\}$	$D_{4,14}^{U}=\{1,2,3,4\}$	$D_{4,14}^{L}=(1,2,3,4)$	$D_{4,13}^{U}=(1,3,4)$	$D_{4,13}^{L}=(1,3,4)$	$D_{4,12}^{U}=\{1,3,4\}$	$D_{4,12}^{L}=\{1,3,4\}$
$D_{5,4}^{U}=\{2\}$	$D_{5,4}^{L}=\{2,3,4\}$	$D_{5,3}^{U}=\{2,3,4\}$	$D_{5,3}^{L}=\{2,3,4\}$	$D_{5,2}^{U}=\{2,3,4\}$	$D_{5,2}^{L}=\{2,4\}$	$D_{5,1}^{U}=\{2,3,4\}$	$D_{5,1}^{L}=\{2,3,4\}$
$D_{5,9}^{U}=\{2,3,4\}$	$D_{5,9}^{L}=\{2,4\}$	$D_{5,8}^{U}=\{2\}$	$D_{5,8}^{L}=\{2\}$	$D_{5,7}^{U}=\{2,3,4\}$	$D_{5,7}^{L}=\{2,3,4\}$	$D_{5,6}^{U}=\{2,3,4\}$	$D_{5,6}^{L}=\{2,3,4\}$
$D_{5,13}^{U}=(2,4)$	$D_{5,13}^{L}=\{2,4\}$	$D_{5,12}^{U}=\{2,3,4\}$	$D_{5,12}^{L}=\{2,4\}$	$D_{5,11}^{U}=\{2,4\}$	$D_{5,11}^{L}=\{2,4\}$	$D_{5,10}^{U}=\{2,4\}$	$D_{5,10}^{L}=(2,4)$
$D_{6,2}^{U}=(2,3)$	$D_{6,2}^{L}=\{2\}$	$D_{6,1}^{u}=\{2,4\}$	$D_{6,1}^{L}=\{2\}$	$D_{5,15}^{U}=\{4\}$	$D_{5,15}^{L}=\{4\}$	$D_{5,14}^{U}=\{2,3,4\}$	$D_{5,14}^{L}=(2,3,4)$
$D_{6,7}^{U}=\{1\}$	$D_{6,7}^{L}=\{1\}$	$D_{6,5}^{U}=\{1\}$	$D_{6,5}^{L}=\{1\}$	$D_{6,4}^{U}=\{2\}$	$D_{6,4}^{L}=\{\}$	$D_{6,3}^{U}=\{1,2\}$	$D_{6,3}^{L}=\{1,2\}$
$D_{6,11}^{U}=\{1\}$	$D_{6,11}^{L}=\{1\}$	$D_{6,10}^{U}=\{2\}$	$D_{6,10}^{L}=\{2\}$	$D_{6,9}^{U}=\{1,2\}$	$D_{6,9}^{L}=\{1,2\}$	$D_{6,8}^{U}=\{2\}$	$D_{6,8}^{L}=\{2\}$
$D_{6,15}^{U}=\{1\}$	$D_{6,15}^{L}=\{1\}$	$D_{6,14}^{U}=\{1,2\}$	$D_{6,14}^{L}=\{1,2\}$	$D_{6,13}^{U}=\{\}$	$D_{6,13}^{L}=\{\}$	$D_{6,12}^{U}=\{1\}$	$D_{6,12}^{L}=\{1\}$
$D_{7,4}^{U}=\{\}$	$D_{7,4}^{L}=\{2\}$	$D_{7,3}^{U}=\{1,2\}$	$D_{7,3}^{L}=\{1,2\}$	$D_{7,2}^{U}=\{2,3,4\}$	$D_{7,2}^{L}=\{2,4\}$	$D_{7,1}^{U}=\{2,4\}$	$D_{7,1}^{L}=\{2,4\}$
$D_{7,9}^{U}=\{2\}$	$D_{7,9}^{L}=\{2,4\}$	$D_{7,8}^{U}=\{2\}$	$D_{7,8}^{L}=\{2\}$	$D_{7,6}^{U}=\{2,3,4\}$	$D_{7,6}^{L}=\{2,3,4\}$	$D_{7,5}^{U}=\{1\}$	$D_{7,5}^{L}=\{1\}$
$D_{7,13}^{U}=\{\}$	$D_{7,13}^{L}=\{\}$	$D_{7,12}^{U}=\{4\}$	$D_{7,12}^{L}=\{4\}$	$D_{7,11}^{U}=\{\}$	$D_{7,11}^{L}=\{4\}$	$D_{7,10}^{U}=\{2\}$	$D_{7,10}^{L}=\{2\}$
$D_{8,2}^{U}=(1,2,3,4)$	$D_{8,2}^{L}=(1,2,3,4)$	$D_{8,1}^{U}=\{1,2,3,4\}$	$D_{8,1}^{L}=(1,2,3,4)$	$D_{7,15}^{U}=\{1\}$	$D_{7,15}^{L}=\{1\}$	$D_{7,14}^{U}=\{2,4\}$	$D_{7,14}^{L}=(2,4)$
$D_{8,6}^{U}=(1,3,4)$	$D_{8,6}^{L}=\{1,3,4\}$	$D_{8,5}^{U}=\{1,3,4\}$	$D_{8,5}^{L}=\{1,3,4\}$	$D_{8,4}^{U}=\{1,4\}$	$D_{8,4}^{L}=\{1,4\}$	$D_{8,3}^{U}=\{1,2,3,4\}$	$D_{8,3}^{L}=(1,2,3,4)$
$D_{8,11}^{U}=(1,3,4)$	$D_{8,11}^{L}=\{1,3,4\}$	$D_{8,10}^{U}=\{1,3,4\}$	$D_{8,10}^{L}=\{1,3,4\}$	$D_{8,9}^{U}=(1,3,4)$	$D_{8,9}^{L}=\{1,3,4\}$	$D_{8,7}^{U}=\{1,3,4\}$	$D_{8,7}^{L}=\{1,3,4\}$
$D_{8,15}^{U}=\{1,3,4\}$	$D_{8,15}^{L}=\{1,3,4\}$	$D_{8,14}^{U}=\{1,2,3,4\}$	$D_{8,14}^{L}=\{1,3,4\}$	$D_{8,13}^{U}=(1,3,4)$	$D_{8,13}^{L}=\{1,3,4\}$	$D_{8,12}^{U}=\{1,3,4\}$	$D_{8,12}^{L}=\{1,3,4\}$
$D_{9,4}^{U}=\{\}$	$D_{9,4}^{L}=\{\}$	$D_{9,3}^{U}=\{1,2,3\}$	$D_{9,3}^{L}=\{1,2,3\}$	$D_{9,2}^{U}=\{2,3,4\}$	$D_{9,2}^{L}=\{2,4\}$	$D_{9,1}^{U}=\{2,3,4\}$	$D_{9,1}^{L}=\{2,3\}$
$D_{9,8}^{U}=\{2\}$	$D_{9,8}^{L}=\{2\}$	$D_{9,7}^{U}=\{1,3,4\}$	$D_{9,7}^{L}=\{1,3\}$	$D_{9,6}^{U}=\{3,4\}$	$D_{9,6}^{L}=\{3,4\}$	$D_{9,5}^{U}=\{1\}$	$D_{9,5}^{L}=\{1,3\}$
$D_{9,13}^{U}=\{\}$	$D_{9,13}^{L}=\{1\}$	$D_{9,12}^{U}=\{3,4\}$	$D_{9,12}^{L}=\{3,4\}$	$D_{9,11}^{U}=\{\}$	$D_{9,11}^{L}=\{\}$	$D_{9,10}^{U}=\{\}$	$D_{9,10}^{L}=\{\}$
$D_{10,2}^{U}=(2,3,4)$	$D_{10,2}^{L}=\{2,3,4\}$	$D_{10,1}^{U}=\{1,2,3,4\}$	$D_{10,1}^{L}=\{1,2,3,4\}$	$D_{9,15}^{U}=\{1,2\}$	$D_{9,15}^{L}=\{1\}$	$D_{9,14}^{U}=\{2,3,4\}$	$D_{9,14}^{L}=\{3,4\}$
$D_{10,6}^{U}=(1,3,4)$	$D_{10,6}^{L}=\{1,3,4\}$	$D_{10,5}^{U}=\{1,3\}$	$D_{10,5}^{L}=\{1,3\}$	$D_{10,4}^{U}=\{\}$	$D_{10,4}^{L}=\{\}$	$D_{10,3}^{U}=\{1,2,3\}$	$D_{10,3}^{L}=\{1,2,3\}$
$D_{10,11}^{U}=(1,4)$	$D_{10,11}^{L}=\{1,4\}$	$D_{10,9}^{U}=\{1,2,3,4\}$	$D_{10,9}^{L}=(1,2,3,4)$	$D_{10,8}^{U}=\{2\}$	$D_{10,8}^{L}=\{2\}$	$D_{10,7}^{U}=\{1,3,4\}$	$D_{10,7}^{L}=\{1,3,4\}$
$D_{10,15}^{U}=(1)$	$D_{10,15}^{L}=\{1\}$	$D_{10,14}^{U}=(1,2,3,4)$	$D_{10,14}^{L}=(1,2,3,4)$	$D_{10,13}^{U}=\{1,3\}$	$D_{10,13}^{L}=\{1,3\}$	$D_{10,12}^{U}=\{1,4\}$	$D_{10,12}^{L}=(1,3,4)$
$D_{11,4}^{U}=(2)$	$D_{11,4}^{L}=\{2\}$	$D_{11,3}^{U}=\{1,2,3\}$	$D_{11,3}^{L}=\{1,2,3\}$	$D_{11,2}^{U}=\{2,3,4\}$	$D_{11,2}^{L}=\{2,3,4\}$	$D_{11,1}^{U}=\{2,3,4\}$	$D_{11,1}^{L}=\{2,3,4\}$
$D_{11,8}^{U}=(2)$	$D_{11,8}^{L}=\{2\}$	$D_{11,7}^{U}=\{1,2,3,4\}$	$D_{11,7}^{L}=\{1,2,3\}$	$D_{11,6}^{U}=\{2,3,4\}$	$D_{11,6}^{L}=\{2,3,4\}$	$D_{11,5}^{U}=\{1,3\}$	$D_{11,5}^{L}=\{1,3\}$
$D_{11,13}^{U}=\{2,3\}$	$D_{11,13}^{L}=\{2,3\}$	$D_{11,12}^{U}=\{1,3,4\}$	$D_{11,12}^{L}=\{1,3,4\}$	$D_{11,10}^{U}=\{2,3\}$	$D_{11,10}^{L}=\{2,3\}$	$D_{11,9}^{U}=\{1,2,3,4\}$	$D_{11,9}^{L}=(1,2,3,4)$
$D_{12,2}^{U}=(2,3)$	$D_{12,2}^{L}=\{2,4\}$	$D_{12,1}^{U}=\{2,3,4\}$	$D_{12,1}^{L}=\{2,3\}$	$D_{11,15}^{U}=\{1\}$	$D_{11,15}^{L}=\{1\}$	$D_{11,14}^{U}=\{1,2,3,4\}$	$D_{11,14}^{L}=\{1,2,3,4\}$
$D_{12,6}^{U}=(2,3,4)$	$D_{12,6}^{L}=\{2,3,4\}$	$D_{12,5}^{U}=\{1\}$	$D_{12,5}^{L}=\{1,3\}$	$D_{12,4}^{U}=\{2\}$	$D_{12,4}^{L}=\{2\}$	$D_{12,3}^{U}=\{1,2\}$	$D_{12,3}^{L}=(1,2,3,4)$
$D_{12,10}^{U}=\{2,3\}$	$D_{12,10}^{L}=\{2\}$	$D_{12,9}^{U}=\{1,2\}$	$D_{12,9}^{L}=\{1,2\}$	$D_{12,8}^{U}=\{2\}$	$D_{12,8}^{L}=\{2\}$	$D_{12,7}^{U}=\{1,2,3\}$	$D_{12,7}^{L}=\{1,2,3\}$
$D_{13,15}^{U}=\{1\}$	$D_{12,15}^{L}=\{1\}$	$D_{12,14}^{U}=\{2,3,4\}$	$D_{12,14}^{L}=(2,3,4)$	$D_{12,13}^{U}=\{2\}$	$D_{12,13}^{L}=\{2\}$	$D_{12,11}^{U}=\{2\}$	$D_{12,11}^{L}=\{2\}$
$D_{13,4}^{U}=(2)$	$D_{13,4}^{L}=\{2\}$	$D_{13,3}^{U}=\{1,2,3,4\}$	$D_{13,3}^{L}=\{1,2,3,4\}$	$D_{13,2}^{U}=\{2,3,4\}$	$D_{13,2}^{L}=\{2,4\}$	$D_{13,1}^{U}=\{1,2,3,4\}$	$D_{13,1}^{L}=\{1,2,3,4\}$
$D_{13,8}^{U}=(2)$	$D_{13,8}^{L}=\{2\}$	$D_{13,7}^{U}=\{1,2,3,4\}$	$D_{13,7}^{L}=\{1,2,3,4\}$	$D_{13,6}^{U}=(1,2,3,4)$	$D_{13,6}^{L}=\{1,2,3,4\}$	$D_{13,5}^{U}=\{1,3\}$	$D_{13,5}^{L}=\{1,3\}$
$D_{13,12}^{U}=\{1,3,4\}$	$D_{13,12}^{L}=\{1,3,4\}$	$D_{13,11}^{U}=\{1,4\}$	$D_{13,11}^{L}=\{1,4\}$	$D_{13,10}^{U}=\{2,4\}$	$D_{13,10}^{L}=\{2,4\}$	$D_{13,9}^{U}=(1,2,3,4)$	$D_{13,9}^{L}=\{2,3,4\}$
$D_{14,2}^{U}=\{2,3\}$	$D_{14,2}^{L}=\{2\}$	$D_{14,1}^{U}=\{2,4\}$	$D_{14,1}^{L}=\{2,3\}$	$D_{13,15}^{U}=\{1,4\}$	$D_{13,15}^{L}=\{1,4\}$	$D_{13,14}^{U}=\{1,2,3,4\}$	$C_{13,14}^{L}=(1,2,3,4)$
$D_{14,6}^{U}=(3,4)$	$D_{14,6}^{L}=\{3,4\}$	$D_{14,5}^{U}=\{1\}$	$D_{14,5}^{L}=\{1\}$	$D_{14,4}^{U}=\{\}$	$D_{14,4}^{L}=\{\}$	$D_{14,3}^{U}=\{1,2\}$	$D_{14,3}^{L}=\{1,2,3\}$
$D_{14,10}^{U}=\{\}$	$D_{14,10}^{L}=\{\}$	$D_{14,9}^{U}=\{1\}$	$D_{14,9}^{L}=\{1,2\}$	$D_{14,8}^{U}=\{\}$	$D_{14,8}^{L}=\{2\}$	$D_{14,7}^{U}=\{1,3\}$	$D_{14,7}^{L}=\{1,3\}$
$D_{14,15}^{U}=\{1\}$	$D_{14,15}^{L}=\{1\}$	$D_{14,13}^{U}=\{\}$	$D_{14,13}^{L}=\{\}$	$D_{14,12}^{U}=\{1\}$	$D_{14,12}^{L}=\{1\}$	$D_{14,11}^{U}=\{\}$	$D_{14,11}^{L}=\{\}$
$D_{15,4}^{U}=(2)$	$D_{15,4}^{L}=\{2\}$	$D_{15,3}^{U}=\{1,2,3,4\}$	$D_{15,3}^{L}=\{1,2,3\}$	$D_{15,2}^{U}=\{2,3,4\}$	$D_{15,2}^{L}=\{2,3,4\}$	$D_{15,1}^{U}=\{1,2,3,4\}$	$D_{15,1}^{L}=(2,3,4)$
$D_{15,8}^{U}=(2)$	$D_{15,8}^{L}=\{2,3\}$	$D_{15,7}^{U}=\{2,3,4\}$	$D_{15,7}^{L}=\{2,3,4\}$	$D_{15,6}^{U}=\{2,3,4\}$	$D_{15,6}^{L}=\{2,3,4\}$	$D_{15,5}^{U}=\{1,2,3\}$	$D_{15,5}^{L}=\{1,2,3\}$
$D_{15,12}^{U}=(2,3,4)$	$D_{15,12}^{L}=\{2,3,4\}$	$D_{15,11}^{U}=\{2,3,4\}$	$D_{15,11}^{L}=\{2,3,4\}$	$D_{15,10}^{U}=(2,3,4)$	$D_{15,10}^{L}=\{2,3,4\}$	$D_{15,9}^{U}=\{3,4\}$	$D_{15,9}^{L}=(2,3,4)$
				$D_{15,14}^{U}=(2,3,4)$	$D_{15,14}^{L}=\{2,3,4\}$	$D_{15,13}^{U}=\{2,3\}$	$D_{15,13}^{L}=\{2,3\}$

$$NI^{L}_{k,l}=\begin{vmatrix}
- & 0.2992 & 0.2332 & 0 & 0.1550 & 0.6 & 0.1305 & 0 & 0.1113 & 0 & 0.0171 & 0.0743 & 0 & 0.3206 & 0.1244\\
1 & - & 0.9524 & 0.025 & 0.8095 & 0.9057 & 1 & 0 & 1 & 0.1724 & 0.5174 & 0.6854 & 0.2421 & 1 & 0.7156\\
1 & 1 & - & 0 & 0.0333 & 1 & 0.5155 & 0 & 0.6446 & 0.0789 & 0.33 & 0 & 0 & 1 & 0.0244\\
1 & 1 & 1 & - & 1 & 1 & 1 & 0.3886 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
1 & 1 & 1 & 0.3576 & - & 1 & 1 & 0.3702 & 1 & 0.5139 & 0.4570 & 1 & 0.3981 & 1 & 0.0577\\
1 & 0.4382 & 0.4360 & 0 & 0.2127 & - & 0.1765 & 0.1690 & 0.1707 & 0.0281 & 0.0055 & 0.1111 & 0 & 0.2047 & 0.1725\\
1 & 0.5907 & 1 & 0.0417 & 0.1428 & 1 & - & 0.2581 & 0.3476 & 0.0962 & 0.1141 & 0.3333 & 0 & 1 & 0.0099\\
1 & 1 & 1 & 1 & 1 & 1 & 1 & - & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
1 & 0.8283 & 1 & 0 & 0.5294 & 1 & 1 & 0.0466 & - & 0 & 0 & 0.3882 & 1 & 1 & 0.176\\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 0.3882 & 1 & - & 1 & 1 & 1 & 1 & 1\\
1 & 1 & 1 & 0.2262 & 1 & 1 & 1 & 0.3393 & 1 & 0.5763 & - & 1 & 0.6418 & 1 & 1\\
1 & 1 & 1 & 0.1852 & 0.5532 & 1 & 1 & 0.3226 & 1 & 0.3509 & 0.0496 & - & 0.0978 & 1 & 0.2698\\
1 & 1 & 1 & 0.2807 & 1 & 1 & 1 & 0.3889 & 0.8977 & 0.8611 & 1 & 1 & - & 1 & 1\\
1 & 0.5078 & 0.5362 & 0 & 0.3235 & 1 & 0.4335 & 0.0702 & 0.2357 & 0 & 0 & 0.1736 & 0 & - & 0.2576\\
1 & 1 & 1 & 0.4305 & 1 & 1 & 1 & 0.4070 & 1 & 0.6349 & 0.8955 & 1 & 0.4454 & 1 & -
\end{vmatrix} \tag{29}$$

$$NI^{U}_{k,l}=\begin{vmatrix}
- & 0.2277 & 0.2767 & 0 & 0.2531 & 0.1505 & 0.2024 & 0 & 0.1698 & 0 & 0.0089 & 0.1053 & 0 & 0.0382 & 0\\
1 & - & 1 & 0.0109 & 1 & 1 & 1 & 0 & 1 & 0.1712 & 0.4027 & 1 & 0.2323 & 1 & 0.6967\\
1 & 0.7240 & - & 0 & 0.0675 & 1 & 0.9482 & 0 & 1 & 0.0191 & 0.3877 & 1 & 0 & 1 & 0\\
1 & 1 & 1 & - & 1 & 1 & 1 & 0.3037 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
1 & 0.8531 & 1 & 0.1576 & - & 1 & 1 & 0.9355 & 1 & 0.2734 & 0.5804 & 1 & 0.1679 & 1 & 0.0653\\
1 & 0.6275 & 0.3910 & 0.0094 & 0.3551 & - & 0.4253 & 0.1604 & 0.2979 & 0.0329 & 0.0124 & 0.3465 & 0 & 0.8347 & 0.2874\\
1 & 0.5098 & 1 & 0 & 0.2449 & 1 & - & 0.2118 & 0.3538 & 0.0432 & 0 & 1 & 0 & 0.5454 & 0.0272\\
1 & 1 & 1 & 1 & 1 & 1 & 1 & - & 1 & 1 & 1 & 1 & 1 & 1 & 0.2308\\
1 & 0.9011 & 0.7654 & 0 & 0.4628 & 1 & 1 & 0.5545 & - & 0 & 0 & 1 & 0 & 1 & 0.6651\\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 0.3062 & 1 & - & 1 & 1 & 0.3333 & 1 & 1\\
1 & 1 & 1 & 0.0686 & 1 & 1 & 1 & 0.8031 & 1 & 0.3173 & - & 1 & 0.5204 & 1 & 1\\
1 & 0.8898 & 0.6143 & 0.0464 & 0.3003 & 1 & 1 & 0.2776 & 0.4840 & 0.6551 & 0.0229 & - & 0.0194 & 1 & 0.2040\\
1 & 1 & 1 & 0.1264 & 1 & 1 & 1 & 0.4825 & 1 & 1 & 1 & 1 & - & 1 & 1\\
1 & 0.4769 & 0.2828 & 0 & 0.4151 & 1 & 0.5936 & 0 & 0.375 & 0 & 0 & 0.4 & 0 & - & 0.3271\\
1 & 1 & 1 & 0.2115 & 1 & 1 & 1 & 1 & 1 & 0.3610 & 0.6642 & 1 & 0.2622 & 1 & -
\end{vmatrix} \tag{30}$$

Then the final discordance matrix is calculated by considering lower and upper limits using formula (16) as follows:

$$NI=\begin{vmatrix}
- & 0.2634 & 0.2549 & 0 & 0.2040 & 0.3752 & 0.1664 & 0 & 0.1405 & 0 & 0.013 & 0.0898 & 0 & 0.1794 & 0.0622\\
1 & - & 0.9762 & 0.0179 & 0.9047 & 0.9528 & 1 & 0 & 1 & 0.1718 & 0.4600 & 0.8427 & 0.2372 & 1 & 0.7061\\
1 & 0.862 & - & 0 & 0.0504 & 1 & 0.7318 & 0 & 0.7323 & 0.4040 & 0.3588 & 0.5 & 0 & 1 & 0.0122\\
1 & 1 & 1 & - & 1 & 1 & 1 & 0.3461 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
1 & 0.9265 & 1 & 0.2576 & - & 1 & 1 & 0.6528 & 1 & 0.3936 & 0.5187 & 1 & 0.283 & 1 & 0.0615\\
1 & 0.5328 & 0.4135 & 0.0047 & 0.2839 & - & 0.3009 & 0.1647 & 0.2343 & 0.0305 & 0.0089 & 0.2288 & 0 & 0.5197 & 0.2299\\
1 & 0.5502 & 1 & 0.0208 & 0.1938 & 1 & - & 0.2349 & 0.3507 & 0.0697 & 0.0570 & 0.6666 & 0 & 0.7727 & 0.0185\\
1 & 1 & 1 & 1 & 1 & 1 & 1 & - & 1 & 1 & 1 & 1 & 1 & 1 & 0.6154\\
1 & 0.8647 & 0.8827 & 0 & 0.4961 & 1 & 1 & 0.3005 & - & 0 & 0 & 0.6941 & 0.5 & 1 & 0.4205\\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 0.3472 & 1 & - & 1 & 1 & 0.6666 & 1 & 1\\
1 & 1 & 1 & 0.2948 & 1 & 1 & 1 & 0.5712 & 1 & 0.4468 & - & 1 & 0.5811 & 1 & 1\\
1 & 0.9449 & 0.8071 & 0.1158 & 0.4267 & 1 & 1 & 0.3001 & 0.742 & 0.503 & 0.0362 & - & 0.0586 & 1 & 0.2369\\
1 & 1 & 1 & 0.2035 & 1 & 1 & 1 & 0.4357 & 0.9488 & 0.9305 & 1 & 1 & - & 1 & 1\\
1 & 0.4923 & 0.46 & 0 & 0.3693 & 1 & 0.5135 & 0.0351 & 0.3053 & 0 & 0 & 0.2868 & 0 & - & 0.2923\\
1 & 1 & 1 & 0.321 & 1 & 1 & 1 & 0.7035 & 1 & 0.4979 & 0.7798 & 1 & 0.3538 & 1 & -
\end{vmatrix} \tag{31}$$

Step 6: Specifying the effective concordant matrix (using formulas (18), (20) and (21)). The average value of the concordance matrix (using formula (19)) and the effective matrix are shown in (32) and (33), respectively.

$$\bar{I} = \frac{\sum_{k=1}^{15}\sum_{1=1}^{15} I_{k,1}}{15*14} = \frac{52.5}{15*14} = 0.25 \quad (32)$$

$$f_{k,1} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (33)$$

Step 7: Specifying the effective discordant matrix (using formulas (23), (24)). The average value of the discordant matrix (using formula (22)) and the effective matrix are shown in (34) and (35), respectively.

$$N\bar{I} = \frac{\sum_{k=1}^{15}\sum_{1=1}^{15} NI_{k,1}}{15*14} = \frac{133.7866}{15*14} = 0.6371 \quad (34)$$

$$g_{k,1} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (35)$$

Step 8: Specifying the global and effective matrix (using the formula (25)) is shown in (36):

Table 6: Ranking the alternatives

Node	No. of outputs	No. of inputs	Rank	Node	No. of outputs	No. of inputs	Rank
1	13	0	1	9	7	4	6
2	5	3	8	10	2	10	12
3	8	2	4	11	3	9	10
4	0	13	15	12	7	4	7
5	5	7	9	13	2	11	13
6	13	0	2	14	11	2	3
7	8	3	5	15	2	7	11
8	0	11	14				

$$f_{k,1} * g_{k,1} = h_{k,1} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (36)$$

Step 9: Neglecting non-effective alternatives (using formulas (26) and (27)) and ranking the alternatives. Based on matrix h, the ranking can be determined without drawing any graph. Drawing the graph in the current problem is complicated, because there are many alternatives. In fact, the number of outputs for each node can be found by using matrix h. In the case that the numbers of outputs for two alternatives are the same, the option having fewer inputs is preferred for sorting purposes, since it will be less dominated by other options. Ranking the alternatives is shown in Table 6.

CONCLUSION

Everyday, we need to make decisions both in the personal and professional activities. Therefore, increasing the power of decision making using the various decision methods is highly paid attention. The decision method used in the current research is the ELECTRE method. This method was developed to be used when the data are of interval nature. Considering the nature of the data, the existing steps in ELECTRE method were reviewed and matched to the current conditions. Then a numerical example was solved to prove the correctness of the proposed method. The results show the ranking of

15 bank branches. Also the comparison of the results with the ones of the TOPSIS method shows that both methods are extremely similar and prove each other. This is a rare situation and proves the accuracy of the proposed method.

Also, the contribution of the present study is to use and develop ELECTRE method which is one of the multi attribute decision making method in ranking various alternatives for deterministic and exact data. Therefore, when we face with interval data and it is impossible to use existing methods for uncertain, inexact and fuzzy data (for data like a time interval, etc.). It is needed to use a new method for ranking. Since, the ELECTRE method is based on deterministic and exact data. The method presented in this study will allow the users to rank their existing alternatives more efficiency and easily.

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