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## Fault Diagnosis Using Adaptive Technique

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**Abstract:** In this study, by use of adaptive technique, a new method for fault detection and isolation is investigated. In the proposed method, estimation of fault signal obtains that it provides significant information about fault characteristics including size and severity of the fault, which are essential for many applications. The proposed technique is examined on a model of an aircraft and reconstructed fault signal is obtained. The simulation results are compared with the results achieved by use of sliding mode technique. Simulation results and comparison illustrate the capability of the proposed method.

**Key words:** Fault detection and isolation, adaptive observer, sliding mode observer, eigenstructure assignment

### INTRODUCTION

Extensive class of FDI methods are developed analytically on the basis of mathematical models describing systems. In the last 30 years, a few researches have been performed in this region (Patton and Chen, 1995). A model-based FDI produces residual so that one can find fault through it. Therefore this method does not detect the fault directly and this can be mentioned as a main problem. For suitable residual production, different suggestions have been proposed. For example, one can refer to parity space (Ding *et al.*, 2002) and residual production on the basis of observer (Commault *et al.*, 2003). Observer based residual generation estimates the fault through output or state estimation of the system using a proper observer. The residual here means fault estimation. When the residual is evaluated, fault signal is constructed and separated from the residual (Emami Naeini *et al.*, 1988). For this purpose, fixed and adaptive threshold level is used. In presence of uncertainty or noise and in order to prevent from false alarms, selected FDI scheme must be robust. It means it has to distinguish the fault and unknown input impacts (Chen and Patton, 1999; Ibaraki *et al.*, 2005; Chen and Seif, 2006).

The fault estimation approaches imposed by use of observers are commonly direct ways providing fault information and isolation. This type method provides an estimate of the size and severity of the fault, which are important in many applications. For example, one can refer to observer design by use of sliding mode method. This method attracted attention of many researchers due to its more robustness in comparison with design based observer of Luenberger (Edwards and

Spurgeon, 1994; Edwards *et al.*, 2000; Chen *et al.*, 2005). For a class of linear systems, a canonical form is suggested and then input and output faulty signals are estimated using sliding mode observer (Edwards *et al.*, 2000). Problem associated with sliding mode observers in separation of actuator faults is solved in (Chen *et al.*, 2005) using a bank of sliding mode observers.

In this study a canonical form together with adaptive technique are employed to design an adaptive observer for FDI applications (Edwards *et al.*, 2000). Simulation results presented in this study show that the adaptive observer is robust against parameters uncertainties. An adaptive observer is designed for a model of an aircraft and its simulation results are compared with those of the sliding mode given in Edwards *et al.* (2000). This comparison illustrates robustness and eligibility of the proposed method and it can be suggested as an alternative method for FDI design with some confidences.

### PROBLEM FORMULATION

Consider the following system:

$$\begin{cases} \dot{X}(t) = AX(t) + Bu(t) + Df_a(t) \\ y(t) = CX(t) + f_s(t) \end{cases} \quad (1)$$

where,  $A \in R^{n \times n}$ ,  $B \in R^{n \times m}$ ,  $C \in R^{p \times n}$ ,  $D \in R^{n \times q}$ , with  $q < p < n$ ,  $X(t) \in R^n$  is the state vector,  $u(t) \in R^m$  is the input vector and  $y(t) \in R^p$  is the output vector. Also  $f_a(t)$  and  $f_s(t)$  represent the actuator and sensor faults, respectively with limited value. Here,  $u(t)$  and  $y(t)$  are available. The matrices  $C$  and  $D$  are assumed to be of full rank and we have the following relation for system:

- Rank (CD) = q
- Invariant zeros of (A,D,C) must lie in the open LHP

In this case on the basis of (Edwards *et al.*, 2000), one can find linear transformation like T so that after its application in (1), the following equation can be obtained (Z = TX):

$$\begin{cases} \dot{z}_1(t) = A_{11}z_1(t) + A_{12}z_2(t) + B_1u(t) \\ \dot{z}_2(t) = A_{21}z_1(t) + A_{22}z_2(t) + B_2u(t) + D_2f_a(t) \\ y(t) = z_2(t) + f_s(t) \end{cases} \quad (2)$$

where,  $z_1(t) \in \mathbb{R}^{(n-p)}$ ,  $z_2(t) \in \mathbb{R}^p$  and  $A_{11}(t) \in \mathbb{R}^{(n-p) \times (n-p)}$  is stable matrix. There are the following relations between system 1 and 2:

$$\begin{cases} TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \\ TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \\ TD = \begin{bmatrix} 0 \\ D_2 \end{bmatrix} \\ CT^{-1} = \begin{bmatrix} 0 & I_p \end{bmatrix} \end{cases} \quad (3)$$

### OBSERVER DESIGN WITH SLIDING MODE

For system (2), consider observer equation as follows:

$$\begin{cases} \dot{\hat{z}}_1(t) = A_{11}\hat{z}_1(t) + A_{12}\hat{z}_2(t) + B_1u(t) - A_{12}e_y(t) \\ \dot{\hat{z}}_2(t) = A_{21}\hat{z}_1(t) + A_{22}\hat{z}_2(t) + B_2u(t) - (A_{22} - A_{22}^s)e_y(t) + v \\ \hat{y}(t) = \hat{z}_2(t) \end{cases} \quad (4)$$

where,  $A_{22}^s$  is a stable design matrix that should be determined by designer. The discontinuous vector v and error vector  $e_y$  are defined as follows:

$$e_y(t) = \hat{y}(t) - y(t) \quad (5)$$

$$v = \begin{cases} -\rho \|D_2\| \frac{P_2 e_y}{\|P_2 e_y\|} & \text{if } e_y \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

where,  $P_2 \in \mathbb{R}^{p \times p}$  Lyapunov Matrix for

$$A_{22}^s \left( (A_{22}^s)^T P_2 + P_2 A_{22}^s < 0 \right)$$

and the scalar  $\rho$  is selected so that:

$$\|f_a(t)\| < \rho \quad (7)$$

If we want to write observer relation for Eq. 1, it is enough to apply linear transformation

$$\hat{X} = T^{-1}\hat{Z} = T^{-1} \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix}$$

on relation 4:

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) - T^{-1} \begin{bmatrix} A_{12} \\ A_{22} - A_{22}^s \end{bmatrix} e_y(t) + T^{-1} \begin{bmatrix} 0 \\ I_p \end{bmatrix} v \quad (8)$$

In Edwards *et al.* (2000), it is shown that one can estimate  $f_a(t)$  and  $f_s(t)$  in term of v approximately:

$$v_{eq} \rightarrow D_2 f_a(t) \Rightarrow f_a(t) \approx (D_2^T D_2)^{-1} D_2^T v_{eq} = -\rho \|D_2\| (D_2^T D_2)^{-1} D_2^T \frac{P_2 e_y}{\|P_2 e_y\| + \delta} \quad (9)$$

$$v_{eq} \approx -(A_{22} - A_{21}A_{11}^{-1}A_{12})f_s(t) \quad (10)$$

where, scalar  $\delta$  is small positive and it has been assume that  $f_s(t)$  has slow changes.

### OBSERVER DESIGN WITH ADAPTIVE TECHNIQUES

**Theorem:** For system (2) and observer (4) and by assuming stability of matrix  $A_{22}^s$ , by use of vector:

$$(v)^T = -2\alpha e_y^T P_2 \quad (11)$$

In finite time, one can make error estimation is quadratically stable if fault is fixed or limited and has slow varying. In relation (11)  $P_2 \in \mathbb{R}^{p \times p}$  Lyapunov Matrix for

$$A_{22}^s \left( (A_{22}^s)^T P_2 + P_2 A_{22}^s < 0 \right)$$

and  $\alpha$  is design constant.

**Prove:** At first, by assuming  $f_a(t) = 0$  and defining

$$e_y(t) = \hat{y}(t) - y(t), \quad e_1(t) = \hat{z}_1(t) - z_1(t)$$

and

$$e_2(t) = \hat{z}_2(t) - z_2(t)$$

we have:

$$\dot{e}_1(t) = A_{11}e_1(t) \quad (12)$$

$$\dot{e}_y = \dot{e}_2 = A_{21}e_1 + A_{22}^s e_y + v - D_2 f_a \quad (13)$$

With regard to stability of matrix  $A_{11} \in \mathbb{R}^{(n-p) \times (n-p)}$ , stability of  $e_1(t)$  is proved easily. By defining  $\tilde{v} = v - D_2 f_s(t)$  and considering stability  $e_1(t)$ , for proving stability of its output estimation error, we rewrite:

$$\dot{e}_y(t) = A_{22}^s e_y(t) + \tilde{v} \tag{14}$$

By defining Lyapunov function as:

$$V = e_y^T P_2 e_y + \frac{1}{2\alpha} \tilde{v}^T \tilde{v} \tag{15}$$

And defining

$$(A_{22}^s)^T P_2 + P_2 A_{22}^s = -Q$$

which Q is positive definite matrix and using (11), we have for its derivation of V:

$$\begin{aligned} \dot{V} = e_y^T & \left( (A_{22}^s)^T P_2 + P_2 A_{22}^s \right) e_y + 2e_y^T P_2 \tilde{v} \\ & + \frac{1}{\alpha} (\tilde{v})^T \dot{\tilde{v}} = -e_y^T Q e_y + \left( 2\alpha e_y^T P_2 + (\tilde{v})^T \right) \frac{\tilde{v}}{\alpha} = -e_y^T Q e_y < 0 \end{aligned} \tag{16}$$

Therefore  $e_y(t)$  is stable and tends to zero.

With regard to the fact that estimation error  $e_1(t)$  and  $e_y(t)$  become zero and use of relation (13) in steady state, one can reach the following result:

$$v_{eq} - D_2 f_s(t) \rightarrow 0 \Rightarrow f_s(t) \approx -2\alpha (D_2^T D_2)^{-1} D_2^T P_2 \int e_y(t) dt \tag{17}$$

Now, for state which  $f_a = 0$  and  $f_s$  is available, we rewrite the relations:

$$\begin{cases} y(t) = z_2(t) + f_s(t) \\ e_y(t) = e_2(t) - f_s(t) \\ \dot{e}_y(t) = \dot{e}_2(t) - \dot{f}_s(t) \end{cases} \tag{18}$$

By use of the above relation, one can rewrite relations (12) and (13) as follow:

$$\dot{e}_1(t) = A_{11} e_1(t) + A_{12} f_s(t) \tag{19}$$

$$\dot{e}_y = A_{21} e_1 + A_{22} e_y + A_{22} f_s - \dot{f}_s + v \tag{20}$$

It is found that  $f_s$  and  $\dot{f}_s$  appear as disturbance in equation. With regard to the fact that value of fault is zero or it has been assumed to have slow change, therefore by selecting big value of  $\alpha$ , estimation error can tend to zero. With this assuming in steady state, one can reach the following relation like (10):

$$v_{eq} \approx -(A_{22} - A_{21} A_{11}^{-1} A_{12}) f_s(t) \rightarrow f_s \approx 2\alpha (A_{22} - A_{21} A_{11}^{-1} A_{12})^{-1} P_2 \int e_y(t) dt \tag{21}$$

It is clear that if matrix  $(A_{22} - A_{21} A_{11}^{-1} A_{12})$  is nonsingular, one can calculate fault through the above relation; otherwise due to defect of rank in matrix, one can not use directly the above said relation. This occurs in aircraft model which is mentioned further.

### SIMULATION RESULTS

Here, in order to show ability of method, linear model of longitudinal movement of aircraft has been studied:

$$\begin{cases} \dot{X}(t) = AX(t) + BU(t) + Df_s(t) \\ Y(t) = CX(t) + f_s(t) \end{cases} \tag{22}$$

Where:

$$A = \begin{bmatrix} -0.062 & 0.2859 & 0 & -9.81 & 0 & 0.0125 & 0 \\ -0.562 & -2.3298 & 32.9799 & 0 & 0 & 0 & 5.3170 \\ -0.070 & -0.4526 & -0.0499 & 0 & 0 & 0 & -13.5789 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 33 & 0 & 0 & 0 \\ -16.85 & 0 & 0 & 0 & 0 & -1.1968 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 130.0813 \\ 20 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{23}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$U = \begin{cases} \eta & \text{Elevator Angle} \\ T_H & \text{Throttle} \end{cases}$$

$$X = \begin{cases} x_1 & \text{Forward velocity} \\ x_2 & \text{Downward velocity} \\ x_3 & \text{Pitch rate} \\ x_4 & \text{Pitch angle} \\ x_5 & \text{Height} \\ x_6 & \text{Thrust} \\ x_7 & \text{Elevator actuator mode} \end{cases}$$

By application of Eigenstructure assignment method of Wang *et al.* (2005) and considering closed loop roots according to below, state feedback gain  $U(t) KX(t)$  is calculated as follows:

$$K = \begin{bmatrix} 0.0950 & -0.0368 & 0.4398 & 2.3644 & 0.0815 & 0.0003 & -0.7621 \\ 0.0984 & 0.0004 & -0.0006 & -0.0218 & -0.0103 & -0.0140 & 0.0003 \end{bmatrix} \quad (24)$$

By use of algorithm (Edwards *et al.*, 2000), one can find linear transformation so that equation can be converted to canonical form (2):

$$TAT^{-1} = \begin{bmatrix} -2.3298 & 5.3170 & 32.9799 & 0 & -0.5620 & 0 & 0 \\ 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ -0.4526 & -13.5789 & -0.0499 & 0 & 0.0700 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 33 & 0 \\ 0.2859 & 0 & 0 & 0 & -0.0620 & -9.8 & 0.0125 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -16.8501 & 0 & -1.1968 \end{bmatrix}$$

$$TB = \begin{bmatrix} 0 & 0 \\ 20 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 130.0813 \end{bmatrix}, TD = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, CT^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (25)$$

By selecting  $A_{22}^s = \text{diag}\{-5, -8, -10, -12, -14\}$ , general form of observer is obtained. For two schemes mentioned for observer,  $P_2 = \text{eye}(5)$  has been considered which holds in relation

$$(A_{22}^s)^T P_2 + P_2 A_{22}^s < 0$$

For the observer designed on the basis of sliding mode, value of  $\rho = 75$  and for observer designed on basis of adaptive method, value of  $\alpha = 50$  has been considered. Now, if there is any change in equilibrium state to the extent of  $\text{Irad sec}^{-1}$  in pitch rate, simulation will have been performed in different states. ( $f_i$  is indicator of fault in  $i$ th of sensor and  $f_j$  is indicator of fault in  $j$ th of actuator):

- By assuming  $f_2(t) = 2u(t-2)$  (Fig. 1) and  $f_1 = u(t-2)$  (Fig. 2) for two observers designed with sliding mode and adaptive methods, simulations are done and reconstructed fault signals have been drawn (Fig. 3, 4)
- With above assumptions of (A) and 20% of changes in parameters of system, simulations have been repeated (Fig. 5, 6)
- By assuming  $f_2(t) = r(t-5) - 2r(t-10) + r(t-15)$  (Fig. 7) and  $f_1 = 0.1 [r(t-5) - 2r(t-10) + r(t-15)]$  (Fig. 8) for two

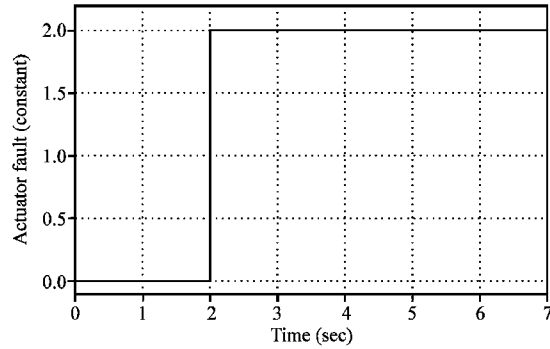


Fig. 1: Fault signal occurred in second actuator (constant fault)

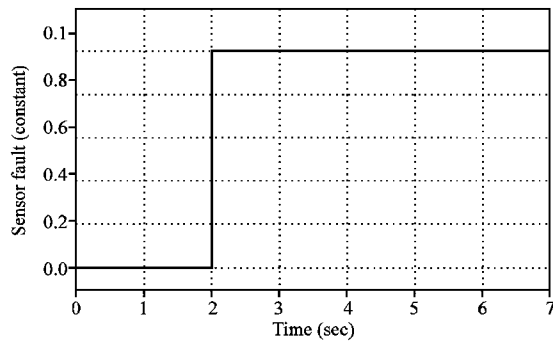


Fig. 2: Fault signal occurred in first sensor (constant fault)

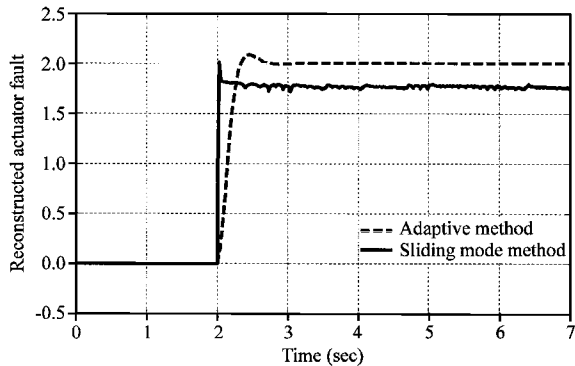


Fig. 3: Reconstructed fault signal for state that constant fault has occurred in second actuator

designed with sliding mode and adaptive methods, simulations are done and reconstructed fault signals have been drawn (Fig. 9, 10)

- With above assumptions of (C) and 20% of changes in parameters of system, simulations have been repeated (Fig. 11, 12)
- By assuming

$$f_{a_2}(t) = 5[e^{-0.05t} + \frac{t}{4t+10}]u(t-2) \text{ (Fig. 13) and}$$

$$f_{s_1} = [e^{-0.05t} + \frac{t}{4t+10}]u(t-2) \text{ (Fig. 14)}$$

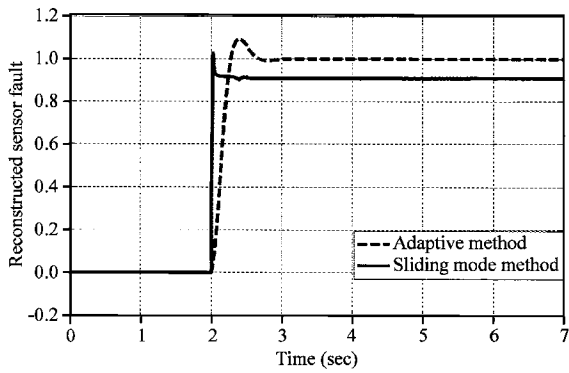


Fig. 4: Reconstructed fault signal for state that constant fault has occurred in first sensor

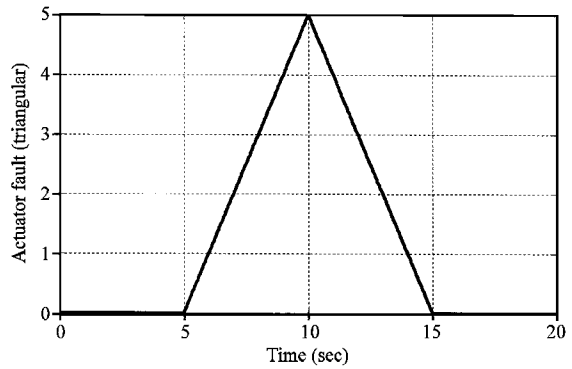


Fig. 7: Fault signal occurred in second actuator (triangular fault)

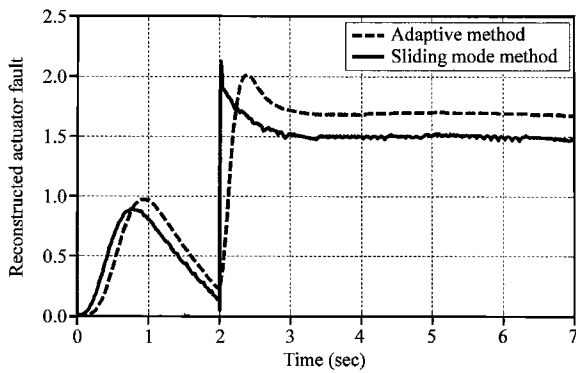


Fig. 5: Reconstructed fault signal for state that 20% of changes have been made in parameters of system and constant fault occurred in second actuator

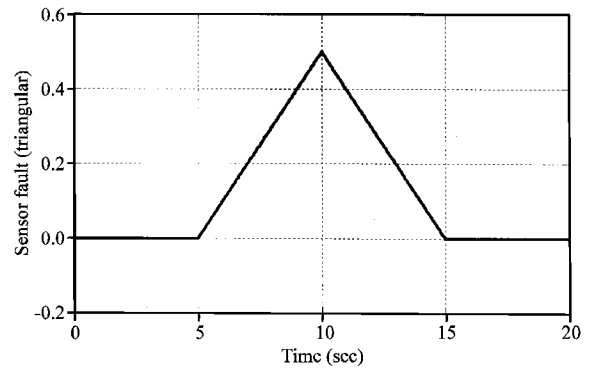


Fig. 8: Fault signal occurred in first sensor (triangular fault)

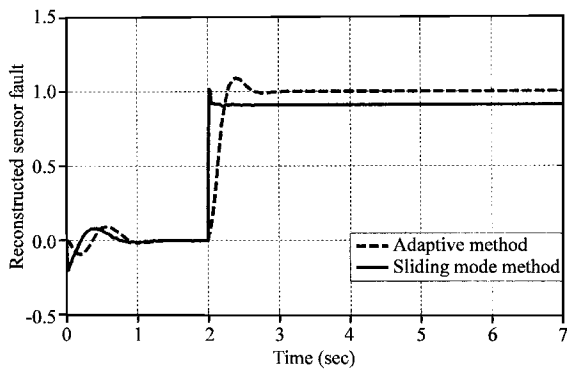


Fig. 6: Reconstructed fault signal for state that 20% of changes have been made in parameters of system and constant fault occurred in first sensor

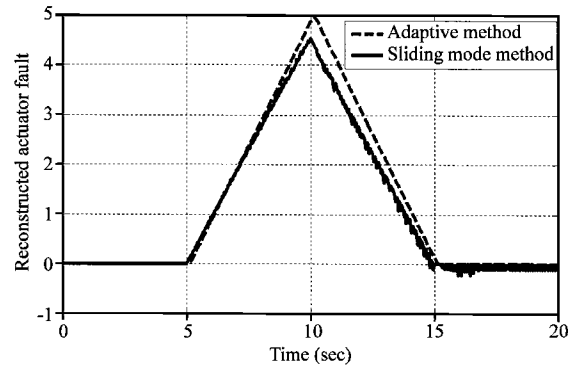


Fig. 9: Reconstructed fault signal for state that triangular fault has occurred in second actuator

- With above assumptions of (E) and 20% of changes in parameters of system, simulations have been repeated (Fig. 17, 18)

for two observers designed with sliding mode and adaptive methods, simulations are done and reconstructed fault signals have been drawn (Fig. 15, 16)

For reconstruction of fault signal, relation (9) and (10) have been used for design of observer on the basis of sliding mode and relation (17) and (21) have been used for

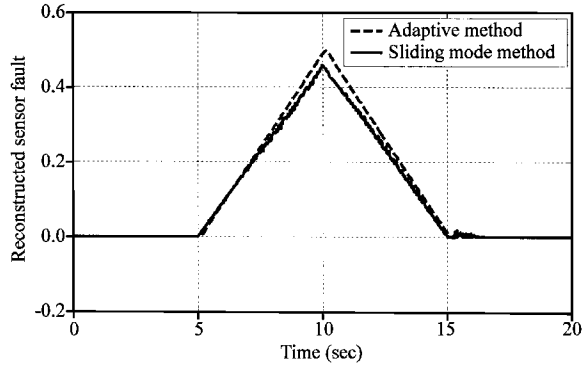


Fig. 10: Reconstructed fault signal for state that triangular fault has occurred in first sensor

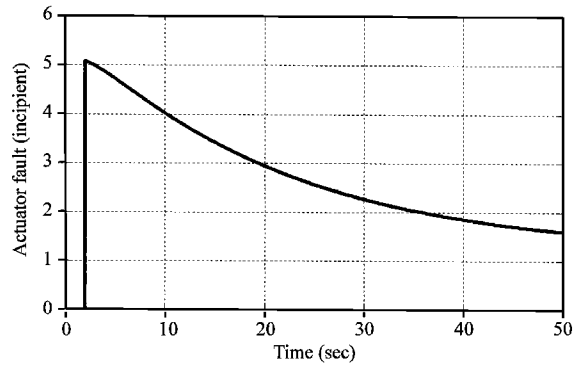


Fig. 13: Fault signal occurred in second actuator (incipient fault)

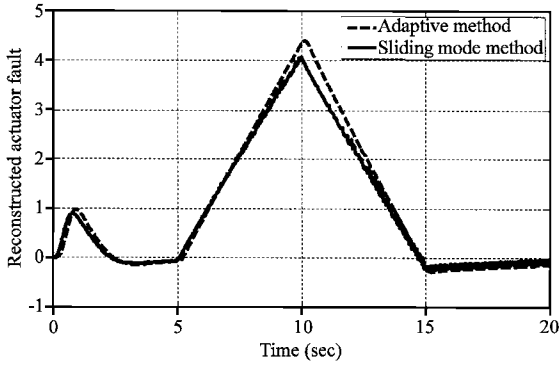


Fig. 11: Reconstructed fault signal for state that 20% of changes have been made in parameters of system and triangular fault occurred in second actuator

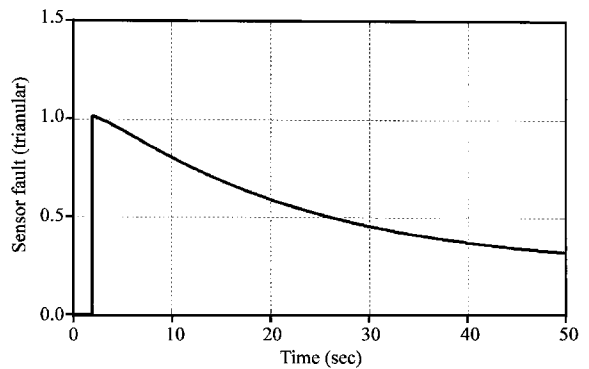


Fig. 14: Fault signal occurred in first sensor (incipient fault)

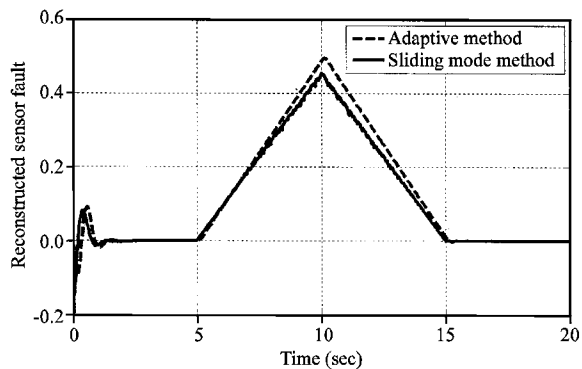


Fig. 12: Reconstructed fault signal for state that 20% of changes have been made in parameters of system and triangular fault occurred in first sensor

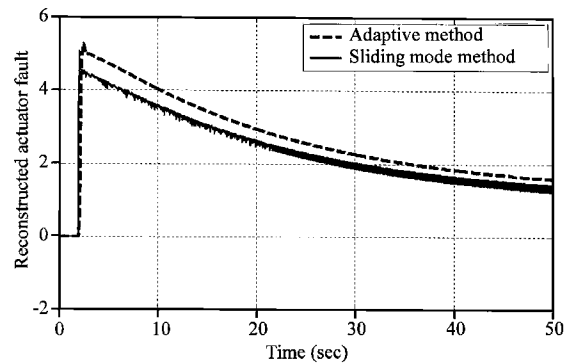


Fig. 15: Reconstructed fault signal for state that incipient fault has occurred in second actuator

design of observer on the basis of adaptive method. According to (10) and (21), it is clear that none of the two methods can recognize fault in second sensor due to defect of rank in matrix  $(A_{22}-A_{21}A_{11}^{-1}A_{12})$ . Result of simulation show simplicity and ability of the mentioned method.

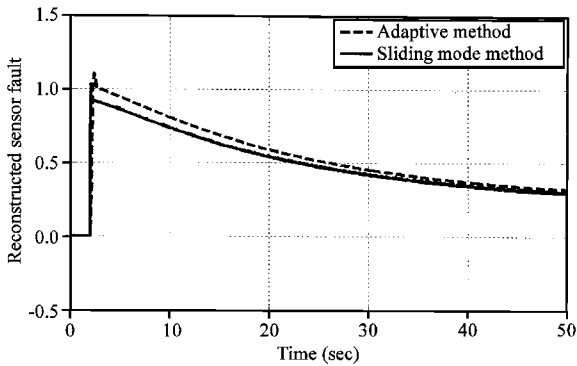


Fig. 16: Reconstructed fault signal for state that incipient fault has occurred in first sensor

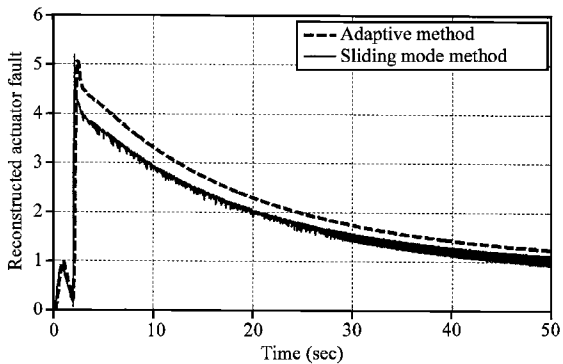


Fig. 17: Reconstructed fault signal for state that 20% of changes have been made in parameters of system and incipient fault occurred in second actuator

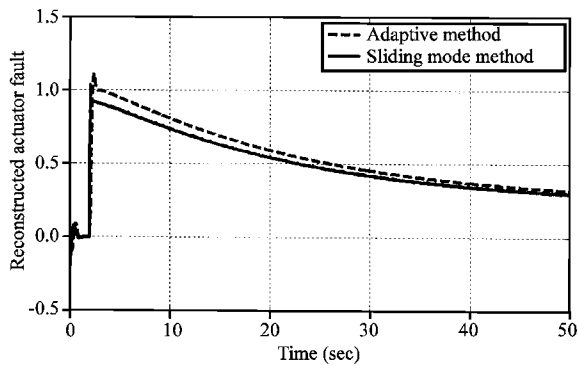


Fig. 18: Reconstructed fault signal for state that 20% of changes have been made in parameters of system and incipient fault occurred in first sensor

**CONCLUSION**

In this study, by use of adaptive technique a novel FDI is imposed. This method can reconstruct a

class of actuator and sensor faults. In order to show capability of the recommended method, simulation results have been compared with those obtained by SMO method on a model of an aircraft. Simulation results confirm the capability of the mentioned method to reconstruct fault signal, so SMO can be replaced by adaptive observer with some advantages for fault detection and isolation purposes.

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