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Quantum Entanglement Implementation Using Interferometric Electro-Optic Modulator and Coupled Mode Theory

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Abstract: In this study, an all optical method will be proposed for quantum entanglement implementation. Universal set of quantum gates will be realized by using Coupled mode theory, interferometric electro-optic modulator and Y-junction beam splitter. Normal modes in waveguides are used as quantum bits and coupled mode equation is derived for optical waveguide modes. This all optical technique can be used to perform any quantum computation. The proposed universal gates have potential of being more compact and easily realized compared to other optical implementations. This method is based on planar lightwave circuit technology and it is suitable for integrated optics.

Key words: Quantum gate, quantum communication, quantum computation, qubit, integrated optics

INTRODUCTION

Quantum gates and entanglement are important part of quantum communication and computation (Prevedel et al., 2007). Many researchers work on theories of quantum computation, such as entanglement, quantum decoherence, quantum cryptography and quantum error correction. Now Quantum computation and communication move closer to being a reality (Benenti et al., 2004; Benenti and Strini, 2007; Nielsen and Chuang, 2000). For transition from theory to applications, we need to realize quantum computer by using the implementation of quantum gates. Standard versions of Planar Lightwave Circuit (PLC) technology are well documented and have been practiced since late 1980's. They combine significant features of optical fiber and integrated circuit technologies. Basically, light-guiding channels, similar in function to optical fibers, are defined on a silicon platform. These are fabricated by depositing sequential glass layers onto silicon wafers. Typically, an intermediate core layer with an elevated refractive index is patterned using photolithography and dry-etching. This patterned structure becomes the light-guiding channel. This baseline technology is well suited to the fabrication of passive devices such as couplers which depend on the proximity, controlled spacing and path lengths of parallel waveguides. The silicon chip provides a robust support for the waveguides. Such PLC components are interfaced to fiber optic networks via edge attached fibers. This technology supports complex and versatile photonic integration. Implementation of quantum gates by using all optical circuits is very important in all optical communication networks. Since the trend for photonics integration is toward smaller and more densely-packed components, it is necessary to be able to model these small components perfectly (Okamoto, 1999; Doerr, 2006; Suzuki and Sugita, 2005).

Coupled mode theory: Coupled mode theory is used in optical filters and switches. In these devices, two coupled waveguides that the coupling between them is controlled by applied voltage are used. The waveguides are made from electro-optics material, that its permittivity is controlled by voltage. By controlling the permittivity, we can transfer power from one limb to another.

The set of coupled differential equations that we have just derived can be used to analyze transferred energy from one optical waveguide to another, when the guiding structures are brought into proximity. Consider two parallel waveguides, 1 and 2 in Fig. 1, for which the total field solutions can be written as linear combinations of individual waveguide modes:

\[ E(x,y,z) = a(z)E_0^1(x,y) + b(z)E_0^2(x,y) \]

\[ H(x,y,z) = a(z)H_0^1(x,y) + b(z)H_0^2(x,y) \]

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Where, $K = K_{ab} = K_{ba}$, $\beta = \beta_a = \beta_b$, have been used. Complete power transfer occurs for synchronous coupling. For $K_1 = (2n+1)\pi/2$, complete power transfer occurs; this is called a cross state. For $K_1 = n\pi$, there is no power transfer from guide 1 to guide 2, which called a parallel state.

Power transferring between guide 1 and 2 is shown in Fig. 2. As can be seen in this figure, if the length of coupler is $L = \pi/2K$, then the power of guide 1 transfer to guide 2 completely.

### INTERFEROMETRIC ELECTRO-OPTIC MODULATOR

The basic electro-optic interferometer is shown in Fig. 3. In this device, a single input is split between two waveguides. The space between the waveguides is designed suitably so that the evanescent coupling does not take place. The two outputs are added together in a single output guide where the two waves interfere, with amount of interference depending upon the difference in propagation time along the two limbs.

The propagation constant of the two guides can be changed differentially by using the electro-optic effect to change the guide refractive indices. A DC bias can be applied to equalize the phase difference of the two limbs in the absence of a modulation voltage. Interferometric modulator using LiNbO$_3$ with bandwidths up to 40 GHz have been reported and they have a considerable future in optical communication and computation (Tamir, 1988).

The transfer function matrix of this element is:

$$U = \begin{bmatrix} \cos\phi/2 & j\sin\phi/2 \\ j\sin\phi/2 & \cos\phi/2 \end{bmatrix}$$

Where:

- $\phi = \text{Phase difference between the two limbs of modulator that adjusted by applied voltage}$

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Fig. 3: Interferometric electro-optic modulator
Another way for producing phase difference between two limbs of modulator is using of Kerr-like nonlinear waveguide. In the Kerr-like medium, intensity of electromagnetic fields changes refractive index of the waveguide and provides phase shift.

**QUANTUM CIRCUIT FOR QUANTUM ENTANGLEMENT**

We can model entangled photons by using quantum gates (Cerf et al., 1998). This model includes Hadamard and CNOT gates (Fig. 4). If the input of this circuit is one of states $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$, then four different entangled states, called Bell states, are produced at the output. In the first state, suppose that the input is $|00\rangle$.

Hadamard gate converts state $|0\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Therefore, the state at the input of CNOT gate is:

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

(6)

At the output of the CNOT gate, this state becomes,

$$\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = |\Phi^+\rangle$$

(7)

If we consider the other three states $|01\rangle$, $|10\rangle$, $|11\rangle$ as input, then the output of circuit is one of the following Bell states,

$$|01\rangle \rightarrow \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) = |\Psi^+\rangle;$$

$$|10\rangle \rightarrow \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) = |\Phi^-\rangle;$$

$$|11\rangle \rightarrow \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) = |\Psi^-\rangle;$$

(8)

These states are entangled states and are the basis of the quantum computation and communication. From Fig. 4, we see that quantum entanglement implementation consist of realization of quantum gates. We propose the new method for integrated optics implementation of quantum entanglement.

**ALL OPTICAL IMPLEMENTATION OF QUANTUM NOT GATE**

Qubits can be realized by the two normal modes of the dual-mode waveguides, such as the zero logical state $|0\rangle$ encoded into one normal mode, TM$_{0}$ and the logical one $|1\rangle$ given by other orthogonal normal mode, TM$_{1}$ (Fig. 5). A qubit's state space consists of all superpositions of the basic normal modes $|0\rangle$ and $|1\rangle$.

As mentioned above, in this study, an all optical method is proposed for implementation of quantum gates. Realization of quantum NOT gate using interferometric electro-optic modulator is shown in Fig. 6. By applying the suitable $V_{not}$ to the electrodes, we can adjust phase difference between the two limbs of the modulator.

With due attention to Eq. 5, the relation between input and output of interferometric electro-optic modulator is:

$$\begin{bmatrix}
|\psi_{i}\rangle \\
|\psi_{o}\rangle
\end{bmatrix} =
\begin{bmatrix}
\cos(\frac{\phi}{2}) & j\sin(\frac{\phi}{2}) \\
{j\sin(\frac{\phi}{2})} & \cos(\frac{\phi}{2})
\end{bmatrix}
\begin{bmatrix}
|0\rangle \\
|1\rangle
\end{bmatrix}.$$  

(9)

If $\phi = 0$, then all inputs are unchanged at the gate output, but if $\phi$ is adjusted to the value of $\pi$, the modulator acts as a quantum NOT gate. All inputs to $|0\rangle$ appear as the $|1\rangle$ output and vice versa, extra an additional phase. Superposition states are generated by adjusting the phase difference. For example by choosing $\phi = \pi/2$, we have the following states,

$$|\psi_{i}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|\psi_{o}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

(10)

The Beam Propagation Method (BPM) simulation results for quantum NOT gate have been shown in Fig. 7-9.
Fig. 6: Realization of quantum NOT gate using interferometric electro-optic modulator. The width of waveguides is 12 \( \mu \text{m} \) and the length of gate is 2.8 cm.

Fig. 7: Quantum NOT: \( |0\rangle \rightarrow |1\rangle \). (a) Optical field amplitude and (b) output electric field profile.

Fig. 8: Quantum NOT: \( |1\rangle \rightarrow |0\rangle \). (a) Optical field amplitude and (b) output electric field profile.
ALL OPTICAL IMPLEMENTATION OF HADAMARD GATE

The quantum Hadamard gate operates on a single qubit. It is represented by the following equation (Andruch and Ali, 2004).

\[ H = \begin{bmatrix}
0 & 1 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix} \begin{bmatrix}
|0\rangle \\
|1\rangle
\end{bmatrix} \]

(11)

An integrated optics beam splitter will be proposed for realization of this gate. Beam splitter is a basic element of many optical fiber communication systems, often providing a Y-junction by which signals from separate sources can be combined, or the received power can be divided between two or more channels. A passive Y-junction beam splitter is shown in Fig. 10. Unfortunately, the power transmission through such a splitter decreases sharply with increasing half angle θ, the power is radiated into the substrate. The passive Y-junction beam splitter finds application where equal power division of incident beam is required.

We consider waveguide branch that supports the two lowest order normal modes namely TM₀ and TM₁ (|0⟩ and |1⟩). Transferred powers between these modes in the branches are described by the coupled mode equations. Because of linearity of these equations and superposition properties of Maxwell’s equations, the solution can be obtained for each local normal mode independently. The achieved solutions are superimposed at the output branches. Normal TM₀ and TM₁ modes (|0⟩ and |1⟩) with equal powers and phases are applied on an ideal power dividing branch. Considering each normal mode independently, the output mode amplitudes in branch 1 are same and have values equal to \( \frac{1}{\sqrt{2}} \), but in branch 2, the output amplitudes of TM₀ and TM₁ modes
Fig. 12: (a) The amplitude of optical filed propagation for state $|0\rangle$, (b) the electrical field profile amplitude of input state $|0\rangle$, (c) the amplitude of optical filed propagation for state $|1\rangle$ and (d) the electrical field profile amplitude of input state $|1\rangle$.

Fig. 13: (a) The amplitude of optical filed propagation for superimpose state and (b) the electrical field profile amplitude of input state $|\frac{1}{2}\rangle$ and $|\frac{1}{2}\rangle$, respectively. Therefore, the output of branch 1 is the same as the output of Hadamard gate when its input is $|0\rangle$. Also, the output of branch 2 is the same as the output of Hadamard gate, when its input is $|1\rangle$. The relation between the output and input of this element is:
By comparing the Eq. 11 and 12, we can see that the output of this element is exactly same as the Hadamard gate.

The Beam Propagation Method (BPM) simulation results for this gate are shown in Fig. 11-13.

**ALL OPTICAL IMPLEMENTATION OF CONTROLLED NOT GATE**

In the quantum controlled NOT gate, if input control bit is |0>, then the control bit and the target bit do not change. In other case, for input control bit |0>, the target bit changes as below:

\[
\begin{align*}
|0\rangle & \rightarrow |1\rangle \\
|1\rangle & \rightarrow |0\rangle
\end{align*}
\]

(13)

Both the coupled mode theory and the interferometric electro-optic modulator are used for implementation of this gate. The planar lightwave integrated optics of this gate is shown in Fig. 14. There is a coupling region in this scheme. The length of this coupler is designed for power transferring between the two couplers for TM, mode. The electro-optic material such as LiNbO₃ is used in limbs of modulator. Two electrodes are placed on two limbs. When the optical power is detected at the coupler output, the applied voltages on the electrodes are adjusted as the phase difference π is created between the two limbs of the modulator. If there is not any optical power on the output branch of the coupler, then the applied voltages on electrodes are adjusted as the phase difference 0 is created between the two limbs of the modulator. When |0>

![CNOT gate realization](image1)

**Fig. 14:** The CNOT gate realization. The width of waveguides is 12 µm. The length of coupler is 200 µm, separation between coupler waveguides is 11 µm and gate length is 2.8 cm.

![CNOT gate profiles](image2)

**Fig. 15:** The CNOT quantum gate: |00⟩→|00⟩. (a) optical field amplitude, (b) input electric field profiles and (c) output electric field profiles
Fig. 16: The CNOT quantum gate: $|01\rangle \rightarrow |01\rangle$. (a) optical field amplitude, (b) input electric field profiles and (c) output electric field profiles.

Fig. 17: The CNOT quantum gate: $|10\rangle \rightarrow |11\rangle$. (a) optical field amplitude, (b) input electric field profiles and (c) output electric field profiles.
Fig. 18: The CNOT quantum gate: $|11\rangle \rightarrow |10\rangle$. (a) optical field amplitude, (b) input electric field profiles and (c) output electric field profiles

Fig. 19: All optical method to perform quantum entanglement for integrated optics circuit
is present at the control bit, the intensity of the qubit is never coupled into the output of coupler. Therefore the control and target qubits are left unchanged. When \(|1\rangle\) is present at the control bit, the intensity of the qubit is coupled into the output of coupler. Thus a phase shift of \(\pi\) is created between the two limbs of the modulator and the states of the target bit will be flipped, namely \(|0\rangle \rightarrow |1\rangle\) and \(|1\rangle \rightarrow |0\rangle\).

The Beam Propagation Method (BPM) simulation results for this gate are shown in Fig. 15-18.

**ALL OPTICAL IMPLEMENTATION OF QUANTUM ENTANGLEMENT**

We proposed the schemes for all optical implementation of quantum gates in the previous sections. We can achieve quantum entanglement by combination of these quantum gates. We propose a fully optical method to perform quantum entanglement. This proposal is shown in Fig. 19. The Y-junction beam splitter is used as a Hadamard gate. The upper branch state of the beam splitter is equal to the output of the Hadamard gate, when its input is \(|0\rangle\) and state of the other branch is the same as the output of Hadamard gate when its input is \(|1\rangle\).

The output of each branch of the Y-junction beam splitter is used for the input control bit of each CNOT gate. This scheme is consist of two parts. Bell states \(|\psi^+\rangle\) and \(|\Psi^-\rangle\) are generated by the upper part and \(|\phi^+\rangle\) and \(|\Phi^-\rangle\) are generated by the other part.

In conclusion, we proposed an all optical method for implementation of the quantum gates and entanglement. By using planar lightwave technology, all single qubit and double qubit quantum logic gates are feasible. By using a dual mode waveguide interferometric electro-optic modulator, directional couplers and Y-junction beam splitter, we propose a fully optical method to perform quantum gates and entanglement as the basis of quantum communication and computation.

**REFERENCES**


