Loss-minimization Scheme in Modified DTC-SVM for Induction Motors with Torque Ripple Mitigation

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Abstract: Direct Torque Control (DTC) is known to produce quick and robust torque response for AC drives. However, during steady state, notable torque, flux and current pulsations occur which will be reflected in speed response, speed estimation error and also in increased acoustic noise. This study presented a modified direct torque and flux control based on Space Vector Modulation (SVM) for induction motor drives. This method based on control of stator flux components. A new strategy is proposed to minimize the torque ripple of induction motor in which, the stator flux level is selected in accordance with the efficiency optimized motor performance. Simulation results show consistency and the good performance of the proposed control strategy.

Key words: Direct torque control, induction motor, loss minimization, space vector modulation, torque ripple

INTRODUCTION

Direct Torque Control (DTC) of induction motor drives offers high performance in terms of simplicity in control and fast torque response. Principle of the classical DTC is decoupled control of flux and torque using hysteresis control of flux and torque error and flux position. In this method, switching look-up table is included for selection of voltage vectors feeding the induction motor (Takahashi and Noguchi, 1986). However, during steady state operation, the DTC produces high level of torque ripple and variable switching frequency of inverter.

Development of DTC for overcoming the drawbacks of classical DTC is voltage modulation application replacing look-up table of the voltage vector selection. The voltage modulation is based on space vector modulation (SVM) with constant switching frequency. The SVM strategy is based on space vector representation of the converter AC side voltage and has become very popular because of its simplicity. Contrary to conventional Pulse Width Modulation (PWM) method, in the SVM method there is no separate modulators for each phase (Hebeter et al., 1992).

Alternatively, SVM method is incorporated with direct torque control (so-called DTC-SVM) for induction motor drives to provide a constant inverter switching frequency. Several methods with DTC-SVM are presented in the literature. The first method is based on deadbeat control derived from the torque and stator flux errors (Hebeter et al., 1992). It offers good steady state and dynamic performance. However, this technique has a limitation since it is computationally intensive. Another method is based on fuzzy logic and artificial neural network for decoupled stator flux and torque control (Grabowski et al., 2000). Good steady state and dynamic response are achieved. Another method uses two PI controllers instead of hysteresis controllers for generating direct and quadrature components of voltage from stator flux and torque errors, respectively (Lasca et al., 2000; Lai and Chan, 2001). This method provides good transient performance, robustness and reduced torque ripple. However, in any of above methods, an analytical approach is not presented to direct reducing torque ripple.

Another objective in induction motor control is to achieve maximum efficiency. Choosing the level of flux in the induction motor remains an open problem for the perspective of maximizing motor efficiency and many researchers continue to work on this problem. Numerous operation schemes have been proposed by many researchers concerning the optimal choice of flux level for a given operating point. In low-frequency operation, core loss (hysteresis and eddy current losses) is rather low compared with copper loss. As the speed goes up, the contribution of the eddy current loss increases and finally becomes dominant (Lorenz and Yang, 1992). The technique allowing efficiency improvement can be divided into two categories. The first category is the loss-model-based approach (Lorenz and Yang, 1992), which consists of computing losses by using the machine model and selecting a flux level that minimizes the losses. The second category is the search controller (Kirschen et al.,
1985), in which the flux is decreased until the input power settles down to the lowest value for a given torque and speed. Important drawbacks of the search controller are the slow convergence and high torque ripples. The less-model-based approach is fast and does not produce torque ripple. However, the accuracy depends on the accurate modeling of the motor and the losses. Different loss models for loss minimization in induction motor can be found in the literature. In model-based loss minimization algorithms, the leakage inductance of stator and rotor are usually neglected to simplify the loss model and minimization algorithm. However, with these simplified models, the exact loss minimization cannot be achieved, especially in high-speed operation of the motor, since a large voltage drop across the leakage inductance is neglected (Lim and Nam, 2004). In this study, a simplified loss model without neglecting the inductance, which presented by Lim and Nam (2004), is used.

The main contribution of this study is torque ripple minimization along with efficiency optimization of induction motor which is based on the analytical determination of torque ripple in DTC-SVM. Torque ripple minimization is achieved by optimum selection of switching pattern in SVM and efficiency optimization is achieved by optimum selection of stator flux level in DTC.

INDUCTION MACHINE MODEL

The dynamic behavior of induction machine is described using the following equations in terms of space vectors in rotor flux oriented frame.

\[
\bar{V}_s = R_s\bar{i}_s + \frac{d\bar{\psi}_s}{dt} + j\omega_s\bar{\psi}_s
\]

\[
0 = R_e\bar{i}_r + \frac{d\bar{\psi}_r}{dt} + j(\omega_s - \omega_e)\bar{\psi}_r
\]

\[
\bar{\psi}_s = L_\phi\bar{i}_s + M_{\phi}\bar{i}_r
\]

\[
\bar{\psi}_r = L_m\bar{i}_r + M_{\psi}\bar{i}_s
\]

where, \(R_s, R_e\) represent the stator and rotor resistance. \(L_s, L_m, M\) are the self and mutual inductances. \(\omega_s\) is the rotor angular speed expressed in electrical radians and \(\omega_e\) is the speed of rotor flux oriented frame.

The electromagnetic torque is expressed as:

\[
T_e = \frac{3pM}{2\sigma L_s L_r}(\bar{\psi}_s \times \bar{\psi}_r)
\]

In Eq. 5 the symbol \(\times\) represents scalar vector product.

Eliminating \(\bar{\psi}_r\) from Eq. 1-4, leads to the state variable form of the induction machine equations with stator and rotor fluxes as state variables as:

\[
\begin{bmatrix}
\frac{d\bar{\psi}_s}{dt} \\
\frac{d\bar{\psi}_r}{dt}
\end{bmatrix} = \begin{bmatrix}
\frac{-R_s L_s}{L_s L_r - M^2} - j\omega_s & \frac{R_s M}{L_s L_r - M^2} \\
\frac{R_e M}{L_s L_r - M^2} & \frac{-R_e L_e}{L_s L_r - M^2} - j(\omega_s - \omega_e)
\end{bmatrix} \begin{bmatrix}
\bar{\psi}_s \\
\bar{\psi}_r
\end{bmatrix} + \begin{bmatrix}
V_s \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\bar{\psi}_{s(k+1)} \\
\bar{\psi}_{r(k+1)}
\end{bmatrix} = \begin{bmatrix}
\bar{\psi}_{s(k)} + \frac{d\bar{\psi}_s}{dt}\Delta t \\
\bar{\psi}_{r(k)} + \frac{d\bar{\psi}_r}{dt}\Delta t
\end{bmatrix}
\]

\[
T_{e(k+1)} = \frac{3pM}{2\sigma L_s L_r}(\bar{\psi}_{s(k+1)} \times \bar{\psi}_{r(k+1)})
\]


\[
\Delta T_{e(k)} = -T_e \frac{R_e}{L_s} + \frac{R_e L_e}{L_s L_r} \frac{\Delta \omega}{\sigma}
\]
\[
\Delta T_e = \frac{3pM}{2\sigma L_v L_r} [(\frac{V_{s1}^*}{\sqrt{3}} - j\alpha_\mu \phi_{m1}) \phi_{m1}] a
\]
(15)

From Eq. 13-15, torque variation can be written as:
\[
\Delta T = T_{e1} - T_e = \frac{3pM L_v}{2\sigma L_v L_r} \phi_{m1} [V_{s1} - \alpha_\mu \phi_{m1} - k \phi_{m1}] \]
(16)

Where:
\[
k = \frac{1}{\sigma} \frac{R_{o1} + R_{o2}}{L_v} \]
(17)

Equation 16 clearly shows the effects of the applied voltage vector and stator and rotor flux on the torque variation.

**PROPOSED SWITCHING PATTERN OF SVM FOR TORQUE RIPPLE MITIGATION**

As shown in Fig. 1, the switching pattern of inverter control includes eight switching states, made up of six active and two zero switching states. Active vectors divide the plane into six sectors, where the reference voltage vector \((V_{s1}^*\) with \(\gamma\) angle) is obtained by switching on (for the proper time) two adjacent vectors.

The vector patterns associated with the switching states are expressed by Lai and Chan (2001):

\[
t_a = t_s \frac{2}{\sqrt{3}} \sin(\frac{\pi}{3} - \gamma) \]
(18)

\[
t_b = t_s \frac{2}{\sqrt{3}} \sin(\gamma) \]
(19)

\[
t_c = t_b - t_a - t_b \]
(20)

\[a = \frac{3V^*}{2V_a} \]

In this study, for minimizing torque ripple, time \(t_b\) is divided into two parts: \(t_{b1}\) and \(t_{b2}\). In proposed strategy for space vector modulation, switching pattern is shown in Fig. 2.

From Eq. 16, torque slope during \(t_a\) and \(t_b\) can be written as:
\[
S_a = \frac{3pM L_v}{2\sigma L_v L_r} \phi_{m1} [V_{s1} - \alpha_\mu \phi_{m1} - k \phi_{m1}] \]
(21)

where, \(i\) is A and B and torque slope during \(t_{b1}\), \(t_{b2}\) can be written as:
\[
S_b = \frac{3pM L_v}{2\sigma L_v L_r} \phi_{m1} [\alpha_\mu \phi_{m1} + k \phi_{m1}] \]
(22)

Figure 3 shows the torque ripple during sampling time \(t_s\).

According to Fig. 3, the square of the RMS value of torque ripple during \(t_s\) can be expressed as:
\[
T_{rip}^2 = (\frac{S_a^2}{2S_a} - \frac{S_b^2}{2S_b})t_{b1} - \frac{T_{b2} - T_{b1} - V_y^*}{S_b} + (\frac{S_a^2}{2S_a} - \frac{S_b^2}{2S_b})t_{b2} - \frac{T_{b2} - T_{b1} - V_y^*}{S_b} \]
(23)

The optimum value of \(t_{b1}\) to minimize the torque ripple can be obtained by the partial derivatives of Eq. 23 with respect to \(t_{b1}\). By solving Eq. 24, proper time \(t_{b1}\) can be obtained from Eq. 25.

**Fig. 2: Proposed switching pattern**

**Fig. 3: Torque variation during \(t_s\)**
\[
\frac{\partial \theta_{\omega}}{\partial \theta_{\phi}} = 0
\]  
\[
t_{\phi} = \frac{L_p}{L_{\phi}} i_p
\]  

\section*{LOSS MINIMIZATION SOLUTION}

In this study, a simplified induction motor model with iron loss is used. Equivalent circuit of this model in the stationary reference frame is shown in Fig. 4. This model in synchronous frame is described by following equations:

\[
V_u = R_u i_u + L_u \frac{di_u}{dt} + L_u \frac{di_m}{dt} - \omega_L L_n i_m
\]  
\[
V_p = R_p i_p + L_p \frac{di_p}{dt} + L_p \frac{di_m}{dt} + \omega_L L_n i_m
\]  
\[
0 = R_s i_s + L_s \frac{di_s}{dt} + L_s \frac{di_m}{dt} - \omega_L L_n i_m
\]  
\[
0 = R_s i_s + L_s \frac{di_s}{dt} + L_s \frac{di_m}{dt} + \omega_L L_n i_m
\]  
\[
i_u = i_u - \frac{\omega_L}{R_u} (L_m \frac{di_m}{dt} - \omega_L L_n i_m) + i_m
\]  
\[
i_p = i_p - \frac{\omega_L}{R_p} (L_m \frac{di_m}{dt} + \omega_L L_n i_m) + i_m
\]  

where, \(\omega_L = \omega_s - \omega_m\) and S is slip of motor and \(R_m\) is core loss equivalent resistance.

It is assumed that the rotor flux oriented scheme is utilized. The rotor flux oriented scheme is realized by aligning the reference frame on the rotor flux axis in which, \(\Phi_\phi = 0\). In the steady state condition, \(i_u = 0\) and:

\[
\omega_L = \frac{L_u R_s i_m}{R_s \Phi_\phi}
\]

The total motor losses can be expressed as:

\[
P_{loss} = R_u i_u^2 + R_p i_p^2
\]

Where:

\[
R_s = R_s + \frac{\omega_L^2 M^2}{R_u}
\]

\[
R_p = R_p + \frac{\omega_L^2 M^2}{R_s L_r^2} + \frac{R_s M^2}{L_i^2}
\]

The electromagnetic torque in the rotor flux oriented scheme at steady state is given by:

\[
T_r = k_t i_s i_p
\]

Where:

\[
k_t = \frac{3p M^2}{4L_r}
\]

From Eq. 34 and 37, the cost function is defined by:

\[
J(i_u, i_p) = \lambda (k_t i_s i_p - T_r)
\]

The optimal solution for Eq. 38 is (Lim and Nam, 2004):

\[
i_u = \left( \frac{R_s T_r^2}{R_s \Phi_\phi} \right)^{1/4}
\]

\[
i_p = \left( \frac{R_p T_r^2}{R_p \Phi_\phi} \right)^{1/4}
\]

From Eq. 39, 40, the optimum flux level is obtained from:

\[
\begin{bmatrix}
\Phi_u \\Phi_p
\end{bmatrix} = \begin{bmatrix}
\Phi_u \\Phi_p
\end{bmatrix} = \begin{bmatrix}
R_s \Phi_\phi \\Phi_\phi
\end{bmatrix}
\]

\[
\Phi_{opt} = \sqrt{\Phi_u^2 + \Phi_p^2}
\]

\section*{PROPOSED STRATEGY AND SIMULATION RESULTS}

Figure 5 shows the block diagram of the proposed strategy for DTC-SVM of induction motor. In this method,
Table 1: Parameters of induction motor

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>2 kW</td>
</tr>
<tr>
<td>Number of poles</td>
<td>2</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>0.399 Ω</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>2.7 mH</td>
</tr>
<tr>
<td>Magnetizing inductance</td>
<td>56.6 mH</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>0.3338 Ω</td>
</tr>
<tr>
<td>Rotor leakage inductance</td>
<td>3.8 mH</td>
</tr>
<tr>
<td>Rated speed</td>
<td>3500 rpm</td>
</tr>
<tr>
<td>Core loss equivalent resistance</td>
<td>650 Ω</td>
</tr>
<tr>
<td>DC Link voltage</td>
<td>500 V</td>
</tr>
</tbody>
</table>

the reference torque ($T_{ref}$) is obtained from speed PI controller. The optimum components of stator flux in rotor flux reference frame are calculated from reference speed and $T_{ref}$. These components are transformed in stationary reference frame. The reference voltage components are calculated from:

$$ V_d = R_{ld} + \frac{\Phi_{qref} - \Phi_{qest}}{\Delta t} \quad (43) $$

$$ V_\alpha = R_{l\alpha} + \frac{\Phi_{qref} - \Phi_{qest}}{\Delta t} \quad (44) $$

These components in stationary frame are applied to space vector modulator. The values of $t_a$, $t_b$, $t_c$, $t_d$ are calculated based on the reference flux and applied to space vector modulator.

The rating and parameters of the standard induction motor are given in Table 1. The rated power of motor is 2 kW and the rated speed is 3500 rpm.

The validity of the proposed method is shown by comparing the simulation results of proposed DTC-SVM and conventional DTC-SVM.

As shown in Fig. 6a and b the torque ripple in proposed method is reduced compared with conventional method. Total Harmonic Distortion (THD) of torque in proposed method is 2.89% and in conventional method is 4.12%. Both methods are simulated in constant sampling frequency ($f_s = 10$ KHz).

![Figure 5: Block diagram of proposed method](image)

![Figure 6: Simulation results in torque ripple](image)

Figure 7a shows the optimal stator flux that is calculated for step load change from 10 to 5 Nm and Fig. 7b shows the speed of motor. The estimated stator flux is calculated by:

$$ \Phi_{s\alpha} = \int (V_{s\alpha} - R_{s\alpha}) dt \quad (45) $$

$$ \Phi_{s\beta} = \int (V_{s\beta} - R_{s\beta}) dt \quad (46) $$
Fig. 7: Simulation results in (a) Optimal stator flux and (b) speed of motor.

Fig. 8: Simulation results in the input power of motor at: (a) Rated stator flux and (b) optimal stator flux.

For validation of loss-minimization in proposed method, Fig. 8 shows the input power of motor when the rated flux is applied to DTC and when the optimal flux is applied to DTC (for constant load of $P_{out} = 800$ W).

As shown in Fig. 8, when the optimal flux is applied to DTC, the input power of motor reduced about 13.6% and efficiency of motor improved about 15%. Efficiency improvement of the induction motor for different rotor speeds and torques has been illustrated in Table 2.

This research project was conducted from August 2009 to March 2010 in Isfahan University of Technology, Isfahan, Iran.

**CONCLUSION**

In this study, a new strategy for direct torque control of induction motor based on space vector modulation is presented. The features of the proposed method include, first, minimizing the torque ripple of motor by optimal selection of switching pattern in SVM and second, minimizing the loss of motor by optimal selection of stator flux level. The proposed method is evaluated using simulation result which shows very good performance of the method.

**REFERENCES**


