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System Identification using Orthonormal Basis Filters

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Abstract: The widely used dynamic models for identification of linear time invariant systems in process industries are Auto Regressive with Exogenous Input (ARX) and Finite Impulse Response (FIR) models. Their popularity is due to their simplicity in developing the model. However, they need very large amount of data to reduce variance error, in addition ordinary ARX model structures lead to inconsistent model parameters. Orthonormal Basis Filter (OBF) model structures permit incorporation of prior knowledge of the system in the form of one or more poles, which renders it the capacity to capture the system dynamics with a few number of parameters (parsimonious in parameters). In addition, the resulting OBF models are consistent in parameters. The model parameters can be easily developed using linear least square method. In this study, OBF model development for simulation and real case studies is presented.

Key words: System identification, orthonormal basis filters, ARX model, FIR model

INTRODUCTION

Models of real systems are used, practically, in all fields of science and engineering. In engineering, models are required for the design and development of new processes and for analyzing and improving existing processes. In process industries, models are used in controller design, optimization and fault detection and diagnosis. Models are extensively used in advanced process control design and implementations. Nearly all optimal control design techniques rely on the use of the model of the system to be controlled. In Model Predictive Controllers (MPC), models are used to predict the future values of the output which is used in calculating the optimum input values. The process of developing system models from experimental data is known as system identification.

A general linear dynamic model consists of deterministic and stochastic parts. According to this general model, the output is the sum of the input u (k) and noise e (k) filtered by their respective filters (Ljung, 1999; Nelles, 2001). Equation 1 represents the general linear model shown in Fig. 1.

$$y(k) = \frac{B(q)}{F(q)A(q)}u(k) + \frac{C(q)}{A(q)D(q)}e(q) \quad (1)$$

This general model leads to a much complicated model where parameter estimation is usually difficult; therefore it is most commonly simplified by making assumptions on the polynomials A, B, C, D and F. Some of the most commonly used linear models derived from this general model.

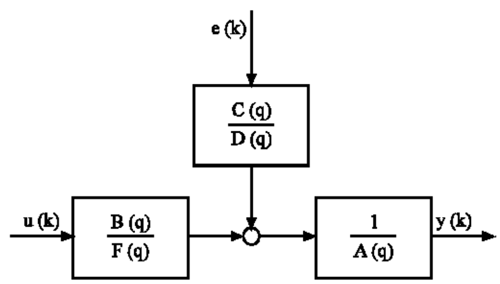

Fig. 1: Block diagram for the general linear model

Auto regressive with exogenous input (ARX): Autoregressive with exogenous input (ARX) model is derived from the general linear model by assuming C(q) = D(q) = F(q) = 1. ARX models are very popular in industrial application because of the simplicity in estimating the model parameters (Nelles, 2001).

$$y(k) = \frac{B(q)}{A(q)}u(k) + \frac{1}{A(q)}e(q) \quad (2)$$

Auto regressive moving average with exogenous input (ARMAX): The ARMAX structure is derived from the general linear model by assuming D(q) = F(q) = 1. The parameters of the ARMAX model are calculated by nonlinear optimization or by extended least square method.

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$$y(k)=\frac{B(q)}{A(q)}u(k)+\frac{C(q)}{A(q)}e(q) \quad (3)$$

Output Error (OE): The output error structure does not include a noise model where A(q) = C(q) = D(q) = 1. Estimation of the model parameters involves nonlinear optimization.

$$y(k)=\frac{B(q)}{F(q)}u(k)+e(q) \quad (4)$$

Box Jenkins (BJ): The Box Jenkins structure is the most flexible among the linear model structures. It is derived from the general structure by assuming A(q) = 1 (Nelles, 2001).

$$y(k)=\frac{B(q)}{F(q)}u(k)+\frac{C(q)}{D(q)}e(q) \quad (5)$$

Finite Impulse Response (FIR): The finite impulse response model is the simplest of the linear models. It is a linear combination of delay filters, q^{-1}, q^{-2}, ...

$$y(k)=B(q)u(k)+e(q) \quad (6)$$

The FIR and ARX models are the most popular linear models in process industries. It is because the model parameters can be easily estimated using linear least square method. However, both models have major drawbacks. FIR model requires large number of parameters (non-parsimonious) to accurately capture system dynamics and ARX model, for most practical systems, results in inconsistent parameters (Nelles, 2001). When model parameters are non-parsimonious, large input-output data set is required to minimize variance errors in model parameters. When a model is inconsistent in parameters, there will be a systematic error (bias) in the estimated model parameters that cannot be removed by increasing the number of data points.

The ARMAX is the next commonly used model structure. Its model parameters can be estimated using nonlinear optimization or extended least square method. However, the common denominator dynamics A(q) may not describe many practical problems, where the noise is not correlated with the input. BJ models are the most flexible of all the linear models. However, their application is very limited due to the difficulty in estimating the model parameters (Nelles, 2001). Estimation of BJ model parameters involves non-linear optimization and because of the large number of parameters, it is rarely applied in Multiple-Input Multiple-Output (MIMO) systems. One common problem in all the linear models is that prior knowledge of time delay is required to accurately estimate the model parameters.

Recently, there has been a significant progress in system identification based on Orthonormal Basis Filters (OBF) and their implementation in MPC and fault tolerant control (Patwardhan and Shah, 2005; Patwardhan *et al*., 2006). The OBF models allow incorporation of a priori knowledge of system dynamics into the model and due to this, they can accurately capture the dynamics with a fewer number of parameters. Unlike ARX models, OBF models do not have the parameter inconsistency problem. OBF models are parsimonious in parameters compared to FIR and step response models (Nelles, 2001; Patwardhan and Shah, 2005; Van den Hof *et al*., 2005). The parameters of OBF models can be easily determined using linear least square method and time delays can also be easily estimated and incorporated into the models (Patwardhan and Shah, 2005).

The present study compares the accuracy of FIR, ARX and OBF models in two case studies, viz., (1) a simulated example, a SISO system and (2) a pilot-scale distillation column, a MIMO system.

A brief introduction to various basis filters used in OBF based system identification is provided in the next section together with techniques for the estimation of time delay and model parameters.

Orthonormal basis filters: The OBF models can be considered as a generalization of FIR models in which the filters q^{-1}, q^{-2}, ... are replaced with more complex orthonormal basis filters which allow incorporation of a prior knowledge of the system (Patwardhan and Shah, 2005; Van den Hof *et al*., 2000; Wahlberg, 1991). Two filters, f_m and f_n, are said to be orthonormal if they satisfy the property.

$$\langle f_m(q), f_n(q)\rangle=\begin{cases}1 & (m=n)\\ 0 & (m\neq n)\end{cases} \quad (7)$$

where <,> represents the inner product defined on the set of all stable transfer functions. Thus, a stable system, G(q), can be approximately represented by a finite-length generalized Fourier series expansion as:

$$G(q)=\sum_{i=1}^{n}L_i f_i(q) \quad (8)$$

where, q is forward shift operator, $\{L_i\}_{i=1,2,...}$ is model parameters and f(q) is orthonormal basis filters for the system G(q).

One of the important steps in OBF model development is the selection of an appropriate type of orthonormal basis filter. The various types of orthonormal basis filters are discussed below.

Laguerre filter: The Laguerre filters are first-order lag filters with one real pole. They are, therefore, more appropriate for well damped processes (Nelles, 2001; Patwardhan and Shah, 2005; Van den Hof *et al.*, 2005). The Laguerre filters are given by:

$$f_i = \sqrt{(1-p^2)}\frac{(1-pq)^{i-1}}{(q-p)^i}, \qquad |p|<1 \tag{9}$$

where, p is pole (estimated).

Kautz filter: Kautz filters allow the incorporation of a pair of conjugate complex poles; they are, therefore, effective for modeling weakly damped processes (Nelles, 2001; Patwardhan and Shah, 2005; Van den Hof *et al.*, 2005). The Kautz filters are defined by

$$f_{2i-1} = \frac{\sqrt{(1-a^2)(1-b^2)}}{q^2+a(b-1)q-b}g(a,b,q,i) \tag{10}$$

$$f_{2i} = \frac{\sqrt{(1-b^2)}(q-a)}{q^2+a(b-1)q-b}g(a,b,q,i) \tag{11}$$

where

$$g(a,b,q,i) = \left(\frac{-bq^2+a(b-1)q+1}{q^2+a(b-1)-b}\right)^{i-1} \tag{12}$$

$-1 < \alpha < 1$ and $-1 < b < 1$ $n = 1, 2, ...$

Generalized orthonormal basis filter: Heuberger *et al.* (1995) introduced the generalized orthonormal basis filters and showed the existence of orthogonal functions that, in a natural way, are generated by stable linear dynamic systems and form an orthonormal basis for the linear signal space l_2^n. They showed that pulse, Laguerre and Kautz filters are generated from inner functions and their minimal balanced realization. Ninness and Gustafsson (1997) unified the construction of orthonormal basis filters. The GOBF filters are formulated as:

$$f_i(q,p) = \frac{\sqrt{1-|p_i|^2}}{(q-p_i)}\prod_{j=1}^{i-1}\frac{(1-p_j^* q)}{(q-p_j)} \tag{13}$$

where $p \equiv \{p_j : j = 1,2,...,n_b\}$ is an arbitrary sequence of poles inside the unit circle appearing in complex conjugate pairs.

Markov-OBF: When a system involves time delay and an estimate of the time delay is available, Markov-OBF can be used. The time delay in Markov-OBF is included by placing some of the poles at the origin (Patwardhan and Shah, 2005). For a SISO system with dead time equal to d samples, the basis function can be selected as:

$$f_i = z^{-i} \text{ for } i = 1, 2, ..., d \tag{14}$$

$$f_{i+d}(q,p) = \frac{\sqrt{1-|p_i|^2}}{(q-p_i)}\prod_{j=1}^{i-1}\frac{(1-p_j^* q)}{(q-p_j)}z^{-d} \tag{15}$$

for $i = 1, 2, ..., N$

Estimation of time delay: Patwardhan and Shah (2005) presented a two-step method for estimating time delays from step response of GOBF models. In the first step, the time delays in all input-output channels are assumed zero and the model is identified with GOBF. In GOBF models, the time delay is approximated by a non-minimum phase zero and the corresponding step response is an inverse response. The time delay is then estimated from a tangent line drawn at the point of inflection.

A similar approach to determine the time delay is presented by Tufa *et al.* (2008). In this method, the time delay estimated by the previous method is divided into apparent and contributed time delays. The apparent time delay represents the true time delay and the contributed time delay represents the time delay due to the tail of the sigmoidal response curve which is significant for higher order systems. The latter method gives more accurate estimation of time delay when the order of the system is high.

Estimation of GOBF poles: Finding an appropriate estimate of the poles for the filters is an important step in estimating the parameters of the OBF models. Arbitrary choice of poles may lead to a non-parsimonious model unless an iterative technique is used. Van den Hof *et al.* (2000) showed that for a SISO system with poles $\{a_j^0 : |a_j^0| < 1 \text{ for } j = 1,2,...,n_0\}$, the rate of convergence of the model parameters is determined by the lowest magnitude of eigen value:

$$\rho = \max_j \prod_{k=1}^{n} \left|\frac{a_j^0 - p_k}{1-\bar{p}_k a_j^0}\right| \tag{16}$$

Therefore, a good approximation by a small number of parameters can be obtained by choosing a basis for which ρ is small. It is shown that the poles determined by Van den Hof *et al.* (2005) method closely match the dominant poles of the system (Patwardhan and Shah, 2005; Wahlberg, 1991).

Model parameter estimation: Once the dominant poles of the system and the types of filters are determined, the model parameters can be estimated using linear least square method. The parameter vector, θ, of the model are then calculated by the linear least square (Eq. 17).

$$\theta = (X^T X)^{-1} X^T y \tag{17}$$

where, θ is model parameters, X is the regressor matrix and y is output sequence.

The regressor matrix, X, is formed by filtering the input sequence u(k) with the corresponding filters $f_i(q, p)$ and arranging them in a matrix form as shown in Eq. 18.

$$X = \begin{bmatrix} u_{f1}(m) & u_{f2}(m-1) & \dots & u_{fm}(1) \\ u_{f1}(m+1) & u_{f2}(m) & \dots & u_{fm}(2) \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ u_{f1}(N-1) & u_{f2}(N-2) & \dots & u_{fm}(N-m) \end{bmatrix} \tag{18}$$

where, $u_{fi}(k) = f_i(q, p)\, u(k)$

If an estimate of the dominant pole is not available, an iterative technique can be employed where an arbitrary sequence of poles can be used as a starting point and better estimates of the dominant poles are obtained from the noise-free step response of the GOBF model. The iterative technique for estimating the poles and the deterministic part of the OBF model is explained by Tufa *et al*. (2008).

PRESENT STUDY

In this study, the advantages of OBF models over FIR and ARX models are illustrated through a case study by simulation and OBF and OBF plus ARMA noise models are then developed for a real plant case study of a MIMO system.

Case study 1: In this case study, OBF, ARX and FIR models are developed from the same data set generated by simulation from a system represented by Fig. 1 using SIMULINK. The input is a 'PRBS' data set generated using the *idinput* function in MATLAB. The model of the system to be identified is given by Eq. 19.

$$y(k) = q^{-1}\frac{1-1.5q^{-1}+0.48q^{-2}+0.6q^{-3}}{1-2.657q^{-1}+2.3523q^{-2}-0.6939q^{-3}}u(k) + \frac{1-0.6q^{-1}}{1-1.4q^{-1}+0.48q^{-2}}e(k) \tag{19}$$

The prediction capability of the various models are compared using the Percentage Prediction Error (PPE) defined by Eq. 20.

$$(PPE)_i = \frac{\sum_{i=1}^{n}(y_i(k)-\hat{y}_i(k))^2}{((y_i(k)-\bar{y}_i(k))^2}\times 100 \tag{20}$$

where $\bar{y}_i$ represents the mean value of measurements $\{y_i(k)\}$ and $\hat{y}_i(k)$ the predicted value of $y_i(k)$.

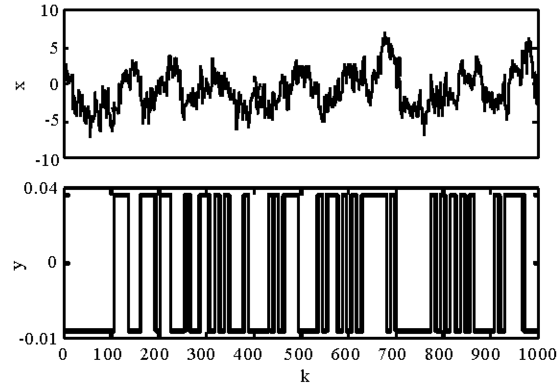

Fig. 2: The input-output data used for model development for the case of 1000 data points

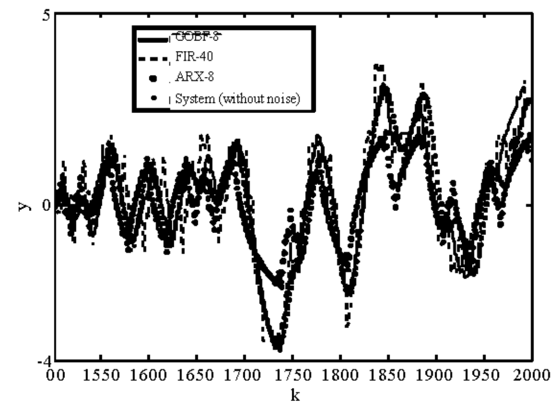

Fig. 3: The prediction using 500-validation datapoints of the GOBF, FI and ARX models compared to the actual output using 1000 data points for model development

Two thousand data points are generated and the 1000 data points shown in Fig. 2 are used for model development while the remaining 1000 data points are used for validation. Five hundred of the validation data are depicted in Fig. 3.

The prediction of the GOBF, FIR and ARX models with number of model parameters 8, 40 and 8, respectively, compared to the response of the original system without noise (simulation in all cases), for the validation data, is given in Fig. 3. It is seen from Fig. 3, that the GOBF model is much closer to the response of the original system than the other two models.

The percentage prediction error, PPE, for each model type developed with 500, 1000, 2000, 3000 and 4000 data points for GOBF, FIR and ARX models with various number of model parameters is given in Table 1. For the comparison, the number of parameters for the GOBF

Table 1: Percentage prediction error of the various models with different number of parameters and data points

	PPE						
No. of data points	OBF 8	FIR 30	FIR 40	FIR 60	ARX 4-4	ARX 8-8	ARX 16-16
500	23.5	36.46	26.00	18.1	56.59	26.80	7.17
1000	6.97	11.30	4.74	3.21	61.23	28.59	7.60
2000	4.71	10.66	4.06	2.34	76.48	41.46	8.09
3000	1.98	10.79	3.09	1.22	70.91	43.56	10.32
4000	1.41	10.39	2.98	0.78	73.78	42.53	11.03

model is fixed at 8. It is observed from the table that, FIR model requires between 40 and 60 parameters to describe the model as accurate as GOBF model with 8 parameters. In addition, as the number of data points increases the percentage prediction error decreases for the GOBF model. In the case of ARX model, increasing the number of model parameters improves the accuracy. However, the accuracy does not improve with increasing number of data points. This shows the inconsistency problem of ARX models, in that, the bias of the parameters cannot be eliminated by increasing the number of model parameters.

Case study 2: In this case study, a GOBF model is developed for a binary distillation column. The distillation column is a part of a reaction-separation system where the product stream from the reactor becomes the feed stream for the distillation column. Isopropyl Alcohol (IPA) is dehydrogenated in the catalytic packed bed tubular reactor. The products from the reactor, acetone and hydrogen, together with un-reacted IPA are cooled in a plate heat exchanger and sent to a vapor-liquid separator where hydrogen is separated from condensed acetone and IPA. This acetone-IPA mixture is stored in an intermediate storage vessel and fed to the distillation column for separation. The bottom product of the column consisting mainly of IPA is recycled back to the reactor. In the present study, the distillation column is operated alone with acetone-IPA mixture as the feed and the product streams are recombined. A snapshot of the 5.5 m high distillation column is shown in Fig. 4. The major dimensions and nominal operating conditions of the distillation column are given in Table 2.

The input sequences are designed as a low frequency Pseudo Random Binary Signal (PRBS) generated using the *idinput* function in MATLAB with band [0 0.04] and levels 18 22 kg h^{-1} and 0.4 0.8 L min^{-1} for steam and reflux flow rates, respectively. Four thousand data points are collected with a sampling interval of 5 sec. The first three thousand data points are used for model identification and the rest are used for validation. The input-output data used for identification of the distillation column is depicted in Fig. 5.

Table 2: Major dimensions and nominal operating conditions of the distillation column

Description	Values
Height	5.5 m
Diameter	0.15 m
Number of trays	15
Type of tray	Bubble cap
Tray spacing	35 cm
Tray numbering	Bottom to top
Feed tray	Tray 7
Feed rate	0.5 l min
Reflux flow rate	0.7 l min
Steam flow rate	20 kg h^{-1}
Distillate flow rate	0.3 l min
Bottom product flow rate	0.3 l min
Bottom product flow rate	0.2 l min
Feed composition, mole fraction	0.1824 (acetone)
Bottom temperature	80.5°C
Top temperature	72.7°C
Column pressure	1.013 bar

Fig. 4: The pilot-scale distillation column

GOBF model: The transfer function of the distillation column is given in the following form:

$$\begin{bmatrix} T_{14} \\ T_1 \end{bmatrix} = \begin{bmatrix} \hat{G}_{11} & \hat{G}_{12} \\ \hat{G}_{21} & \hat{G}_{22} \end{bmatrix} \begin{bmatrix} F_{st} \\ F_R \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{21}$$

where, G_{ij} is GOBF models, H_i is Stochastic part of the model and e_1, e_2 is innovation sequences.

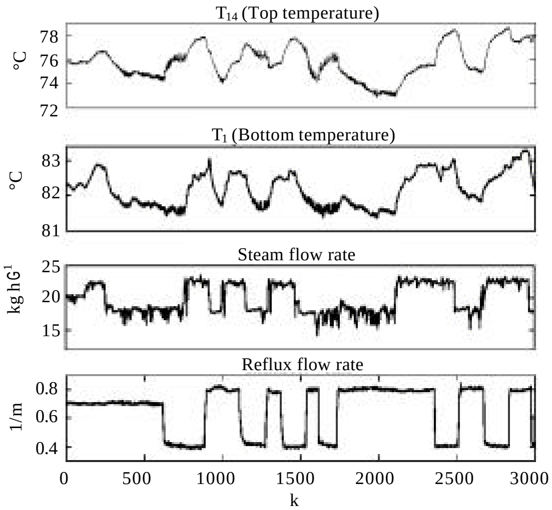

Fig. 5: The input-output sequence used for identification for changes in (a) steam flow rate and (b) reflux flow rate in the pilot scale distillation column

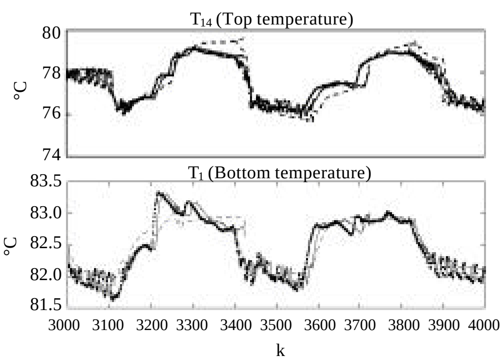

Fig. 6: The prediction of the GOBF model with (____) and without noise model (-----) compared to the original (···) measured data for the distillation column

Individual GOBF models with eight terms each are developed with alternating poles 0.7788 and 0.8187. The estimated GOBF model parameters corresponding to the transfer functions are:

L_{11} = [0.0235 0.0084 0.0128 0.0074 0.0044 0.0094 -0.0069 0.0211]
L_{12}= [0.0192 0.1217 -0.0681 0.1262 -0.0676 0.0066 -0.0627 -0.0443]
L_{21}= [-0.0055 0.0302 -0.0048 0.0222 0.0027 0.0354 -0.0213 0.0583]
L_{22}= [-0.8270 -0.0936 -0.7039 0.2749 -0.0700 -0.3731 0.6136 -0.7887]

The GOBF and the GOBF plus noise model, are compared to the actual measured output in Fig. 6.

The result shows that a GOBF model can capture the dynamics of the distillation column with good accuracy and MIMO systems can be easily developed using GOBF model.

CONCLUSION

The OBF models capture the dynamics of linear systems with much smaller number of parameters than FIR models. GOBF model structures enable consistent parameter estimation, while ARX leads to inconsistent parameter estimation. GOBF model parameters are estimated using linear least square method. It is shown that if the system involves time delay, then an iterative procedure can be employed to simultaneously estimate the delay time and the model parameters. In addition, it is demonstrated that GOBF model can be extended to develop MIMO model for a real pilot scale distillation column. It is also illustrated that the stochastic part of the model can be developed using the residual sequence obtained from the noise-free GOBF model.

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