



Journal of Applied Sciences

ISSN 1812-5654

science
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Stochastic Programming with Tractable Mean-risk Objectives for Refinery Planning under Uncertainty

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Abstract: The application of Information Technology (IT) and Information Systems (IS) have been crucial in enhancing the operating flexibility and resiliency of refineries. In particular, the Process Systems Engineering (PSE) community has been instrumental in carrying out a key role in extending the systems engineering boundaries from mere chemical process systems to the incorporation of business process systems with consideration for risk. Thus, this study considers a robust framework for the economic and operational risk management of a refinery under uncertainty by extending an existing two-stage stochastic program with fixed recourse via scenario analysis. The problem is mathematically formulated as a two-stage stochastic nonlinear program with a tractable mean-risk structure in the objective function. Two measures of risk are considered, namely the metrics of Mean-Absolute Deviation (MAD) and Conditional Value-at-Risk (CVaR). The scenario analysis approach is adopted to represent uncertainties in three types of stochastic parameters, namely prices of crude oil and commercial products, market demands and production yields. However, a large number of scenarios are required to capture the stochasticity of the problem. Therefore, to circumvent the problem of the resulting large-scale model, we implement a Monte Carlo simulation approach based on the Sample Average Approximation (SAA) technique to generate the scenarios. A statistical-based scenario reduction strategy is applied to determine the minimum number of scenarios required yet still able to compute the true optimal solution for a desired level of accuracy within the specified confidence intervals. The proposed model is illustrated through a representative numerical example, with computational results demonstrating how risk-averse-and risk-inclined solutions in the face of uncertainty can be attained in a risk-conscious model.

Key words: Two-Stage stochastic programming, refinery planning, Mean-Absolute Deviation (MAD), Conditional Value-at-Risk (CVaR)

INTRODUCTION

Stochastic programming has emerged as one of the most prominent operation research models for optimization involving uncertainty. Refinery planning problems are subject to uncertainty in many factors, which primarily includes fluctuations in prices of crude oil and saleable products, market demand for products and production yields. The risk terms in Khor *et al.* (2008) are handled using the metric mean-absolute deviation. After obtaining the first model with MAD as risk measurement, the second model is developed in which the risk terms are performed by CVaR. A comparison is performed between the two models to assess which of these two risk measures is superior, both computationally and conceptually, in capturing the economic and operating risk in the planning of a refinery. However, there are large numbers of scenarios that create difficulty to handle various circumstances. For example, there may be more than thousands of cases happening. It is hard to predict

and control numerous scenarios. Therefore, it is necessary to find the minimum number of scenarios to capture all the circumstances. Monte Carlo simulation approach based on the Sample Average Approximation (SAA) technique is applied in this work to generate the minimum number of scenarios which present for thousands cases. The two-stage stochastic program with risk management as presented by Khor *et al.* (2008) is formulated as:

$$\max z = E[z_0] - \theta_1 V(z_0) - E_s - \theta_3 W \quad (1)$$

where, $E[z_0]$ is the expectation of the original objective function z_0 with random price coefficients; θ_1 and θ_3 are weights representing the risk factors; $V(z_0)$ is the sampling variance of z_0 ; E_s is the expected recourse penalty and W is the MAD-based risk measure. In this work, we extend model Eq. 1 by considering MAD as the risk measure in place of variance. Further, we investigate the viability of the risk measure Conditional Value-at-Risk (CVaR), which

has gained wide attention in computational finance, within the domain of refinery planning. All the uncertain parameters are assumed to be discrete random variables.

**MONTE CARLO SIMULATION APPROACH
BASED ON SAMPLE AVERAGE APPROXIMATION
(SAA) METHOD**

In this study, we adopt the Monte Carlo simulation approach for scenario generation based on the Sample Average Approximation (SAA) method (You *et al.*, 2009; Shapiro, 2000; Shapiro and Homem-de-Mello, 1998). The procedure involved is as follows:

- **Step 1:** A relatively small number of scenarios (for example, 50 scenarios) with their associated probabilities are randomly and independently generated for the uncertain parameters of prices, demands and yields. This is accomplished by employing the Monte Carlo simulation approach based on the SAA technique. (This data is otherwise obtained from plant historical data.) The resulting stochastic model (a linear program) with the objective function given in Eq. 2 is solved to determine the optimal stochastic profit with its corresponding material flowrates:

$$\max E_z = E_{z_0} - E_\xi \tag{2}$$

where,

$$E_{z_0} = \sum_{i \in I} \sum_{s \in S} p_s c_{i,s} X_i$$

and

$$E_\xi = \sum_{i \in I} \sum_{s \in S} p_s \xi_{i,s} = \sum_{i \in I} \sum_{s \in S} p_s \left[\begin{aligned} &(c_i^+ z_{i,s}^+ + c_i^- z_{i,s}^-) \\ &+ \sum_{k \in K} (q_{i,j}^+ y_{i,k,s}^+ + q_{i,j}^- y_{i,k,s}^-) \end{aligned} \right]$$

- **Step 2:** The Monte Carlo sampling variance estimator is determined using the optimal stochastic profit and flowrates computed in step 1.

$$S(n) = \sqrt{\frac{\sum_{s=1}^n (E_z - z_{i,s})^2}{S-1}} \tag{3}$$

where,

$$z_{i,s} = \sum_{i \in I} (c_{i,s} X_i + \xi_{i,s})$$

- **Step 3:** The lower- and upper-confidence limits of the 95% confidence interval H of $1-\alpha$ are computed as follows:

$$\left[E_z - \frac{z_{\alpha/2} S(n)}{\sqrt{S}}, E_z + \frac{z_{\alpha/2} S(n)}{\sqrt{S}} \right] \tag{4}$$

- **Step 4:** The minimum number of scenarios N that is theoretically required to obtain an optimal solution is determined using the relation below:

$$N = \left[\frac{z_{\alpha/2} S(n)}{H} \right]^2 \tag{5}$$

where, the standard normal random variable $z_{\alpha/2} = 1.96$ at confidence interval $(1-\alpha) = 95\%$.

**MODEL FORMULATION OF STOCHASTIC
REFINERY PLANNING MODEL WITH MEAN-
ABSOLUTE DEVIATION (MAD) AS RISK MEASURE**

The risk metric Mean-Absolute Deviation (MAD) is employed as a measure of deviation from the expected profit (Konno and Yamazaki, 1991). It is defined as follows:

$$MAD(x) = E \left| \sum_{j=1}^n R_j x_j - E \sum_{j=1}^n R_j x_j \right| \tag{6}$$

In this study, the rate of return R in Eq. 6 refers to unit cost of materials (crude oil and refinery products) and the amount of money x_j invested in an asset j refers to the refinery production amount. Therefore, the formulation of the MAD-based risk measure for price uncertainty becomes:

$$MAD(z_0) = \sum_{s \in S} p_s \left| \sum_{i \in I} c_{i,s} X_{i,s} - \sum_{i \in I} \sum_{s \in S} p_s c_{i,s} X_{i,s} \right| \tag{7}$$

while for demands and yields uncertainty, it is given by:

$$MAD_\xi = \sum_{s \in S} p_s \left| \sum_{i \in I} \xi_{i,s} - \sum_{i \in I} \sum_{s \in S} p_s \xi_{i,s} \right| \tag{8}$$

**FORMULATION OF STOCHASTIC REFINERY
PLANNING MODEL WITH CONDITIONAL
VALUE-AT-RISK (CVAR) AS RISK MEASURE**

CvaR, also termed as mean excess loss, mean shortfall or tail VaR, is a risk assessment technique that is originally intended to be employed for reducing the

probability that an investment portfolio will incur high losses. It offers the advantage of a linear programming formulation for determining the optimal solution of financial planning problems that explicitly minimizes loss or risk. CVaR is performed by taking the likelihood (at a specific confidence level, e.g., 0.95 or 0.99) that a specific loss will exceed the metric known as Value-at-Risk (VaR). From a mathematical point of view, CVaR is derived by taking a weighted average between VaR and the losses exceeding VaR. For a discrete probability distribution function, CVaR can be defined as follows (Rockafellar and Uryasev, 2002, 2000):

$$F_\alpha(x, VaR) = CVaR = VaR + \frac{1}{1-\alpha} \sum_{i \in I} \sum_{s \in S} p_s (f(x, y_{i,s}) - VaR) \quad (9)$$

Using CVaR as the risk metric yields the following form of the objective function:

$$\max z = E[z_0] - \theta_1 CVaR_{z_0} - E_{s'} - \theta_3 CVaR_\xi \quad (10)$$

where, $CVaR_{z_0}$ is the risk measure imposed by the recourse costs to handle price uncertainty.

$$CVaR_{z_0} = VaR_1 + \frac{1}{1-\alpha} \sum_s p_s (c_{i,s} x_{i,s} - VaR_1) \quad (11)$$

where, $CvaR_\xi$ is the risk measure imposed by the recourse costs to handle uncertainty in demands and yields.

$$CVaR_\xi = VaR_2 + \frac{1}{1+\alpha} \sum_{i \in I} \sum_{s \in S} p_s \left(\begin{matrix} c_i^+ z_{i,s}^+ \\ + c_i^- z_{i,s}^- \end{matrix} + \sum_{k \in K} \begin{matrix} q_{i,j}^+ y_{i,k,s}^+ \\ + q_{i,j}^- y_{i,k,s}^- \end{matrix} \right) - VaR_2 \quad (12)$$

Substituting Eq. 10 and 11 into 9, we obtain a two-stage stochastic programming model with meanBrisk objective in which the risk measures are assessed by CVaR.

NUMERICAL EXPERIMENTS

We illustrate the risk modeling approach proposed in this paper on the numerical example taken from Khor *et al.* (2008) and provide major details on the implementation using GAMS/CONOPT3.

Solving two-stage stochastic program with MAD as risk measure: The expectation of the objective function value is given by the original objective function itself: The corresponding expression for expected profit is formulated for the 13 scenarios that has been randomly generated.

Solving two-stage stochastic program with CVaR as risk measure: The following is the procedure for developing a loss distribution in order to determine the value for the parameter VaR.

- **Step 1:** The objective value of deterministic profit for each of the 13 scenarios is computed (i.e., multiplication of flowrate and the corresponding price per unit flowrate)
- **Step 2:** The probability of each scenario is randomly generated using Monte Carlo simulation based on pseudorandom number generation
- **Step 3:** The computed values in step 1 are sorted in ascending order
- **Step 4:** The plot of cumulative distribution function against the sorted deterministic profit values is

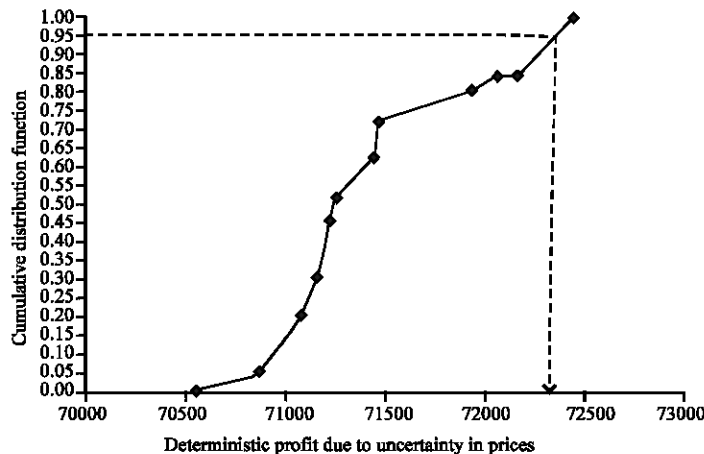


Fig. 1: Loss distribution to determine VaR_1

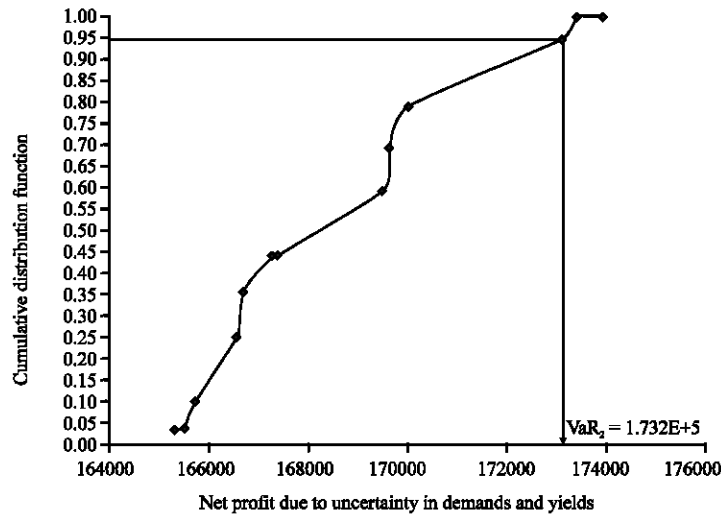


Fig. 2: Loss distribution to determine VaR_2

Table 1: Summary of computational results

Monte Carlo sampling variance estimator $S(n)$	489.4
Lower bound of confidence interval H	965.3
Upper bound of confidence interval H	1237
Range of confidence interval H	271.3
Minimum number of scenarios N	13
Optimal solution for MAD-based model	\$681.95/day
Optimal solution for CVaR-based model	\$20 800.66/day

Table 2: Computational statistics of GAMS implementation for determining optimal solutions of MAD- and CVaR-based meanBrisk stochastic program

Solver	GAMS/CONOPT3
No. of continuous variables	281
No. of single equations	145
CPU time/resource usage	(trivial)
No. of iterations	20 (using MAD)
	101 (using CVaR)

developed to obtain a representation of the loss distribution. At confidence interval of $(1-\alpha) = 0.95$, we can read off the value of VaR from the loss distribution plot, as depicted in Fig. 1 and 2, which represents the penalty for uncertainty in prices and in both demands and yields, respectively. The computational statistics and a summary of the main computational results are provided in Table 1 and 2, respectively

CONCLUSION

In this study, we have proposed a stochastic nonlinear programming model with recourse that incorporates the risk metrics of MAD and CVaR to handle economic and operational risk management in refinery planning problems under uncertainty in prices, demands and yields. The model involves the application of Monte Carlo simulation approach based on the Sample Average Approximation (SAA) method to determine the minimum

number of scenarios required to obtain an optimal solution. Therefore, the main idea of this study is to combine the scenario reduction technique of SAA with the novel application of the computationally-attractive risk metric CvaR to refinery planning problems. The model is applied to a representative refinery planning problem in the literature to illustrate its computational performance.

ACKNOWLEDGMENTS

Author is grateful to Universiti Teknologi PETRONAS for the full support of this study and acknowledges Thi Huynh Nga Nguyen, his undergraduate research assistant, for her contribution in the computational work.

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