Stochastic Programming with Tractable Mean-risk Objectives for Refinery Planning under Uncertainty

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Abstract: The application of Information Technology (IT) and Information Systems (IS) have been crucial in enhancing the operating flexibility and resiliency of refineries. In particular, the Process Systems Engineering (PSE) community has been instrumental in carrying out a key role in extending the systems engineering boundaries from mere chemical process systems to the incorporation of business process systems with consideration for risk. Thus, this study considers a robust framework for the economic and operational risk management of a refinery under uncertainty by extending an existing two-stage stochastic program with fixed recourse via scenario analysis. The problem is mathematically formulated as a two-stage stochastic nonlinear program with a tractable mean-risk structure in the objective function. Two measures of risk are considered, namely the metrics of Mean-Absolute Deviation (MAD) and Conditional Value-at-Risk (CVaR). The scenario analysis approach is adopted to represent uncertainties in three types of stochastic parameters, namely prices of crude oil and commercial products, market demands and production yields. However, a large number of scenarios are required to capture the stochasticity of the problem. Therefore, to circumvent the problem of the resulting large-scale model, we implement a Monte Carlo simulation approach based on the Sample Average Approximation (SAA) technique to generate the scenarios. A statistical-based scenario reduction strategy is applied to determine the minimum number of scenarios required yet still able to compute the true optimal solution for a desired level of accuracy within the specified confidence intervals. The proposed model is illustrated through a representative numerical example, with computational results demonstrating how risk-averse-and risk-inclined solutions in the face of uncertainty can be attained in a risk-conscious model.

Key words: Two-Stage stochastic programming, refinery planning, Mean-Absolute Deviation (MAD), Conditional Value-at-Risk (CVaR)

INTRODUCTION

Stochastic programming has emerged as one of the most prominent operation research models for optimization involving uncertainty. Refinery planning problems are subject to uncertainty in many factors, which primarily includes fluctuations in prices of crude oil and saleable products, market demand for products and production yields. The risk terms in Khor et al. (2008) are handled using the metric mean-absolute deviation. After obtaining the first model with MAD as risk measurement, the second model is developed in which the risk terms are performed by CVaR. A comparison is performed between the two models to assess which of these two risk measures is superior, both computationally and conceptually, in capturing the economic and operating risk in the planning of a refinery. However, there are large numbers of scenarios that create difficulty to handle various circumstances. For example, there may be more than thousands of cases happening. It is hard to predict and control numerous scenarios. Therefore, it is necessary to find the minimum number of scenarios to capture all the circumstances. Monte Carlo simulation approach based on the Sample Average Approximation (SAA) technique is applied in this work to generate the minimum number of scenarios which present for thousands cases. The two-stage stochastic program with risk management as presented by Khor et al. (2008) is formulated as:

$$\max z = E[z_o] - \theta V(z_o) - E_r - \theta W$$  \hspace{1cm} (1)

where, \(E[z_o]\) is the expectation of the original objective function \(z_o\) with random price coefficients; \(\theta\) and \(\theta\) are weights representing the risk factors; \(V(z_o)\) is the sampling variance of \(z_o\); \(E_r\) is the expected recourse penalty and \(W\) is the MAD-based risk measure. In this work, we extend model Eq. 1 by considering MAD as the risk measure in place of variance. Further, we investigate the viability of the risk measure Conditional Value-at-Risk (CVaR), which
has gained wide attention in computational finance, within
the domain of refinery planning. All the uncertain
parameters are assumed to be discrete random
variables.

**MONTE CARLO SIMULATION APPROACH BASED ON SAMPLE AVERAGE APPROXIMATION (SAA) METHOD**

In this study, we adopt the Monte Carlo simulation approach for scenario generation based on the Sample Average Approximation (SAA) method (You et al., 2009; Shapiro, 2000; Shapiro and Homem-de-Mello, 1998). The procedure involved is as follows:

- **Step 1:** A relatively small number of scenarios (for example, 50 scenarios) with their associated probabilities are randomly and independently generated for the uncertain parameters of prices, demands and yields. This is accomplished by employing the Monte Carlo simulation approach based on the SAA technique. (This data is otherwise obtained from plant historical data.) The resulting stochastic model (a linear program) with the objective function given in Eq. 2 is solved to determine the optimal stochastic profit with its corresponding material flowrates:

\[
\text{max } E_x = E_n - E_i
\]

where,

\[
E_n = \sum_{i \in I} \sum_{c \in C} p_{ci} c_i x_i
\]

and

\[
E_i = \sum_{i \in I} \sum_{c \in C} p_{ci} c_i x_i = \sum_{i \in I} \sum_{c \in C} \left[ (c_i z_i^* + c_i z_i^*) + \sum_{k \in K} (q_{ik} x_{ik}^* + q_{ik} y_{ik}^*) \right]
\]

- **Step 2:** The Monte Carlo sampling variance estimator is determined using the optimal stochastic profit and flowrates computed in step 1:

\[
S(n) = \sqrt{\frac{\sum_{k=1}^{K} (E_k - z_{i,k})^2}{S - 1}}
\]

where,

\[
z_{i,k} = \sum_{c \in C} (c_i x_i + z_i^*)
\]

- **Step 3:** The lower- and upper-confidence limits of the 95% confidence interval H of 1-α are computed as follows:

\[
\left[ E_i - \frac{z_{2.5} S(n)}{\sqrt{S}}, E_i + \frac{z_{2.5} S(n)}{\sqrt{S}} \right]
\]

- **Step 4:** The minimum number of scenarios N that is theoretically required to obtain an optimal solution is determined using the relation below:

\[
N = \left[ \frac{z_{2.5} S(n)}{H} \right]^2
\]

where, the standard normal random variable \(z_{2.5}\) = 1.96 at confidence interval (1-α) = 95%.

**MODEL FORMULATION OF STOCHASTIC REFINERY PLANNING MODEL WITH MEAN-ABSOLUTE DEVIATION (MAD) AS RISK MEASURE**

The risk metric Mean-Absolute Deviation (MAD) is employed as a measure of deviation from the expected profit (Konn and Yamazaki, 1991). It is defined as follows:

\[
\text{MAD}(x) = \mathbb{E} \left| \sum_{j \in J} R_j x_j - \mathbb{E} \sum_{j \in J} R_j x_j \right|
\]

In this study, the rate of return \(R\) in Eq. 6 refers to unit cost of materials (crude oil and refinery products) and the amount of money \(x_j\) invested in an asset \(j\) refers to the refinery production amount. Therefore, the formulation of the MAD-based risk measure for price uncertainty becomes:

\[
\text{MAD}(\sigma) = \sum_{c \in C} p_c \left| \sum_{i \in I} s_i x_{i,c} - \sum_{i \in I} \sum_{c \in C} p_{ci} c_i x_{i,c} \right|
\]

while for demands and yields uncertainty, it is given by:

\[
\text{MAD}_t = \sum_{c \in C} p_c \left| \sum_{i \in I} s_i - \sum_{i \in I} \sum_{c \in C} p_{ci} c_i \right|
\]

**FORMULATION OF STOCHASTIC REFINERY PLANNING MODEL WITH CONDITIONAL VALUE-AT-RISK (CVAR) AS RISK MEASURE**

CVar, also termed as mean excess loss, mean shortfall or tail VaR, is a risk assessment technique that is originally intended to be employed for reducing the
probability that an investment portfolio will incur high losses. It offers the advantage of a linear programming formulation for determining the optimal solution of financial planning problems that explicitly minimizes loss or risk. CVaR is performed by taking the likelihood (at a specific confidence level, e.g., 0.95 or 0.99) that a specific loss will exceed the metric known as Value-at-Risk (VaR). From a mathematical point of view, CVaR is derived by taking a weighted average between VaR and the losses exceeding VaR. For a discrete probability distribution function, CVaR can be defined as follows (Rockafellar and Uryasev, 2002, 2000):

\[
F(x, \text{VaR}) = \text{CVaR} = \text{VaR} + \frac{1}{1 - \alpha} \sum_{i=1}^{n} p_i (f(x, y_i) - \text{VaR})
\]  

(9)

Using CVaR as the risk metric yields the following form of the objective function:

\[
\max z = E[z] - \theta_1 \text{CVaR}_z - E_{\epsilon} - \theta_2 \text{CVaR}_\epsilon
\]  

(10)

where, CVaR\_z is the risk measure imposed by the recourse costs to handle price uncertainty.

\[
\text{CVaR}_z = \text{VaR}_z + \frac{1}{1 - \alpha} \sum_{i=1}^{n} p_i (\epsilon_i, x_i, - \text{VaR}_z)
\]  

(11)

where, CVaR\_1 is the risk measure imposed by the recourse costs to handle uncertainty in demands and yields.

\[
\text{CVaR}_\epsilon = \text{VaR}_\epsilon + \frac{1}{1 - \alpha} \sum_{i=1}^{n} p_i \left[ \epsilon_i z_{i,\epsilon} + c_{i,\epsilon} x_{i,\epsilon} + \frac{1}{1 - \alpha} \sum_{j=1}^{m} q_{i,j} y_{i,j} \right]
\]  

(12)

Substituting Eq. 10 and 11 into 9, we obtain a two-stage stochastic programming model with mean-risk objective in which the risk measures are assessed by CVaR.

**NUMERICAL EXPERIMENTS**

We illustrate the risk modeling approach proposed in this paper on the numerical example taken from Khor et al. (2008) and provide major details on the implementation using GAMS/CONOPT3.

**Solving two-stage stochastic program with MAD as risk measure:** The expectation of the objective function value is given by the original objective function itself: The corresponding expression for expected profit is formulated for the 13 scenarios that have been randomly generated.

**Solving two-stage stochastic program with CVaR as risk measure:** The following is the procedure for developing a loss distribution in order to determine the value for the parameter VaR.

- **Step 1:** The objective value of deterministic profit for each of the 13 scenarios is computed (i.e., multiplication of flowrate and the corresponding price per unit flowrate)
- **Step 2:** The probability of each scenario is randomly generated using Monte Carlo simulation based on pseudorandom number generation
- **Step 3:** The computed values in step 1 are sorted in ascending order
- **Step 4:** The plot of cumulative distribution function against the sorted deterministic profit values is

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Fig. 1: Loss distribution to determine VaR.
Fig. 2: Loss distribution to determine VaR$_3$

Table 1: Summary of computational results

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo sampling variance estimator S(0)</td>
<td>488.4</td>
</tr>
<tr>
<td>Lower bound of confidence interval H</td>
<td>965.3</td>
</tr>
<tr>
<td>Upper bound of confidence interval H</td>
<td>1237</td>
</tr>
<tr>
<td>Range of confidence interval H</td>
<td>271.3</td>
</tr>
<tr>
<td>Minimum number of scenarios N</td>
<td>13</td>
</tr>
<tr>
<td>Optimal solution for MAD-based model</td>
<td>$681.95$/day</td>
</tr>
<tr>
<td>Optimal solution for CVaR-based model</td>
<td>$20,800.66$/day</td>
</tr>
</tbody>
</table>

Table 2: Computational statistics of GAMS implementation for determining optimal solutions of MAD- and CVaR-based mean-risk stochastic program

<table>
<thead>
<tr>
<th>Solver</th>
<th>GAMS/CUNOPT3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of continuous variables</td>
<td>281</td>
</tr>
<tr>
<td>No. of single equations</td>
<td>145</td>
</tr>
<tr>
<td>CPU time/resource usage</td>
<td>(trivial)</td>
</tr>
<tr>
<td>No. of iterations</td>
<td>20 (using MAD)</td>
</tr>
<tr>
<td></td>
<td>101 (using CVaR)</td>
</tr>
</tbody>
</table>

devolved to obtain a representation of the loss distribution. At confidence interval of $(1-\alpha) = 0.95$, we can read off the value of VaR$_3$ from the loss distribution plot, as depicted in Fig. 1 and 2, which represents the penalty for uncertainty in prices and in both demands and yields, respectively. The computational statistics and a summary of the main computational results are provided in Table 1 and 2, respectively.

CONCLUSION

In this study, we have proposed a stochastic nonlinear programming model with recourse that incorporates the risk metrics of MAD and CVaR to handle economic and operational risk management in refinery planning problems under uncertainty in prices, demands and yields. The model involves the application of Monte Carlo simulation approach based on the Sample Average Approximation (SAA) method to determine the minimum number of scenarios required to obtain an optimal solution. Therefore, the main idea of this study is to combine the scenario reduction technique of SAA with the novel application of the computationally-attractive risk metric CVaR to refinery planning problems. The model is applied to a representative refinery planning problem in the literature to illustrate its computational performance.

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REFERENCES


