Stretched-exponential Localization Induced by Electric Field in Disordered System with Correlated

A. Yedjou and F. Hamdache
Laboratoire de Physique Des Plasmas Matériaux Conducteurs et Leurs Applications,
Department of Physics, Faculty of Science, Mohamed Boudiaf USTO,
University of Science and Technology of Oran Mohammed Boudiaf USTO,
P.O. Box 1505, El Menour, Oran, Algeria

Abstract: The purpose of this study was to investigate the transport properties of a random dimer model in 1D. A Kronig-Penney model with δ-peak potentials is used to examine how the resonance energy is affected by the electric field. We discussed the influence of an electric field on the nature of the electronic states and compared the result to the case without field. We found that there are important differences, mainly for a large system size L. Localized wave functions have been obtained at particular energies which make transition between extended and stretched-exponential localized states. The most important conclusion so obtained is that the electric field applied to such systems suppresses progressively the effect of the correlation, the transmission coefficient decreases leading to the absence of transport in this kind of electrified chains.

Key words: Disordered system, random potential, electric field, localization, electronic states

INTRODUCTION

There is a growing interest for questions pertaining to wave spread in disordered lattices, which are related to the search of optical or acoustic localization and recently of cold atom localization (Economou and Alkire, 1988; (John and Stephen, 1983; Skiperov et al., 2008). The well-studied case of electronic systems with independent site disorder does not fully cover all cases of such wavelike excitations in complex media. A well-known result of the Anderson model for the site energy is the absence of long-range transport in one dimensional system. All electronic states in one dimension are exponentially localized regardless of the amount of disorder (Anderson, 1958).

Much attention has been paid to special disorder correlations for which new phenomena are expected to appear. For instance, although Anderson localization occurs in one dimension, one finds partial delocalization even for an infinitesimal amount of disorder in the presence of correlations (Datta et al., 1993; Sanchez et al., 1994). A number of recent works dealing with tight-binding Hamiltonian strongly suggest that the occurrence of correlations in neighbour random parameters are not independent with a correlation length (Evengelou, 1990), (Dunlap et al., 1990; Wu and Phillips, 1991). Furthermore, the existence of a mobility edge between extended and localized states was found for 1D random system with weak long-range correlated disorder (Molina, 2005; Esmailpour et al., 2006). Long-range disorder induces the appearance of delocalization and long range transport.

The Random Dimer Model (RDM) can be shown to be an example of the correlated disordered system. In this 1D random model, the site energy takes one out of two possible values, one of which is distributed at random to pairs along the chain, so that the correlation length coincides with lattice spacing. On the basis of this interest the authors claimed that the RDM has $\sqrt{N}$ states which are extended over the whole sample, with N the number of sites in the system. A discrete number of extended states was found numerically (Evengelou and Wang, 1993), (Evengelou and Economou, 1993) and was observed recently in the experiment with semiconductor random superlattices (Bellani et al., 1999). In Kronig Penney model, the electronic field delocalizes the eigenstates where the wave functions decay with a power (Soukoulis et al., 1983; Cota et al., 1985), in this regime, the resistance was checked experimentally. For sufficiently large field strengths, the eigenstates become extended (Markos and Kramer, 1993; Markos and
When the electric field vanishes, it is well-known that the spectrum is then pure and dense), (Abrahams et al., 1979; Landauer, 1970). The wave functions are exponentially localized, with a localization length that decreases with increasing disorder (Mott, 1968). The transmission coefficient has been used successfully to analyze the nature of the electronic states. The effect of exponentially localized eigen-states can be observed in the exponential decreases of the transmission coefficient with the length of the system (Anderson et al., 1980; Thouless, 1974). Moreover, the connection between resistance, or more precisely, conductance and transmission coefficient can be carried out via Landauer formula (Landauer, 1970). One finds:

\[ \ln(1+\rho) = -\frac{\ln T}{L} \]

with \( \rho \) the resistance. Such a behaviour can be expected from the self-averaging of the Lyapunov exponent which is the inverse of the localization length in 1D systems. This characteristic length \( l \) is defined as:

\[ l = -\frac{1}{2L} \ln T \]

This parameter is always positive and describes the spatial scaling properties of a disordered system (Soukoulis and Economou, 1981; Lifshitz et al., 1988).

In this study, we first discussed the delocalization induced by correlations in the Kronig-Penney model. Here, we used an array of \( \delta \)-function potentials with independent random strengths and study numerically the transmission properties for a finite length of the lattice. We derived exact results for the main characteristics of the model using a transfer matrix combined with a Poincaré map approach. Secondly, we examined the size dependence of the transmission coefficient of a linear RDM chain subject to electric field. When \( F = 0 \), the transmission coefficients at particular energy close to 1 and a deep minimum around the resonant energy in the resistance is found, indicating that the localization length of those states is large. For \( F \neq 0 \), we observed that the transmission decreases with increasing \( F \) where \( F \) is the electric strength. However, this minimum disappears and the values of resistance become extremely large. As soon as \( F \) is present, the electron gains the energy \( V(x) = -Fx \). This electrical potential suppresses the resonance energy induced by correlation. That induces a transition between extended and localized behaviour. Finally, we discussed our calculations of the Lyapunov exponent that indicate that all states around the resonance in the presence of the electric field have a localization length smaller than the system size. We expected from this result a mobility edge that depends on the strength of the field in the RDM.

**MODEL**

Here we considered a 1D Kronig-Penney model with random \( \delta \)-function potentials subject to an applied electric field. The problem is defined by the Schrödinger:

\[ \left[ -\frac{d^2}{dx^2} + \sum_{n} \lambda_n \delta(x - n) - Fx \right] \Psi(x) = E \Psi(x) \]  

(1)

where, \( \lambda_n \) is a set of independent random variables that measures the strength of the \( \delta \)-potentials. Here \( E \) is the energy of the electron measured in atomic units and \( \Psi \) is the wave function. We proceed with the problem of the disordered lattice containing a certain number of pair impurities placed randomly. We kept the positions of the \( \delta \)-functions to be regularly spaced \( [x_n = n] \) but we introduced a correlated disorder, for which \( \lambda_n \) takes only two values: \( \lambda \) and \( \lambda' \), where appears only in pairs of neighboring sites (dimer impurities). The electronic potential \( V(x) \) is given by -Fx term in Eq. 1 with F denoting the electric field strength.

In this section, we presented a numerical study of the transmission coefficient of this model. Our approach is inspired by Soukoulis et al. (1983), Flores et al. (1989) who investigated the transmission coefficient and the nature of the electronic states in 1D disordered systems. They found that the transmission coefficient behaves as, with \( f = 1/F \). This reveals power-law localization.

Here, we calculated the transmission coefficient \( T \) the above model (RDM) using the transfer matrix approach. We took an electron impinging from the left of a set of \( \delta \)-function potentials with wave function \( \Psi(x) = e^{i\alpha x} - e^{-i\alpha x} \). The energy of the electron is \( E = q_0^2 \) with \( q_0 \) the momentum of the incident electron. The wave function in the right-hand side of the sample of length \( L \) is \( \Psi(r) = T_{\alpha} e^{i\alpha} \). Here \( q_0 = \sqrt{E^2 + FL} \) with \( L = N + 2 \); \( \alpha \) denotes the momentum of the emerging wave. \( t_{\alpha} \) and \( r_{\alpha} \) are the transmission and the reflection amplitudes of the RDM with \( N \) scatterers respectively between two impurities, we will replace \( V(x) \) by a constant value so that the solution between two impurities are plane wave functions (Soukoulis et al., 1983; Cota et al., 1985).

However, this is valid only when the electric potential between the ends of a sample is infinitesimally small.
The solution of Eq. 1 can be computed recursively for both transmission and reflection amplitude using well-known transfer-matrix techniques (Kirilov and Trott, 1994). Then, the transmission amplitude can be written as:

$$A_n = \left( \alpha_n + \frac{\alpha_{n+1}}{\beta_{n+1}} \right) A_{n+1} - \left( \frac{\beta_n}{\beta_{n+1}} \right) A_n$$ \hspace{1cm} (2)

where $A_n = t_n$ and:

$$\alpha_n = \left[ 1 - i \left( \frac{1}{2q_n} \right) \lambda_n \right] \beta_n - i \left( \frac{1}{2q_n} \right) \lambda_n e^{-\lambda_n}$$ \hspace{1cm} (3)

Equation 2 supplied two boundary conditions, $A_0 = 1$ and $A_1 = 1$ to determine the amplitudes completely. $q_n$ is the momentum of the electron at the site $n$. Finally, the transmission coefficient can be calculated for each chain from:

$$T = \frac{q_n}{q_1} \left| t_n \right|^2$$ \hspace{1cm} (4)

**RESULTS AND DISCUSSION**

We first discussed our numerical results on the transmission coefficient for a RDM and investigated what changes occur when the electric field is applied along the linear chain.

We choose for convenience the length $L = 1000$ and a dimer concentration equal to 20%. We fixed $\lambda = 1$ for the values of potential strength of the host lattice and $\lambda' = 1.5$ for the dimer impurities.

Present results are similar to the ones obtained in by Sanchez et al. (1994), Dunlap et al. (1990), where a unique energy was found in the allowed band (recall their model is a single band) and where a perfect transmission $T = 1$ was seen in the RDM. In such case, the system of electronic transport becomes ballistic. Thus, nondecreasing transmission coefficient for particular energy shows the existence of extended states arrows this one.

In Fig. 1, we showed the transmission coefficient versus energy for intervals near the first resonance. The spectrum of the Kronig Penney model follows the equation $|2q \cos q + \lambda \sin q| \leq 1$ (this is the condition to be able to move in the perfect lattice) when $\lambda$ is fixed. Here, we have averaged the transmission coefficient for 1000 realizations with an accuracy of 1%. We found that around the first resonance $E_r = 3.75$ the transmission coefficient reaches values very close to 1. All realizations show the same peak around $E_r$. It is clear from Fig. 1 that the states close to the resonant energy have good transmission properties, similar to those of the resonant energy.

When $F \neq 0$, there are some important differences with respect to the case $F = 0$. For the same concentration of dimer impurities, we observe that the transmission coefficient decreases for a field as small as $5 \times 10^{-4}$. In this case, the small $F$ will only slightly shift the resonant energy and the transmission coefficient will completely vanish.

We showed in Fig. 2, the resistance of a RDM in both the presence and the absence of an electric field with the same concentration of impurities. The lower curve, corresponding to a dimer model with $F = 0$, exhibits a minimum resistance about ten orders of magnitude below the resistance for $10^{-4}$ (the middle curve). For the $F$ considered in Fig. 2, the curve saturates to essentially a constant value with energy. However, the resistance becomes extremely big compared to the same RDM.

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**Fig. 1:** Plot of resistance versus energy in RDM, with $L = 10^4$ for different values of the electric field

**Fig. 2:** Plot of resistance versus energy in RDM, with $L = 10^4$ for different values of the electric field
without electric field. Such as the localization scenario is quite different in the presence of an electric field.

The dependence of the resistance with system size is useful to study the spatial structure of the electronic states. Exponentially localized states lead to a nonohmic behaviour of the resistance, which increases exponentially with the system size.

In Fig. 3, around the resonance, the resistance has a constant value which indicates that the band of state exist with very good transport for a dimer model without field. In this case, the effect of the correlation in the random dimer potential is dominant, the electron gains more kinetic energy behaving essentially as a free particle in a potential well.

Not only the resonant energy has a low resistance for any length of the chain (lower curve), but also when F is far from zero F = 10, the plot shows a relatively small resistance, exhibits a good behaviour (middle curve). For F = 5.10^{-4} and for large L the resistance rises quickly to large values.

To investigate the nature of electronic states around the resonance, we have analyzed the average scaling of In T with the system size.

In Fig. 4, we showed the results for (In T) versus L for a fixed value of the energy and for different values of field. First, as was done for the resistance, we compared our results to the size dependence of transmission coefficient when the electric field is present. We saw that for F = 0 the curve is flat and (In T) reaches a constant value. We concluded that the states are extended. These extended states are not of the Block-type encountered in periodic solids (Hilke and Flores, 1997; Xiuqing and Xintian, 1997).

When F = 0 on the other hand, we observed three things. For small L<2000 we obtained similar behaviour as for F = 0 However, for increasing F, the value of <In T> changes considerably for relatively small changes of F which suggests exponential decreasing for transmission coefficient, with an exponent that depends on F. For L>700, the electronic states are stretched exponential localized. This means that this phase has a zero measure in the thermodynamic limit. For F = 5.10^{-4} this phase, will diverge for large L. Here, the system will be return to equilibrium.

We investigated the Lyapunov coefficient which represents the inverse of the localization length l. As is shown in Fig. 5, when F = 0, energies close to the resonant energy E_r have γ×10^{-4}. This is in agreement with the notion that delocalization of the electronic states occurs l_r→10^4.

When we increased F, we observed an increase of the Lyapunov exponent that stays much smaller than one. This effect coincides with the standard definition of γ (Lifshitz et al., 1988). The localization can be explained by
Fig. 4: Plot of $\langle \ln T \rangle$ versus length $L$ at the resonant energy $E$, for different values of $F$. $F = 0$ (dark), $F = 10^{-4}$ (red) and $F = 5.10^{-4}$ (blue).

Fig. 5: Plot of Lyapunov exponent versus energy with $L = 10^5$ and for different values of strength of field $F = 0$ (dark), $F = 10^{-4}$ (red) and $F = 5.10^{-4}$ (blue).

The fact that when the electric field strength is increased, the effective potential component as $FN-E$, indicating that the states decay as exponential-law. We concluded that the delocalization-localization can be observed in dimer systems such as the electric field suppresses progressively the effect of the correlation.

**CONCLUSION**

We have studied the effect of electric field on a linear chain with correlated disorder. To analyze the properties of electronic transport, we have used the Kronig-Penny model. Based on the results, we have noted that the electric field impedes the movement of the electrons in the presence of correlation. For relatively small field, we notice that the transmission is stretched exponential-law decaying with the length. This decaying depends on the strength of the electric field.

The electric field has an effect on the resonance energy that carries with it a variation of transmission coefficient which influences the nature of the electronic states. The Lyapunov exponent was also used to analyze the localization length, we have found out that when the electronic field increases, the Lyapunov exponent is saturated by a constant value that is lower then the system’s size, which indicates a localization of the electronic states.

**REFERENCES**


