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Closed-Loop System Identification using OBF-ARMAX Model

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Abstract: Closed-loop identification scheme using OBF-ARMAX model structure is presented. The proposed structure can be used to identify both open-loop stable and open-loop unstable processes that are stabilized by a feedback controller. The algorithm for estimating the model parameters and the formula for the multi-step ahead prediction are derived. The proposed identification scheme is demonstrated using two simulation case studies: One for open-loop stable and another for open-loop unstable. Both case studies demonstrate that the proposed scheme can be effectively used for closed-loop identification of both open-loop stable and open-loop unstable systems that are stabilized by a feedback control system.

Key words: Orthonormal basis filters, closed-loop identification, ARMAX

INTRODUCTION

The stringent environmental and safety requirements and the growing competition in the global market have put a tremendous challenge on process industries. On the other hand, the rapid development in computer and software technology, the advancement of instrumentation and data acquisition facilities and the sustained achievements in the formulation of efficient computational algorithms have brought incredible opportunities. The meeting of these strong challenges and resourceful opportunities has led to the birth of several model based technologies, like model based control systems, online process optimizations and fault detection and diagnosis, to mention a few. At the centre of all these technologies is the mathematical model of the system.

Models are extensively used in advanced process control design and implementations. Nearly all optimal control design techniques rely on the use of model of the system to be controlled. In model predictive control (MPC), models are used to predict the future values of the output which is used in calculating the optimal control move. The process of developing models from experimental data is known as system identification. Currently, there is a significant interest towards orthonormal basis filter (OBF) models because of their advantage both during the development and implementation stages of the models. They are parsimonious in their parameters, the parameters can be easily calculated using linear least square method, they are consistent in parameters and time delays can be easily estimated and incorporated into the model (Van den Hof *et al*., 2005; Heuberger *et al*., 1995; Ninness and Gustafsson, 1997; Patwardhan and Shah, 2005; Nelles, 2001).

Control relevant system identification can be carried out using input-output data either from open-loop or closed-loop tests. When a system identification test is carried out in open loop, in most cases, the noise sequence is not correlated to the input sequence and OBF model identification is carried in a straight forward manner. However, when the system identification test is carried out from closed loop data the input sequence is correlated to the noise sequence (Nelles, 2001; Ljung, 1999; Gilson and van den Hof, 2005). In such cases, if conventional OBF model development technique (Nelles, 2001) is used, the resulting model will not be consistent in parameters (Nelles, 2001; Ljung, 1999; Gilson and van den Hof, 2005; Gaspar *et al*., 1999; Ananth and Chidambaram, 1999). In addition, conventional OBF structures do not include explicit noise model and any unmeasured disturbance cannot be modeled. There are several reasons to prefer to conduct the identification tests in closed-loop. Some of the most compelling reasons are (Nelles, 2001; Ljung, 1999):

- Feedback controller is required to stabilize the process
- Safety and cost consideration may not allow the process to run open-loop
- The model is to be used for the design of improved controller

The Orthonormal Basis Filter plus Autoregressive Moving Average with Exogenous Input (OBF-ARMAX)

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structure proposed in this study leads to consistent deterministic model with explicit noise model.

ORTHONORMAL BASIS FILTER MODELS

The basis of OBF models is the orthonormal filters. There are several OBF filters allowing one or more of the system poles to be introduced into the model. The most commonly used filters are Laguerre, Kautz and Generalized Orthonormal Basis Filters (GOBF). They are defined as follows:

Laguerre:

$$f_i = \sqrt{(1-p^2)}\frac{(1-pq)^{i-1}}{(q-p)^i},\ |p|<1 \quad (1)$$

Kautz:

$$f_{2i-1} = \frac{\sqrt{(1-a^2)(1-b^2)}}{q^2+a(b-1)q-b}g(a,b,q,i) \quad (2a)$$

$$f_{2i} = \frac{\sqrt{(1-b^2)}(q-a)}{q^2+a(b-1)q-b}g(a,b,q,i) \quad (2b)$$

Where:

$$g(a,b,q,i) = \left(\frac{-bq^2+a(b-1)q+1}{q^2+a(b-1)-b}\right)^{i-1} \quad (2c)$$

$-1 < a < 1$ and $-1 < b < 1$ $n = 1, 2, \ldots$

GOBF:

$$f_i(q,p) = \frac{\sqrt{1-|p_i|^2}}{(q-p_i)}\prod_{j=1}^{i-1}\frac{(1-p_j^*q)}{(q-p_j)} \quad (3)$$

where, $p \equiv \{p_j : j = 1, 2, 3, \ldots\}$ is an arbitrary sequence of poles inside the unit circle appearing in complex conjugate pairs.

Lemma *et al*. (2010) showed that a Box Jenkins (BJ) type structure with the OBF deterministic component and an Autoregressive (AR) or Autoregressive Moving Average (ARMA) component can be easily developed. They demonstrated that both weakly-damped and well-damped systems with unmeasured disturbances can be easily modeled using the proposed model structure. The OBF-AR and the OBF-ARMA structures proposed by Lemma *et al*. (2010) are given by Eq. 4 and 5, respectively.

$$y(k) = G_{OBF}(q)u(k) + \frac{1}{D(q)}e(k) \quad (4)$$

$$y(k) = G_{OBF}(q)u(k) + \frac{C(q)}{D(q)}e(k) \quad (5)$$

In AIChE conference, Lemma *et al*. (2009) showed that OBF based process models with explicit noise model can be successfully identified from closed-loop identification data. Two structures were proposed; one for indirect identification and another for direct identification. The indirect identification was based on decorrelation of the input and noise sequences. The structure used in the indirect identification was OBF-ARMA. The direct identification method was based on the fact that the OBF-ARX structure assumes the input and noise sequences are correlated and leads to model that is consistent in parameters. The OBF-ARX structure is given in Eq. 6.

$$y(k) = \frac{G_{OBF}(q)}{A(q)} + \frac{1}{A(q)}e(k) \quad (6)$$

In this study, OBF-ARMAX structure that can be used for direct closed-loop identification of both open-loop stable and open-loop unstable processes that are stabilized by feedback controllers is proposed. It provides more flexible noise model than the OBF-ARX model.

Present work: The proposed OBF-ARMAX structure has a common denominator dynamics for the input and noise model and is represented by Eq. 7. Figure 1 shows the block diagram representation of the OBF-ARMAX structure. As it is pointed out by Ljung (1999), such a model results in a stable predictor even though the system is unstable. This renders the proposed structure the ability to capture the dynamics of both open-stable and open-unstable systems that are stabilized by a feedback controller. The OBF-ARMAX structure confers more flexible noise model

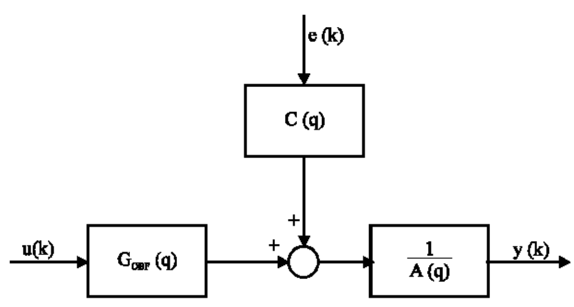

Fig. 1: Block-diagram of a OBF-ARMAX model

than the OBF-ARX structure. The algorithm for estimating the model parameters and the multi-step ahead prediction are formulated in the following parts.

The OBF-ARMAX structure is given by:

$$y(k) = \frac{G_{OBF}(q)}{A(q)}u(k) + \frac{C(q)}{A(q)}e(k) \tag{7}$$

Model parameters estimation:
Rearranging Eq. 7,

$$\hat{y}(k|k-1) = G_{OBF}(q) - (1 - A(q))y(k) + (C(q) - 1)e(k) \tag{8}$$

With A(q) and C(q) monic, expanding Eq. 8,

$$\begin{aligned}\hat{y}(k|k-1) = {} & l_1 u_{f1}(k) + l_2 u_{f2}(k) + \ldots + l_m u_{fm}(k) + \\ & - a_1 y(k-1) - a_2 y(k-2) - \ldots - a_n y(k-n) + \\ & c_1 e(k-1) + c_2 e(k-2) + \ldots + c_n e(k-n)\end{aligned} \tag{9}$$

From Eq. 9 the regressor matrix is formulated for orders m, n, p:

$$X = \begin{bmatrix} u_{f1}(mx) & u_{f2}(mx-1) \ldots u_{fm}(mx-m) & -y(mx-1) & -y(nx-2) \ldots -y(mx-n) & -e(mx-1) & -e(mx-2) \ldots -e(mx-p) \\ . & . \quad . & . & . \quad . & . & . \quad . \\ . & . \quad . & . & . \quad . & . & . \quad . \\ . & . \quad . & . & . \quad . & . & . \quad . \\ u_{f1}(N) & u_{f2}(N-1) \ldots u_{fm}(N-m) & -y(N-1) & -y(N-2) \ldots -y(N-m) & -e(N-1) & -e(N-2) \ldots -e(N-p) \end{bmatrix} \tag{10}$$

Where:
m : Order of the OBF model
n : Order of the A(q)
p : Order of C(q)
mx : Max (n, m, p) + 1
u_{fi} : Input u filtered by the corresponding OBF filter f_i
e(i) : The prediction error

To develop an OBF-ARMAX model, first an OBF-ARX model with high A(q) order is developed. The prediction error e (k) is estimated from this OBF-ARX model and used to form the regressor matrix (Eq. 10). OBF-ARX model development is discussed in detail in (Lemma *et al*., 2009). The parameters of the OBF-ARMAX model are, then, estimated using the regressor matrix (Eq. 10) in the least square Eq. 11. The prediction error and consequently the OBF-ARMAX parameters can be improved by estimating the parameters of the OBF-ARMAX model iteratively.

$$l = (X^T X)^{-1} X^T y \tag{11}$$

Multi−step ahead prediction formula: Consider the OBF-ARMAX model Eq. 12:

$$y(k) = \frac{y_{obf}(k)}{A(q)} + \frac{C(q)}{A(q)}e(k) \tag{12}$$

The i-step ahead prediction is obtained by replacing k with k + i:

$$y(k+i) = \frac{y_{obf}(k+i)}{A(q)} + \frac{C(q)}{A(q)}e(k+i) \tag{13}$$

To calculate the *i*-step ahead prediction, the error term should be divided into current and future parts:

$$y(k+i) = \frac{y_{obf}(k+i)}{A(q)} + \frac{F_i(q)}{A(q)}e(k) + E_i(q)e(k+i) \tag{14}$$

Since e(k) is assumed to be a white noise with mean zero, the mean of $E_i(q)$ e(k+i) is equal to zero and therefore Eq. 14 can be simplified to:

$$\hat{y}(k+i|k) = \frac{y_{obf}(k+i)}{A(q)} + \frac{F_i(q)}{A(q)}e(k) \tag{15}$$

Rearranging Eq. 15:

$$y(k+i) = \frac{y_{obf}(k+i)}{A(q)} + e(k+i)\left(\frac{q^{-i}F_i(q)}{A(q)} + E_i(q)\right) \tag{16}$$

Comparing Eq. 13 to 16, F_i and E_i can be calculated by solving the Diophantine equation:

$$\frac{C(q)}{A(q)} = E_i(q) + \frac{q^{-i}F_i(q)}{A(q)} \tag{17}$$

Rearranging Eq. 12:

$$\frac{1}{A(q)}e(k) = \frac{1}{C(q)}\left(y(k) - \frac{y_{obf}(k)}{A(q)}\right) \quad (18)$$

Using Eq. 18 in 15 to eliminate e(k):

$$\begin{aligned}\hat{y}(k+i\,|\,k) &= \frac{y_{obf}(k+i)}{A(q)} + \frac{F_i(q)}{C(q)}\left(y(k) - \frac{y_{obf}(k)}{A(q)}\right) \\ &= y_{obf}(k+i)\left(\frac{1}{A(q)} - \frac{F_i(q)}{C(q)}\frac{1}{A(q)}\right) + \frac{F_i(q)}{C(q)}y(k)\end{aligned} \quad (19)$$

Rearranging the Diophantine Eq. 17:

$$\frac{E_i(q)}{C(q)} = \frac{1}{A(q)} - \frac{q^{-i}F_i(q)}{C(q)A(q)} \quad (20)$$

Finally using Eq. 20 in 19, the usable form of the i-step ahead prediction formula, Eq. 21 is obtained:

$$\hat{y}(k+i\,|\,k) = \frac{E_i(q)}{C(q)}y_{obf}(k+i) + \frac{F_i(q)}{C(q)}y(k) \quad (21)$$

When OBF-ARMAX model is used for modeling open-loop unstable processes that are stabilized by a feedback controller, the common denominator A(q) that contains the unstable pole does not appear in the predictor Eq. 21. Therefore, the predictor is stable regardless of the presence of unstable poles in the OBF-ARMAX model, as long as the noise model is invertible. Inevitability is required because C(q) appears in the denominator.

CASE STUDIES

Closed loop identification of open-loop stable systems using OBF-ARMAX model: In this closed-loop identification simulation case study, an OBF-ARMAX model of an open-loop stable system is identified from closed-loop test data using the direct identification approach. The deterministic and stochastic components of the system are given by Eq. 22a and b, respectively:

$$G(s) = \frac{0.25e^{-5s}}{(12s+1)(5s+1)} \quad (22a)$$

$$H(z) = \frac{1-0.6z^{-1}}{1-1.342z^{-1}+0.421z^{-2}} \quad (22b)$$

A feedback proportional controller, with K_c=1.0 is used to control the system. The controller gain is chosen so that the closed loop response is stable and gives not more than 25% overshoot. The block diagram of the feedback controlled system is shown in Fig. 2. A white noise sequence with mean -0.0070 and standard deviation 0.0993 is introduced into the system. The Signal to Noise Ratio (SNR) is 6.7350. An external excitation signal, r_1, is used for the purpose of identification.

The excitation signal, r_1, is a 'PRBS' signal generated using the MATLAB function 'idinput' with band (0 0.02) and level (2 -2). Four thousand data points are generated and 3000 of these data points are used for identification while the remaining 1000 data points are used for validation. The changes in the external excitation signal, r_1, system input, u(k) and system output, y(k), are shown in Fig. 3.

OBF-ARMAX model

Pole and number of parameters selection: The dominant pole method to develop parsimonious OBF models is not applicable for OBF-ARMAX structure since the two structures are different. However, the best pole and number of OBF parameters can be estimated by comparing the Percentage Prediction Error (PPE) of various poles and number of OBF parameters. The results of the comparison are presented in Table 1. From Table 1 it is observed that the minimum PPE for OBF-4 (the most parsimonious among the tested) is 6.7929 while the smallest PPE in all the tabulated values is 6.7440 for OBF-7. The difference

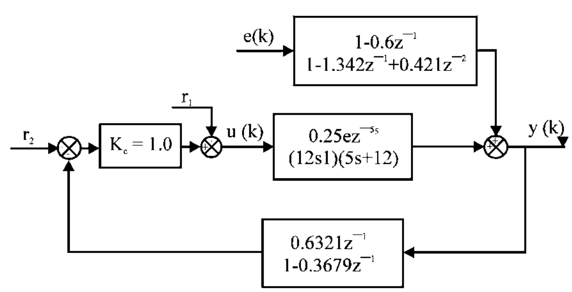

Fig. 2: Block diagram of the closed loop system

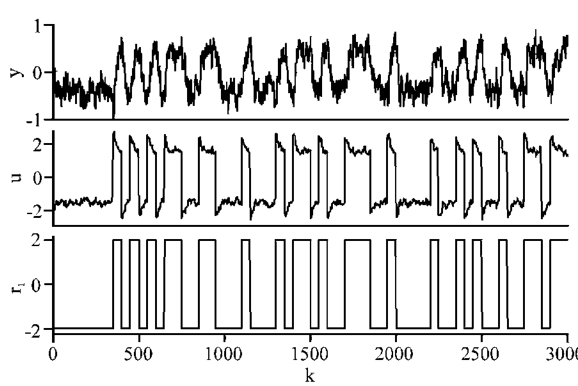

Fig. 3: Data used for system identification of system Eq. 22

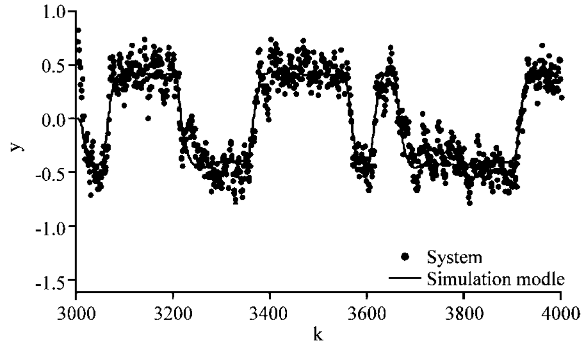

Fig. 4: Output of the simulation model compared to the output of system (Eq. 22) for the validation data points

Table 1: PPE for various poles and number of OBF parameters for $n_D = n_C = 2$ of system (Eq. 22)

Pole	OBF-4	OBF-5	OBF-6	OBF-7	OBF-8
0.7	8.2334	8.6993	6.9257	6.8113	6.7927
0.8	6.9117	6.7680	6.7455	6.7440	6.8056
0.9	6.7929	6.8001	6.7599	6.8777	6.8925
0.91	6.7978	6.8122	6.7551	6.7898	6.8858
0.92	6.8123	6.8303	6.7702	6.7527	6.7920

between the two percentage prediction errors is less than 0.05% which is insignificant and OBF-4 is selected.

The OBF parameters for 4 Laguerre filters and pole of 0.90 is:

$$l = (0.0068\ 0.0079\ -0.0010\ 2.5302e\text{-}004)$$

The denominator polynomial A(q):

$$A(q) = 1 - 0.6401q^{-1} - 0.1268q^{-2}$$

The noise model is:

$$\hat{H}(q) = \frac{1 + 0.1196q^{-1} - 0.0446q^{-2}}{1 - 0.6401q^{-1} - 0.1268q^{-2}} \quad (23)$$

Model validation: The output of the deterministic component the OBF-ARMAX model compared to the system's output for the validation data points is shown in Fig. 4. The PPE of the simulation model compared to the systems output is 16.7642. The spectrum of the noise model (Eq. 22b) compared to the spectrum of the system's noise transfer function (Eq. 23) is shown in Fig. 5. The standard deviation of the residuals of the OBF-ARMAX model is 0.0992. The PPE of the spectrum of the noise model compared to the system's noise transfer function is 2.3659.

The one step-ahead-prediction using the OBF-ARMAX model compared to the system's output for the

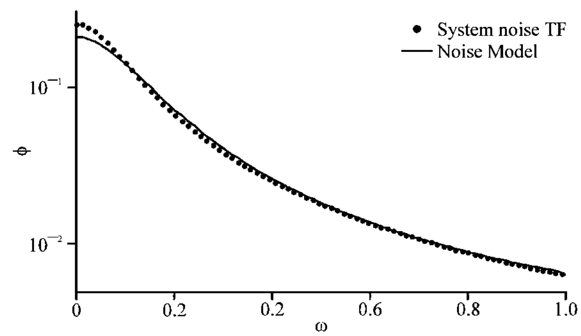

Fig. 5: Spectrum of the noise model compared to the noise transfer function of system (Eq. 22)

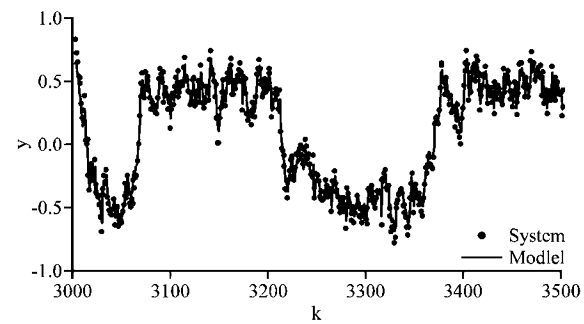

Fig. 6: One-step-ahead prediction of the OBF-ARMAX model compared to the system output for the validation data points for the system (Eq. 22)

validation data points (3001-3500) is shown in Fig. 6 and the PPE is 6.7929. In this case, only 500 points are presented to show the closeness of the system output and the predicted data more clearly.

Residual analysis: The qq-plot and of the residuals with respect to the white noise added to the system is shown in Fig. 7. It is observed from the figure that almost all the points on the qq-plot lie on a straight line with slope equal to one. This shows that the residuals have nearly the same distribution as the white noise. Figure 8 shows the distribution of the residuals compared to the white noise. It is noted from the figure that the two distributions are nearly the same.

The correlation among the residuals for $\tau = 10$:

$$\hat{R} = 10^{-3}[0.2161\ 0.2646\ -0.2554\ 0.0676\ -0.1891\ 0.5927\ -0.6389\ -0.1289\ 0.6668\ 0.1040]$$

The correlation among the residuals which is close to zero also shows that there is no significant correlation among the residuals and the residuals can be considered

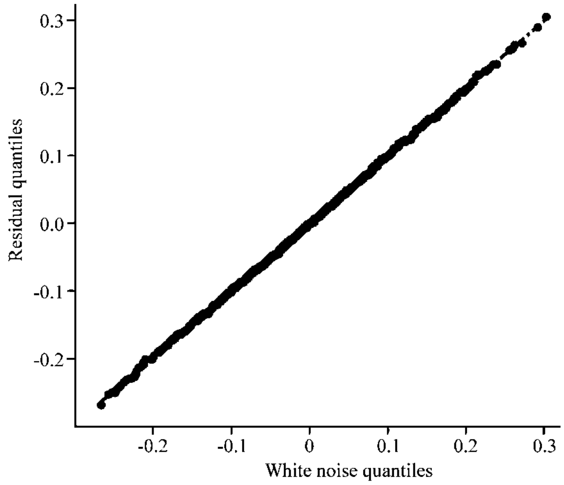

Fig. 7: qq-plot of the residual compared to the white noise for system (Eq. 22)

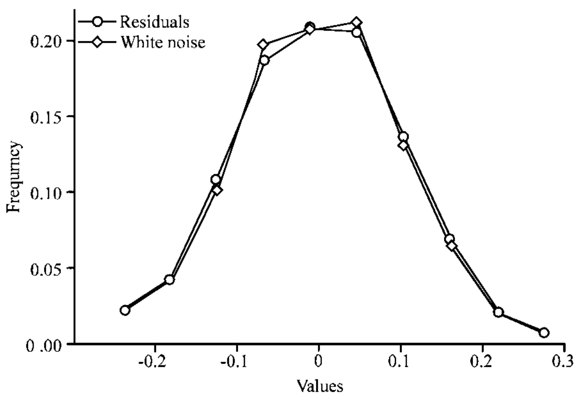

Fig. 8: Distribution of the residuals compared to the white noise for the system (Eq. 22)

white noise. This confirms that the OBF-ARMAX model has captured the model with good accuracy.

Close-loop identification of open-loop unstable process: In this case study, an OBF model with ARMAX structure is used to identify an open-loop unstable process which is stabilized by a feedback control system. The plant and noise transfer functions of the system are given by Eq. 24a and b:

$$G(s) = \frac{0.12e^{-1.2s}}{(15s-1)(7s+1)} \tag{24a}$$

$$H(z) = \frac{1-0.6z^{-1}}{1-1.342z^{-1}+0.421z^{-2}} \tag{24b}$$

The plant transfer function has one RHS pole, 1/15, therefore is open-loop unstable. The system is stabilized

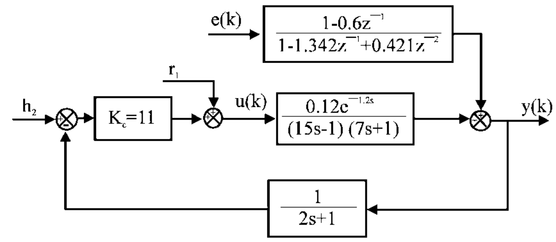

Fig. 9: System stabilized by feedback control system

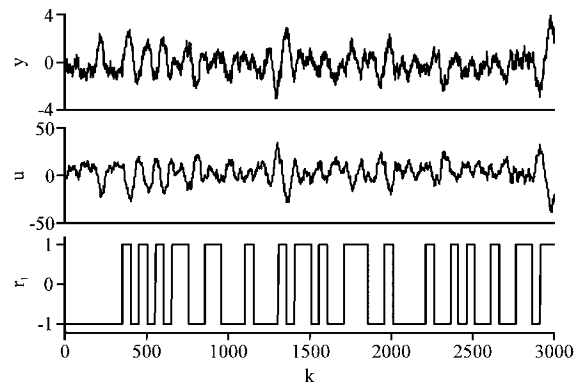

Fig. 10: Data sequences used for identification of system (Eq. 24)

using a proportional feedback controller with $K_c = 11$. A white noise sequence with mean 0.0049 and standard deviation 0.1989 is added to the system and the SNR is 9.9702. An external excitation signal, r_1, is introduced into the system to conduct the identification. The excitation signal is, a PRBS signal generated using the MATLAB function 'idinput' with band (0 0.02) and level of (-1 1). The block diagram of the feedback control system to be identified is shown in Fig. 9.

The excitation signal, the plant input and the plant output used for identification are shown in Fig. 10. Four thousand data points, with a sampling interval of one time unit, are generated using SIMULINK and 3000 of them were used for identification and the remaining 1000 are used for validation.

Pole and number of parameters selection: The same procedure as the previous case is used to determine the number of OBF-parameters and the OBF-pole. OBF model with four numbers of parameters and pole equal to 0.7 is chosen to develop the OBF-ARMAX model (Table 2).

OBF-ARMAX model: The OBF model has four Laguerre filters with pole 0.4 and parameters:

$$l = (0.0028\ -0.0014\ 2.9540e\text{-}004\ 9.5944e\text{-}004)$$

Table 2: PPE for various poles and number of OBF parameters for $n_D = n_C = 2$ for system (Eq. 24)

	Number of OBF parameters			
Pole	4	5	6	7
0.3	6.2929	6.3150	6.2252	6.2007
0.4	6.2835	6.2590	6.1969	6.1892
0.5	6.2624	6.1718	6.2410	6.2448
0.6	6.2154	6.3477	6.3457	6.4538
0.7	6.1931	6.4100	6.3964	6.4847
0.8	6.2722	6.2559	6.2801	6.3174
0.9	6.4841	6.3211	6.3245	6.3227

The denominator polynomial A(q):

$$A(q) = 1 - 1.3696q^{-1} - 0.3289q^{-2}$$

The noise model is:

$$\hat{H}(q) = \frac{1 - 0.4728q^{-1} - 0.0683q^{-2}}{1 - 1.4963q^{-1} - 0.4503q^{-2}} \quad (25)$$

The discrete poles of the noise model that are also shared by the plant model are 1.0590 and 0.3106. It is observed that one of the poles, 1.0590, is outside the unit circle hence it is the unstable pole shared by the plant model and the noise model as the theory requires. Note that the poles of the noise model are shared by the plant model as defined by (Eq. 7).

Model validation: The one-step-ahead prediction by the OBF-ARMAX model compared to the output of the stabilized system for the validation data points is shown in Fig. 9 and the corresponding PPE is 6.1931.

The simulation model and the noise spectrum are irrelevant for such cases, because both are unstable. The accuracy of the model can be checked by residual analysis, as in the case of the OBF-ARX.

Residual analysis: The mean and standard deviation of the residuals of the OBF-ARMAX model are -0.0026 and 0.2160. The qq-plot of the residual of the OBF-ARMAX model with respect to the white noise added to the system for the validation data points are shown in Fig. 10. The distribution of the residuals of the OBF-ARMAX model compared to the white noise added to the system is shown in Fig. 11. It is noted from the figures that, the residual is a normally distributed signal with mean around zero, similar to the white noise. However, it can also be observed that there is small deviation at the intercept qq-plot. This is due to a small increase in the standard deviation

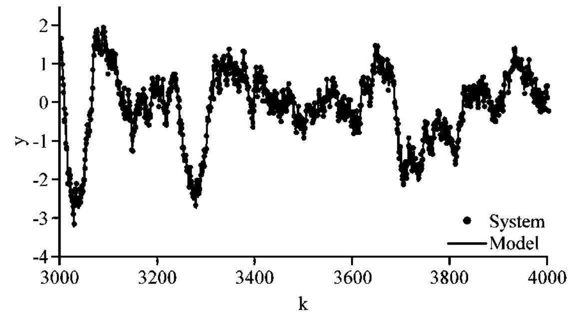

Fig. 11: One-step-ahead prediction of the OBF-ARMAX model compared to the output of the system for the validation data points

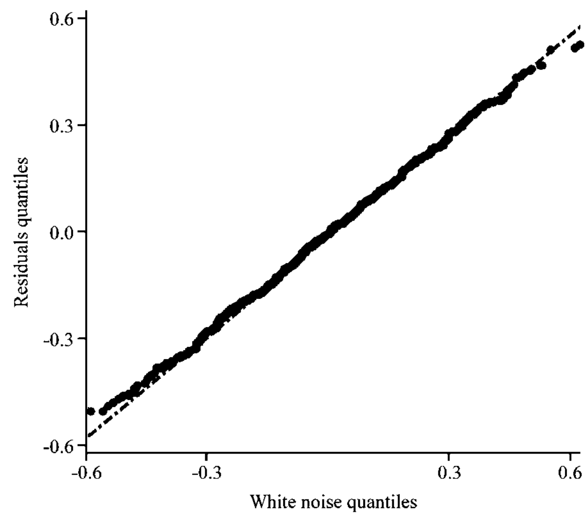

Fig. 12: The qq-plot of the residual of the OBF-ARMAX model with respect to the white noise added into system (Eq. 24)

of the residual as compared to the white noise as can be seen in Fig. 11.

The qq-plant and distribution of the residual of the OBF-ARMAX model are shown in Fig. 12 and 13.

The correlation among the residuals is given by:

$$\hat{R} = (0.0079\ 0.0063\ 0.0042\ 0.0037\ 0.0024\ 0.0057\ -0.0007\ 0.0009\ 0.0047\ 0.0025)$$

In this case study also both the prediction of the validation data and the residual analysis confirm that the OBF-ARMAX model is successfully used to identify open-loop unstable systems that are stabilized by a feedback controller.

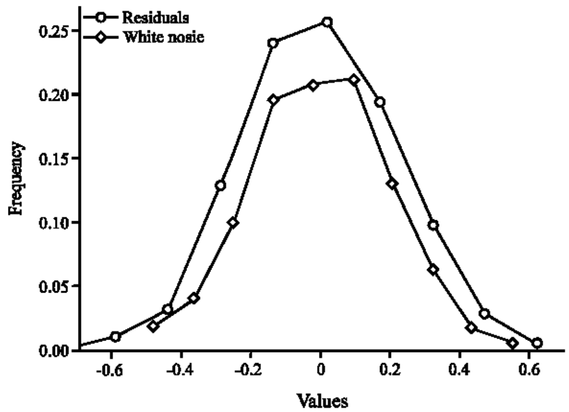

Fig. 13: Distribution of the residual of the OBF-ARMAX model compared to the white noise added into system (Eq. 24)

CONCLUSION

A closed-loop identification scheme based on orthonormal basis filters with OBF-ARMAX structure is presented. The procedure for estimating the model parameters is and the multi-step ahead prediction are derived. It is demonstrated by simulation case studies that the proposed scheme is effective for closed-loop identification of both open-loop stable and open-loop unstable systems that are stabilized by a feedback control system.

REFERENCES

Ananth, I. and M. Chidambaram, 1999. Closed-loop identification of transfer function model for unstable systems. J. Franklin Institute, 336: 1055-1061.

Gaspar, P., Z. Szabo and J. Bokor, 1999. Closed-loop identification using generalized orthonormal basis functions. The 38m ThMO6 14:OO Conference on Decision and Control Phoenix, Arizona USA., December 1999.

Gilson, M. and P.M.J. van den Hof, 2005. Instrumental variable methods for closed-loop system identification. Automatica, 41: 241-249.

Heuberger, P.S.C., P.M.J. Van de Hof and O.H. Bosgra, 1995. A generalized orthonormal basis for linear dynamical systems. IEEE Tran. Automatic Control, 40: 451-465.

Lemma, D.T., M. Ramasamy and M. Shuhaimi, 2009. Closed loop identification using Orthonormal Basis Filter (OBF) and noise models. AIChE Annual Meeting, Nov. 2009, Nasville, USA.

Lemma, D.T., M. Ramasamy, S.C. Patwardhan and M. Shuhaimi, 2010. Development of Box-Jenkins type time series models by combining conventional and orthonormal basis filter approaches. J. Process Control, 20: 108-120.

Ljung, L., 1999. System Identification: Theory for the User. 2nd Edn., Prentice Hall PTR, London, ISBN-10: 0136566952, pp: 672.

Nelles, O., 2001. Nonlinear System Identification. Springerlink, New York.

Ninness, B.M. and F. Gustafsson, 1997. A unifying construction of orthonormal basis for system identification. IEEE Tran. Automatic Control, 42: 515-521.

Patwardhan, S.C. and S.L. Shah, 2005. From data to diagnosis and control using generalized orthonormal basis filters, Part I: Development of state observers. J. Process Control, 15: 819-835.

Van den Hof, P.M.J., P.S.C. Heuberger and B. Wahlberg, 2005. Modeling and Identification with Rational Orthogonal Basis Functions. 1st Edn., Springer, London, ISBN-10: 185233956X, PP: 397.