Sensorless Control of Induction Machines using a Reduced Order Extended Kalman Filter for Rotor Time Constant and Flux Estimation

O. Asseu, Z. Yeo, M. Koffi, M.A. Kouacou and K.E. Ali
Department of Electrical and Electronic Engineering,
Institute National Polytechnique Houphouet Boigny, BP 1093 Yamoussoukro, Côte d'Ivoire

Abstract: This study proposes a novel method to achieve good performance for rotor time constant and flux estimation in induction motor sensorless control, using a reduced order Extended Kalman Filter (EKF) instead of a full-order EKF. This new algorithm uses a reduced order state-space model that is discretized in a particular and innovative way proposed in this study. With this model structure, only the rotor flux components are estimated while the full order EKF also estimates stator current components. Thus, as compared with the full order EKF, this new approach strongly reduces the execution time of the observation and simplifies the tuning of covariance matrices, since, the number of elements to be adjusted is reduced. The satisfying simulations results on Matlab-Simulink environment for a 1.8 kW induction motor, demonstrate the good performance and stability of the proposed reduced order EKF algorithm against parameter variation, modeling uncertainty, measurement and system noises.

Key words: Induction motor, non linear control, reduced order Extended Kalman Filter (EKF)

INTRODUCTION

Sensorless control methods have been making remarkable developments in the most recent year, due to lower cost and greater reliability without mounting problems (Mohanty et al., 2002; Menina et al., 2008). Flux estimation methods are being used that avoid the flux measurement set-up and commercial sensorless vector-controlled drives are already available.

This study presents an innovative strategy to the problem of non-linear estimation of states for Induction Motors (IM) sensorless control.

The naturally structured of non-linear multivariable state of IM models induces the use of robust feedback linearization strategy in order to permit a decoupling and good dynamic stability of the IM variables in a field-oriented (d, q) coordinate so that rotor flux and speed can be separately and independently controlled (Asseu et al., 2008; Mohanty and Patra, 2005; Yazdani-Rahman et al., 2008).

However, this feedback control strategy requires the knowledge of rotor fluxes which are not usually measurable in practice. Also, a variation of the rotor time constant in the IM can induce a lack of field orientation and a state-space coupling, which can involve a degradation of the system. Thus, In order to achieve better dynamic performance, an estimation of rotor time constant and flux is necessary.

An approach, proposed by Hilal et al. (2009), Murat et al. (2007) and Shi et al. (2002) to estimate with success the state variables in an IM in the presence of modeling uncertainty, measurement and system noises (stochastic estimation), is the use of the full-order EKF. This latter provides not only the unmeasurable state variable estimation (fluxes) but also the estimation of the measurable parameters (currents and speed). However, the determination of the measurable parameters estimation imposes some estimation algorithms very long and usually sophisticated. Therefore, in order to reduce the computation rate of the estimation algorithms, the measured parameters estimation is not necessary.

In this study, in order to compare with the full order EKF and respect to the rotor time constant variations in the presence of measurement and system noises, a new approach using a reduced order EKF (REFK) is presented to solve only and specially the problem of the unmeasurable parameters estimation (rotor time constant and flux). Consequently, practical and important improvements are achieved with respect to the well known drawbacks associated to the EKF, like the computational effort for real-time applications, the complexity and the hard tuning of the covariance matrices. In fact, with the
3rd order EKF that is obtained, the dimension of all matrices of the algorithm becomes small enough from a practical point of view.

After a brief review of the IM model, the simulation results for a 1.8 kW induction motor drive system are presented to validate the high robustness of the proposed REKF approach against parameter variations, measurement and system noises.

MATERIALS AND METHODS

Induction motor model: This research project, conducted in the Laboratory of Applied Electrical and Electronic (INPHB Yamoussoukro, Côte d'Ivoire) from May 2009 to December 2009 by a theoretical work, has been confirmed by simulations results on an induction machine.

By assuming that the saturation of the magnetic parts and the hysteresis phenomenon are neglected, the classical dynamic model of the induction motor in a $(d, q)$ synchronous reference frame can be described by De Formel and Louis (2007):

\[
\begin{align*}
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix} &= 
\begin{bmatrix}
R_d I_d + \frac{d}{dt}\Phi_d - \omega_s \Phi_q \\
R_q I_q + \frac{d}{dt}\Phi_q + \omega_s \Phi_d
\end{bmatrix}, \\
\Phi_d &= \frac{V_d}{L_d} + \sigma L_s I_q, \\
\Phi_q &= \frac{V_q}{L_q} + \sigma L_s I_d
\end{align*}
\]

(1a)

\[
\begin{bmatrix}
\Phi_d \\
\Phi_q
\end{bmatrix} = 
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix}
\left[
\begin{bmatrix}
L_d & \frac{L}{L_s} \\
\frac{L}{L_s} & L_q
\end{bmatrix}
\right]
\quad \text{and} \quad
\begin{bmatrix}
\Phi_d \\
\Phi_q
\end{bmatrix} = 
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix}
\left[
\begin{bmatrix}
L_d & \frac{L}{L_s} \\
\frac{L}{L_s} & L_q
\end{bmatrix}
\right]
\]

(1b)

The load mechanical equation is:

\[
J \frac{d\omega_r}{dt} + f_\omega = C_m - C_r, \quad \text{where} \quad C_m = p \frac{L}{L_s} (\Phi_d I_q - \Phi_q I_d)
\]

(1c)

The application of (1a-c) returns a system of fifth-order non-linear differential equation, as state variables the stator currents ($I_{ds}, I_{qs}$), the rotor fluxes ($\Phi_{ds}, \Phi_{qs}$) and the speed ($\omega_r$). Assume that among the state variable, only the stator currents and the speed are measurable.

Thus, the IM model can be rewritten as:

\[
x = f(x_r) + g_r u \quad \text{with} \quad x = [I_{ds} I_{qs} \quad d_1 \quad q_1 \quad \omega_r \quad I_{qm} \quad V_{ds} V_{qs}]
\]

(2)

\[
f_r(x_r) = 
\begin{bmatrix}
-\lambda_1 I_q + \omega_1 I_d - \sigma_1 \Phi_d + \beta_1 \Phi_q \\
-\sigma_1 I_d - \omega_1 I_q - \beta_1 \Phi_d + \lambda_1 \Phi_q \\
\sigma_1 I_d + \omega_1 I_q - \sigma_1 \Phi_d - \omega_1 \Phi_q \\
\sigma_1 I_q + \omega_1 I_d - \sigma_1 \Phi_d - \omega_1 \Phi_q \\
\frac{1}{L_s} \left( I_{qm} \Phi_d - \Phi_q I_d \right) - \frac{p}{J} \omega_r - \frac{f}{J}
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
1 \\
\frac{1}{\sigma_1} \\
0 \\
0 \\
0
\end{bmatrix}
\]

(3)

By choosing a rotating reference frame $(d_x, q_x)$ so that the direction of axis $d$ is always coincident with the direction of the rotor flux representative vector, it is well known that this rotor field orientation in a rotating synchronous reference frame realizes:

\[
\Phi_d = \Phi_r = \text{Constant and } \Phi_q = 0
\]

(3)

From the expressions Eq. 1c, 2 and 3, one can write:

\[
\begin{bmatrix}
\frac{d\omega_r}{dt} \\
\frac{d\Phi_d}{dt} \\
\frac{d\Phi_q}{dt}
\end{bmatrix} = 
\begin{bmatrix}
0 \\
\frac{1}{L_s} \left( I_{qm} \Phi_d - \Phi_q I_d \right) - \frac{p}{J} \omega_r - \frac{f}{J}
\end{bmatrix}
\quad \text{with} \quad I_{qm} = \frac{\Phi_q}{L_s}
\]

(4)

This relation (Eq. 4) shows that the IM dynamic model can be represented as a non-linear function of the rotor time constant. A variation of this parameter can induce, for the IM, a lack of field orientation, performance and stability. Thus, to preserve the reliability and robustness stability under the rotor time constant variation, a robust input-output linearization via feedback control, proposed by Asseu et al. (2009) is used to provide a good regulation and convergence of the rotor flux ($\Phi_r$) and speed ($\omega_r$) for the IM drive. However, since, the resolution of the feedback control for the IM requires the knowledge of the rotor flux value that is not measurable, an on-line estimation of rotor fluxes is necessary.

Thus, in order to take into account the rotor time constant variations, measurement and system noises (stochastic estimation) and then reduce the execution time of the estimation algorithm, this study uses a reduced-order EKF method to provide only the estimation of rotor fluxes and rotor time constant (considering that the currents and speed are already measured).

Reduced-order extended kalman filter model: Let us consider the dynamic model of the IM given by the system Eq. 2. The currents ($I_{ds}, I_{qs}$) and speed ($\omega_r$) estimation is not necessary since, they are measurable. Thus, in order to estimate only the rotor flux ($\Phi_{ds}, \Phi_{qs}$) and rotor time constant ($\omega_r = R/L_s$), a reduced dimensional state vector extended to rotor time constant defined by $x = [\Phi_{ds} \Phi_{qs} \omega_r]^T$ has been introduced. The corresponding three-dimensional extended state space equation obtained is:

\[
x(0) = J(x(0), v(0))
\]

where, $v(i) = [I_{ds}, i_{qm}]^T$ is the new input vector.
where, $\epsilon$ presents the slow variation of $\sigma$. The fact that the state vector only consists of the rotor time constant and flux offers an advantage namely the reduction of the computational volume and complexity. Thus, the rotor time constant and flux can be more easily and rapidly estimated.

For parameter estimation using a REKF, the model structure given by Eq. 5 is directly discretized by means of Euler's approximation (2nd order) proposed in (Lewis, 1992). Thus, the new discrete-time and stochastic nonlinear reduced order model is given by:

\[
\begin{align*}
\mathbf{x}(k+1) &= J(\mathbf{x}(k), v(k)) + n(k) \\
&= \mathbf{x}(k) + \sum_{i=1}^{2} J_1(\mathbf{x}(k), v(k)) + \sum_{i=1}^{2} J_2(\mathbf{x}(k), v(k)) + n(k) \\
y(k) &= h(\mathbf{x}(k)) + r(k)
\end{align*}
\]

with,
\[
\begin{align*}
\mathbf{h}_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\
\mathbf{M}(k) &= \begin{bmatrix} -1 & -1 & \sigma(k) \\ -1 & 1 & 0 \end{bmatrix}
\end{align*}
\]

From the electrical equations Eq. 1a of the IM and the expression Eq. 7, an approximate discrete-time relation (1st order) of the fluxes is given by:

\[
\begin{align*}
\mathbf{J}_1(\mathbf{x}(k), v(k)) &= \begin{bmatrix} (\mathbf{V}_a(k) - \mathbf{E}_a) + \mathbf{T}_a \mathbf{x}(k) + \sigma_x(k) \mathbf{J}_a(k) \\ (\mathbf{V}_b(k) - \mathbf{E}_b) + \mathbf{T}_b \mathbf{x}(k) + \sigma_x(k) \mathbf{J}_a(k) \\ (\mathbf{V}_c(k) - \mathbf{E}_c) + \mathbf{T}_c \mathbf{x}(k) + \sigma_x(k) \mathbf{J}_a(k) \end{bmatrix} \\
\mathbf{J}_2(\mathbf{x}(k), v(k)) &= \begin{bmatrix} 0 \\ \sigma_x(k) \mathbf{J}_a(k) \\ \sigma_x(k) \mathbf{J}_a(k) \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\mathbf{J}(\mathbf{x}(k), v(k)) &= \begin{bmatrix} \mathbf{J}_1(\mathbf{x}(k), v(k)) \\ \mathbf{J}_2(\mathbf{x}(k), v(k)) \end{bmatrix} \\
n(k) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} n(k)
\end{align*}
\]

The expression Eq. 8 is composed of recurrence relations (between the rotor fluxes at the k+1-th instant and their value at the k-th instant) which can be obtained by means of the measurements of the voltage, stator currents and speed, as shown in Eq. 9.

Thus, for the output vector given by: $y(k+1) = x(k+1) + r(k)$, the state vector is estimated in order to minimize the prediction error: $e(k+1) = y(k+1) - \hat{y}(k+1)$ where, $\hat{y}(k+1) = h(\hat{x}(k+1))$ with:

\[
\hat{x}(k+1) = \begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{J}_a(k+1) - \mathbf{J}_a(k) - \mathbf{T}_s \omega(k) \mathbf{J}_a(k) \\ \mathbf{J}_a(k+1) - \mathbf{J}_a(k) + \mathbf{T}_e \omega(k) \mathbf{J}_a(k) \end{bmatrix}
\]

As said in Eq. 6, $r(k)$ represents the noises (white Gaussian noise) present on the currents, voltage or speed transducers, or the errors of discretization and measurement.

Finally, the proposed REKF algorithm, which is a general form is presented in the appendix, applied to the nonlinear model described by Eq. 6 can be defined as follows:

\[
\begin{align*}
[\mathbf{f}(k+1) + \mathbf{L}(k) \mathbf{h}(k)] &= \mathbf{f}(k) + \mathbf{L}(k) \mathbf{h}(k) \\
[\mathbf{g}(k+1) + \mathbf{G}(k)] &= \mathbf{g}(k) + \mathbf{G}(k) \\
[\mathbf{T}(k+1) + \mathbf{P}(k+1)] &= \mathbf{T}(k) + \mathbf{P}(k) \\
[\mathbf{R}(k+1) + \mathbf{R}(k)] &= \mathbf{R}(k) + \mathbf{R}(k)
\end{align*}
\]

where, the prediction vector is:

\[
\hat{x}(k+1) = J(\hat{x}(k), v(k)) = \hat{x}(k) + \mathbf{T}_1 J_1(\hat{x}(k), v(k)) + \sum_{i=1}^{2} J_2(\hat{x}(k), v(k))
\]

with $\hat{x}(k+1) = \begin{bmatrix} \Phi_p(k+1) \\ \Phi_q(k+1) \end{bmatrix}$ and $\hat{\sigma}(k+1)$

$L(k)$ is the Jacobian matrix of partial derivatives of $J(*)$ with respect to $\hat{x}(k)$. From Eq. 12, $L(k)$ is given by:

\[
L(k) = \frac{\partial \mathbf{L}(k)}{\partial \hat{x}(k)}
\]
Thus, once the fluxes and rotor time constant are estimated from the Eq.1c, we can deduce the estimated torque given by:

\[
\hat{C}_{\text{est}}(k) = p_1 L_r \left[ \Phi_\alpha(k) L, (k) - \Phi_\beta(k), I, (k) \right]
\]  

(14)

RESULTS

The proposed REKF algorithm, controlled by a robust feedback linearization strategy (Fig. 1), has been investigated with simulation tests carried out for a 1.8 kW IM by means of SIMULINK in order to illustrate its effectiveness against measurement noise and parameter variation. The nominal electrical parameters of the IM, estimated by means of the identification techniques proposed by Leite et al. (2003) and De Fornel and Louis (2007) are shown in the Table 1.

Thus, the REKF algorithm is implanted in a S-function using C language. In order to evaluate its performances, the comparisons between the observed state variables and the simulated ones have been realized for several operating conditions in the presence of about 15% white noise on the measured currents (Fig. 2) and with a load torque (C_t = 1 N.m).

Thus, the simulations are obtained at first in the nominal case with the nominal parameters of the IM (Table 1) used to realize vector control orientation and the feedback linearization and then in the second case, with 50% of the nominal rotor time constant (\(\alpha_r = 1.5\alpha_{\text{r-nom}}\)) in order to verify the behavior of the proposed REKF algorithms with respect to rotor time constant variation.

Initialization and tuning of the reduced order EKF algorithm: The reduced order EKF is initialized as follows: \(P_{35}(0) = \operatorname{diag}[p_1^2, p_2^2, \ldots, p_5^2, p_6^2, \ldots, p_{10}^2]\) with \(p_1 = 10^4, p_2 = 1\) and \(x(0) = [0 \ 0 \ 0]^T\) with \(\sigma_n = R_n / L_n\).

The system covariance matrix can be adjusted by: \(Q_{35} = \operatorname{diag}[q_1, q_2, \ldots, q_{10}]\) with \(q_1 = 10^2, q_2 = 1\) and the measurement noise covariance matrix has been fixed as follows: \(R_{35} = \operatorname{diag}[r = 10]\).

The three positive gains \(p, q, r\) must be adequately tuned in order to have a good performance, convergence and considerable rapidity of the reduced order EKF. Our proposed Feedback control and REKF operate at 1 m sec sampling period using Euler approximation.

Simulation results: Figure 3 and 4 show the simulation results for a step variation of the rotor flux and speed \(\Phi_{\text{ref}}\) and \(\omega_{\text{ref}}\). One can see that in both nominal (Fig. 3) and non-nominal cases (Fig. 4), the estimated values of fluxes, torque and rotor time constant converge very well to their simulated values. The observed fluxes (Fig. 3a, 4a) indicate the good orientation \(\Phi_\beta\) is constant and \(\Phi_\alpha\) converges to zero) due to a favorable rotor time constant estimation (Fig. 3b, 4b). Also, we can see an absence or a rejection of noises on the fluxes.

Furthermore these results show the good uncoupling between the flux \(\Phi_\alpha\) and the speed \(\omega_\alpha\) because a step

![Fig. 1: Simulation scheme](image-url)

<table>
<thead>
<tr>
<th>Table 1: Nominal parameters of the Induction motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{\text{nom}}) = 1.8 kW</td>
</tr>
<tr>
<td>(f_n = 50) Hz</td>
</tr>
<tr>
<td>(R_n = 5.7) Ω</td>
</tr>
<tr>
<td>(I_n = 0.0504) H</td>
</tr>
</tbody>
</table>

\(I_r = 20.8 / 12\) A | \(p = 2\) |
| \(J = 0.15\) kg m\(^2\) | \(f_r = 0.05\) Nm rad sec\(^{-1}\) |
| \(L_m = 0.1786\) H | \(L_m = 0.1262\) H |
Fig. 2: Presence of white noises on the measured current $I_a$

Fig. 3: (a-d) Nominal case ($R_e = R_a$): Comparison between estimated and simulated values

Fig. 4: (a-d) Non nominal case ($R_e = 1.5R_a$): Comparison between estimated and simulated values

variation in $\omega_v$ ($50\pi$ to $35\pi$ rad sec$^{-1}$), in order to generate a torque change, can not influence on the flux response that remains acceptable (the field orientation is well maintained).

These waveforms illustrate the fast convergence and high performance of the robust feedback decoupling control and REKF algorithm against rotor time constant variations and measurement noises.

CONCLUSION

In this study, a robust feedback linearization strategy and a REKF algorithm are used not only to decouple and
then control independently the rotor flux and the speed (or the generated torque) of the IM in a field-oriented (d, q) coordinate but also to provide the unmeasurable state variable estimation (flux, rotor time constant and torque). A series of simulations tests have been achieved on the induction motor. The results obtained have demonstrated a good performance of this robust decoupling control and REKF algorithm against rotor time constant variations, measured noise and load torque. The main conclusion and contribution of this work is that the well-known drawbacks of the full order EKF, like heavy computational effort for real-time applications, complexity and hard tuning of noise covariance matrices are widely overcome using the proposed reduced order EKF. In fact, the execution time of the REKF algorithm is about half of the full order Extended KF (Asseu et al., 2010). Thus, in the industrial applications, one will appreciate very well the experimental implement of this robust estimator for the reconstitution of the fluxes and the torque as well as the rotor resistance.

APPENDIX

Steps of the EKF algorithm (Blanchard et al., 2007):

Setp 1. $\hat{x}(k+1) = W(k+1)$

Setp 2. $L(k) = \frac{\partial x(k)}{\partial x(k)} \bigg|_{x(k) = \hat{x}(k)}$

Setp 3. $H(k) = \frac{\partial x(k)}{\partial x(k)} \bigg|_{x(k) = \hat{x}(k)}$

Setp 4. $\hat{p}(k+1) = L(k) \hat{p}(k) L(k)^T + Q$

Setp 5. $\hat{h}(k+1) = \hat{h}(k+1) + H(k) \hat{p}(k+1) H(k)^T + R$

Setp 6. $x(k+1) = \hat{x}(k+1) - H(k) \hat{p}(k+1)$

Setp 7. $\hat{h}(k+1) = \hat{h}(k+1) + G(k+1) r(k+1)$

Setp 8. $\hat{p}(k+1) = [I - G(k+1) H(k)] \hat{p}(k+1)$

Setp 9. Increment k and Go to step 1

The EKF algorithm consists of repeated use of step (1-9) for each measurement.

NOMENCLATURES

<table>
<thead>
<tr>
<th>$C_{ar}$, $C_i$</th>
<th>Electromagnetic and load torques (N.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{d}$, $I_{q}$</td>
<td>Stationary frame (d, q)-axis stator currents (A)</td>
</tr>
<tr>
<td>$I_{dr}$, $I_{q}$, $I_{aur}$</td>
<td>Stationary frame (d, q)-axis rotor currents and rotor magnetizing current (A)</td>
</tr>
</tbody>
</table>

REFERENCES


