Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 10 (7): 551-557, 2010
ISSN 1812-5654

Effects of Variable Inflationary Conditions on an Inventory Model with Inflation-Proportional Demand Rate

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Abstract: A time varying inventory model for deteriorating items with allowable shortages is developed in this study. The inflation rates (internal and external) are time-dependent and demand rate is inflation-proportional. The inventory level is described by differential equations over the time horizon and present value method is used. The numerical example is given to explain the results. Some particular cases, which follow the main problem, will discuss and the results will compare with the main model by using the numerical examples. It has been achieved which shortages increases considerably in comparison with the case of without variable inflationary conditions.

Key words: Inventory, time-dependent inflation, inflation-dependent demand, deterioration, shortages

INTRODUCTION

The problem of inventory systems under inflationary conditions has received attention in recent years. Usually, the inflation rates have been assumed constant over the planning horizon. But, many economic factors may affect on the future changes in the inflation rate; such as changes in the world inflation rate, rate of investment, demand level, labor cost, cost of raw materials, rates of exchange, rate of unemployment, productivity level, tax, liquidity, etc. The current study considers time-dependent inflation rates for deterioration items with shortage.

Inventoried goods can be broadly classified into four meta-categories:

- **Obsolescence:** Refers to items that lose their value through time due to rapid changes of technology or the introduction of a new product by a competitor
- **Deterioration:** Refers to the damage, spoilage, dryness, vaporization, etc. of the products
- **Amelioration:** Refers to items whose value or utility or quantity increase with time
- No obsolescence/deterioration/amelioration

If the rate of obsolescence, deterioration or amelioration is not sufficiently low, its impact on modeling of such an inventory system cannot be ignored. There are a few studies for obsolescing and ameliorating items. Moon *et al*. (2005) considered the ameliorating/deteriorating items on an inventory model with time-varying demand pattern. Against obsolescing and ameliorating items, the many researches has founded in the deteriorating inventory area in recent years. Products such as vegetables, fish, medicine, blood, gasoline and radioactive chemicals have finite shelf life and start to deteriorate once they are produced. The deteriorating inventory models under inflationary conditions are studied greatly. In a few of these works, deterioration rate is not constant. For instance, Balkhi (2004a) presented a production lot size inventory model that the production, demand and deterioration rates are known, continuous and differentiable functions of time. Shortages are allowed, but only a fraction of the stock out is backordered and the rest is lost. Lo *et al*. (2007) developed an integrated production-inventory model with assumptions of varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. The most of the inventory systems for deteriorating items are considered a constant deterioration rate which will state in continuance.

Some researches in inflationary inventory systems assumed time-varying demand rate. Yang *et al*. (2001) extended the inventory lot-size models to allow for inflation and fluctuating demand, which is more general than constant, increasing, decreasing and log-concave demand patterns. Other works is performed by Balkhi (2004a).

Several researchers have considered finite replenishment rate for inflationary inventory systems. Wee and Law (1999) derived a deteriorating inventory model under inflationary conditions for determining economic production lot size when the demand rate is a linear decreasing function of the selling price. Balkhi (2004b) proposed two flexible production lot size inventory models for deteriorating items in which the production rate at any instant depends on the demand and the on-hand inventory level at that instant. Another research is performed by Lo *et al.* (2007).

The stock-dependent demand rate models are prepared with some researchers. Hou and Lin (2004) developed an inventory model under inflation and time discounting for deteriorating items with stock-dependent selling rate. The selling rate is assumed to be a function of the current inventory level and the rate of deterioration is assumed to be constant. Liao and Chen (2003) surveyed a retailer's inventory control system for the optimal delay in payment time for initial stock-dependent consumption rate when a wholesaler permits delay in payment. The effect of inflation rate, deterioration rate, initial stock-dependent consumption rate and a wholesaler's permissible delay in payment is discussed. A deterministic Economic Order Quantity (EOQ) inventory model taking into account inflation and time value of money developed for deteriorating items with price-and stock-dependent selling rates by Hou and Lin (2006). An efficient solution procedure is presented to determine the optimal number of replenishment, the cycle time and selling price. Hou (2006) prepared an inventory model for deteriorating items with stock-dependent consumption rate. Maiti *et al.* (2006) proposed an inventory model with stock-dependent demand rate and two storage facilities under inflation and time value of money where the planning horizon is stochastic in nature and follows exponential distribution with a known mean. An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages proposed by Yang *et al.* (2009).

Other efforts in inventory systems under inflationary conditions are performed under the assumption of the permissible delay in payments. Chang (2004) proposed an EOQ model for deteriorating items under inflation when the supplier offers a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. Shah (2006) derived an inventory model by assuming constant rate of deterioration of units in an inventory, time value of money under the conditions of permissible delay in payments. Other models are prepared by Liao and Chen (2003).

The inflationary inventory models with two warehouses are proposed previously. Yang (2004) discussed the two-warehouse inventory problem for deteriorating items with a constant demand rate and shortages. Yang (2006) extended the models introduced in Yang (2004) to incorporate partial backlogging and then compare the 2 two-warehouse inventory models based on the minimum cost approach.

The mentioned studies have considered a constant and well-known inflation rate over the time horizon. Horowitz (2000) discussed a simple EOQ model with a Normal distribution for the inflation rate and the firm's cost of capital. He showed the importance of taking into account the inflation rate and time discounting, especially when the former is relatively high or when there is considerable uncertainty as to either the inflation rate or the marginal cost of capital. Mirzazadeh *et al.* (2009) considered stochastic inflationary conditions with variable probability density functions (pdfs) over the time horizon and the demand rate is dependent to the inflation rates (any arbitrary pdfs can be used). The developed model, also, implicates to finite replenishment rate, finite time horizon, deteriorating items with shortages. The objective is minimization of the expected present value of costs over the time horizon. The numerical example and case study have been provided for evaluation and validation of the theoretical results and some special cases of the model are discussed.

The existing inventory models under inflation and time value of money are numerous. Most of the previous researches consider a constant inflation rate over time and a few models consider stochastic inflationary conditions. Furthermore, in some practical situations, the inflation rate can be well approximated by continuous-time-dependent function. Therefore, this study assumes the inflation is time-dependent. Also, the demand rate is assumed to be inflation-proportional. A numerical example is shown for explaining the theoretical results. Then, the particular cases, which follow the main problem, are discussed corresponding to the situation of:

Case (I) : Constant inflation rates
Case (II) : Identical rate of inflation
Case (III): Without shortages
Case (IV): No deterioration
Case (V) : Identical inflation rate, no shortages and no deterioration
Case (VI): Constant and identical inflation rate, no shortages and no deterioration

THE ASSUMPTION, NOTATIONS AND DESCRIPTION OF THE MODEL

First, the assumptions and notations are explained and then the proposed inventory system is described. The mathematical models in this study are developed based on the following assumptions:

- A constant fraction of the on-hand inventory deteriorates per unit time, as soon as the item is received into inventory

- The internal and external inflation rates are time-varying over the time horizon
- Shortages are allowed and fully backlogged, except for the final cycle
- The replenishment is instantaneous and the replenishment cycle is the same for each period
- The initial inventory level is zero
- The system brings about for prescribed time-horizon of length H

The following notations are used:

i_m = The internal (for m = 1) and the external (for m = 2) inflation rates which are linear time-dependent:

$$i_m = a_m + b_m t \tag{1}$$

where, a_m and b_m are real number. If $b_m<0$ the inflation rate will decrease and if $b_m>0$ the inflation rate will increase over time. If $b_m=0$ the inflation rate is stable that will be discussed in the special cases

r = The interest rate
R_m = The discounted rate net of inflation: $R_m = r-i_m$
$D(i_1,i_2)$ = The demand rate per unit time is a function of inflation rates

$$D(i_1,i_2) = a_0 + b_0 i_1 + c_0 i_2, a_0 > 0 \tag{2}$$

where, a_0, b_0 and c_0 are fixed real number

θ = The constant deterioration rate per unit time ($0<\theta<1$)
p = The purchase cost at time zero
S = The ordering cost per order at time zero
c_{1m} = The internal (for m=1) and external (for m=2) inventory carrying cost per unit per unit time at time zero
c_{2m} = The internal (for m=1) and external (for m=2) shortages cost per unit per unit time at time zero
H = The finite time horizon
T = The interval of time between replenishment
k = The proportion of time in any given inventory cycle which orders can be filled from the existing stock
n = The number of replenishments during time horizon
TVC (n,k)= The total present value of costs over the time horizon

Other notations will be introduced later. The demand rate is dependent to inflation rates and the inflation rates are time-proportional. Therefore, the demand rate can be explained as follow:

$$D(i_1,i_2) = a_0 + b_0 i_1 + c_0 i_2 = (a_0 + b_0 a_1 + c_0 a_2) + (b_0 b_1 + c_0 b_2)t = a + bt, \qquad a > 0 \tag{3}$$

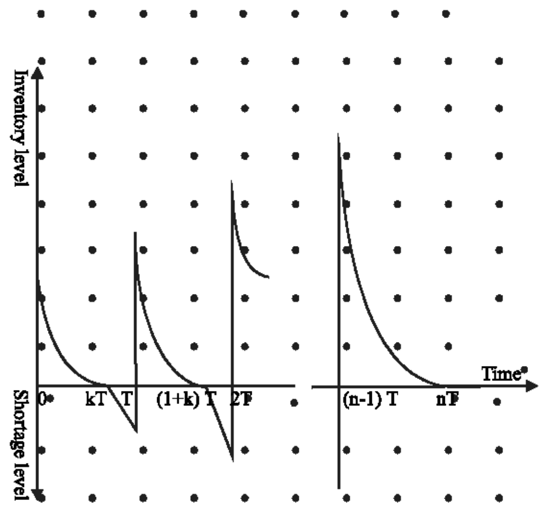

Fig. 1: Graphical representation of the inventory system

The graphical representation of the inventory system is shown in Fig. 1. The time horizon, H, is divided into n equal cycles each of length T so that T=H/n. Initial and final inventory levels are both zero. Each inventory cycle except the last cycle can be divided into two parts: [(j-1)T, (j+k-1)T] and [(j+k-1)T,jT]. During the time interval [(j-1)T,jT], the inventory level leads to zero and shortages occur at time (j+k-1)T. Shortages are accumulated until jT before they are backordered and are not allowed in the last replenishment cycle. The optimal inventory policy yields the ordering and shortage points, which minimize the present value of the total inventory system costs over the time horizon.

THE MATHEMATICAL MODELING AND ANALYSIS

Each inventory cycle, except the last cycle, is divided to the two different parts. During the time interval [(j-1)T, (j+k-1)T], the level of inventory, $I_{1j}(t_{1j})$, gradually decreases mainly to meet demands and partly due to deterioration. Hence, the variation of inventory with respect to time can be described by the following differential equation:

$$\frac{dI_{1j}(t_{1j})}{dt_{1j}} + \theta I_{1j}(t_{1j}) = -D_{1j}(t_{1j}), \qquad (j-1)T \le t_{1j} \le (k+j-1)T \tag{4}$$

The shortages occur at time (k+j-1)T and accumulated until jT before they are backordered. The shortages level to be represented by $I_{2j}(t_{2j})$:

$$\frac{dI_{2j}(t_{2j})}{dt_{2j}} = -D_{2j}(t_{2j}), \qquad (k+j-1)T \le t_{2j} \le jT \tag{5}$$

In the last cycle shortages are not allowed and the inventory level, $I_3(t_3)$, is governed by the following differential equation:

$$\frac{dI_3(t_3)}{dt_3} + \theta I_3(t_3) = -D_3(t_3), \qquad (n-1)T \le t_3 \le nT \tag{6}$$

The boundary conditions are:

$$I_{1j}((k+j-1)T) = 0 \tag{7}$$

$$I_{2j}((k+j-1)T) = 0 \tag{8}$$

$$I_3(nT) = 0 \tag{9}$$

The solution of Eq. 4-6 after apply the boundary conditions and considering Eq. 3 are as follow:

$$I_{1j}(t_{1j}) = \frac{-(a+bt_{1j})}{\theta} + \frac{b}{\theta^2} + \left[\frac{a+b(k+j-1)T}{\theta} - \frac{b}{\theta^2}\right]e^{\theta[(k+j-1)T - t_{1j}]} \tag{10}$$

$$I_{2j}(t_{2j}) = \left[(k+j-1)T - t_{2j}\right]\left[a + \frac{b\left[(k+j-1)\ T\ + t_{2j}\right]}{2}\right] \tag{11}$$

$$I_3(t_3) = \frac{-(a+bt_3)}{\theta} + \frac{b}{\theta^2} + \left[\frac{a+bnT}{\theta} - \frac{b}{\theta^2}\right]e^{\theta[nT - t_3]} \tag{12}$$

The objective of the problem is minimization of the total present value of costs over the time horizon. Consider CP, CH, CS and CR as the present value of costs of purchasing, holding, shortages and replenishment respectively. The total present value of costs over the time horizon (TVC(n,k)) is:

$$TVC(n,k) = CR + CP + CH + CS \tag{13}$$

The detailed analysis is given as follows:

The ordering costs: The replenishment cost occurs at the start of inventory cycle and therefore, the present value of the ordering cost for (j+1)th cycle is:

$$CR_j = Se^{-rjT}e^{(a_1+b_1jT)jT}, \; j = 0,1,2,\ldots,n-1 \tag{14}$$

Therefore, the total ordering cost is:

$$CR = S\sum_{j=0}^{n-1} e^{-(r-a_1)jT}e^{b_1j^2T^2} \tag{15}$$

The purchasing cost: During any given period, the order quantity consists of both demand and deterioration for the relevant period excluding shortage part of the period and the amount required to satisfy the demand during the shortage period in the preceding time interval. For the j-th cycle (j = 1, 2, …, n-1) the present value of the purchase cost can be formulated as follows:

$$\begin{aligned} CP_{j-1} &= pe^{-(r-a_2-b_2(j-1)T)(j-1)T}I_{1j}((j-1)T) + \\ &pe^{-(r-a_2-b_2jT)jT}\left[-I_{2j}(jT)\right], \; j = 1,2,\ldots,n-1 \end{aligned} \tag{16}$$

In the last period shortages are not allowable, therefore, the present value of the purchase cost is:

$$CP_{n-1} = pe^{-(r-a_2-b_2(n-1)T)(n-1)T}I_3((n-1)T) \tag{17}$$

The total purchasing cost over the time horizon would be:

$$CP = \sum_{j=1}^{n} CP_{j-1} \tag{18}$$

The carrying cost: The present value of the inventory carrying cost for the j-th cycle (j = 1, 2, …, n-1) for the mth class (m = 1, 2) is:

$$\begin{aligned} CH_{jm} &= c_{1m}\int_{(j-1)T}^{(j-1+k)T} I_{1j}(t_{1j})e^{-rt_{1j}}e^{(a_m+b_mt_{1j})t_{1j}}\,dt_{1j}, \\ &j = 1,2,\ldots,n-1, m = 1,2 \end{aligned} \tag{19}$$

In the last inventory cycle from similar machinations, we have:

$$CH_{nm} = c_{1m}\int_{(n-1)T}^{nT} I_3(t_3)e^{-rt_3}e^{(a_m+b_mt_3)t_3}dt_3, m = 1,2 \tag{20}$$

Therefore, the total holding cost over the time horizon is:

$$CH = \sum_{m=1}^{2}\sum_{j=1}^{n} CH_{jm} \tag{21}$$

The shortages cost: The shortages cost for j-th cycle can be calculated as follow:

$$\begin{aligned} CS_{jm} &= c_{2m}\int_{(j-1+k)T}^{jT}\left[-I_{2j}(t_{2j})\right]e^{-rt_{2j}}e^{(a_m+b_mt_{2j})t_{2j}}\,dt_{2j}, \\ &j = 1,2,\ldots,n-1, m = 1,2 \end{aligned} \tag{22}$$

Using above equation, the total shortages cost is as follows (note that shortages are not allowable in the last cycle):

$$CS = \sum_{m=1}^{2}\sum_{j=1}^{n-1} CS_{jm} \tag{23}$$

So, the present value of the total costs of the inventory system over the time horizon is obtained using Eq. 15, 18, 21 and 23 that is shown in Eq. 13. The problem is determining n and k so that minimize the total inventory system cost. Optimal value of n and k can be obtained with setting number of replenishments equal to 1 and increasing it. For a given value of n, the necessary condition of optimality sets the first derivative of TVC(n,k) with respect to k equals to zero. After several algebraic operations, the first derivative can be calculated as follow:

$$\begin{aligned}\frac{dTVC(n,k)}{dk} &= p\sum_{j=1}^{n-1}\left[e^{-(r-a_2-b_2(j-1)T)(j-1)T}\left[\frac{e^{-\theta(j-1)T}\left(bT+\left(\frac{aT+b(k+j-1)T^2}{\theta}-\frac{bT}{\theta^2}\right)\right)}{\theta e^{-\theta(k+j-1)T}}\right]\right] \\ &-p\sum_{j=1}^{n-1}\left[\left(b(k+j-1)T^2+aT\right)e^{-(r-a_2-b_2jT)jT}\right]+\sum_{m=1}^{2}\sum_{j=1}^{n-1}\frac{d\left[\int_{(j-1)T}^{(j-1+k)T} I_{1j}(t_{1j})e^{-rt_{1j}}e^{(a_m+b_m t_{1j})t_{1j}}dt_{1j}\right]}{dk} \\ &+\sum_{m=1}^{2}\sum_{j=1}^{n-1}\frac{d\left[\int_{(j-1+k)T}^{jT}\left[-I_{2j}(t_{2j})\right]e^{-rt_{2j}}e^{(a_m+b_m t_{2j})t_{2j}}dt_{2j}\right]}{dk}=0\end{aligned} \tag{24}$$

By increasing n, the objective function leads to the minimum and then by increasing n, the objective function increases monotonously. The iterative methods such as Newton method can be used to solve the model. The second-order condition for a minimum is:

$$\begin{aligned}\frac{d^2TVC(n,k)}{dk^2} &= p\sum_{j=1}^{n-1}\left[e^{-(r-a_2-b_2(j-1)T)(j-1)T}\left[\frac{e^{-\theta(j-1)T}\left(2bT^2+\left(a\theta T^2+b(k+j-1)\theta T^3-bT^2\right)\right)}{e^{-\theta(k+j-1)T}}\right]\right] \\ &-p\sum_{j=1}^{n-1}\left[bT^2e^{-(r-a_2-b_2jT)jT}\right]+\sum_{m=1}^{2}\sum_{j=1}^{n-1}\frac{d^2\left[\int_{(j-1)T}^{(j-1+k)T} I_{1j}(t_{1j})e^{-rt_{1j}}e^{(a_m+b_m t_{1j})t_{1j}}dt_{1j}\right]}{dk^2} \\ &+\sum_{m=1}^{2}\sum_{j=1}^{n-1}\frac{d^2\left[\int_{(j-1+k)T}^{jT}\left[-I_{2j}(t_{2j})\right]e^{-rt_{2j}}e^{(a_m+b_m t_{2j})t_{2j}}dt_{2j}\right]}{dk^2}>0\end{aligned} \tag{25}$$

THE NUMERICAL EXAMPLE

Following example is providing according to the results. Let internal and external inflation rates as follow:

$$i_1 = 0.1 + 0.005t$$
$$i_2 = 0.12 + 0.006t$$

The demand rate, which is a linear function of the inflation rates, is:

Table 1: The optimal solution

n	k	TVC (n,k)	n	k	TVC (n,k)
5	0.575987	71556.80	25	0.608279	67807.31
10	0.596408	68590.77	30	0.609582	67966.94
15	0.603036	67903.47	35	0.610510	68185.08
20	0.606319	67750.90	40	0.611206	68439.60
21*	0.606786*	67750.32*	50	0.612178	69013.81
22	0.607211	67756.50	70	0.613287	70291.92
24	0.607953	67785.68	100	0.614117	72338.11

The minimum cost over the time horizon is 67750.32* for n* = 21 and k* = 0.606786

$$D(i_1, i_2) = 2000 - 1000i_1 - 2000i_2$$

The company interest rate is 20% per annum, the deterioration rate of the on-hand inventory per unit time is 0.01 and the length of time horizon is 10 years.

$r = 0.2$; H=10years; $\theta = 0.01$

The system costs at the beginning of time horizon are $c_{11} = 0.2$; $c_{12} = 0.4$; $c_{21} = 0.8$; $c_{22} = 0.6$; $p = 5$; $S = 100$

Using these parameter values, the optimal solution of the models is obtained and the results are shown in Table 1.

The minimum cost over the time horizon is 67750.32 for n* = 21 and k* = 0.606786. Optimal interval of time between replenishment, T*, equals to H/n* = 0.467 year. The shortages occur after elapsing 60.7% of the cycle time or 103 days.

SOME PARTICULAR CASES

An attempt has been made in this section to study six important special cases of the model.

Case (I): Constant inflation rates: Assume the inflation rates do not change over the time horizon, i.e.,

$$b_m = 0, \quad \text{for } m = 1, 2 \tag{26}$$

Therefore, the demand rate is:

$$D(i_1, i_2) = a_0 + b_0 a_1 + c_0 a_2 \tag{27}$$

The total present value of costs, TVC(n,k), can be obtained with placing Eq. 26 in Eq. 13. The optimal solution in this case, with considering the previous numerical example, is as follows: n* = 22, k* = 0.470016, TVC(n,k) = 59871.78 and T* = 0.454 year. We can see that k decreases considerably in comparison with the main model.

Case (II): Identical rate of inflation: If internal and external inflation rates be identical to each other, the present value of the total cost TVC(n,k) can be obtained by deleting $\sum_{m=1}^{2}$ in Eq. 13 and substituting:

$$c_{1m} = c_1, c_{2m} = c_2, a_m = A, b_m = B, R_m = R, \quad \text{for } m = 1, 2 \tag{28}$$

In the previous numerical example let:

$$i = 0.11 + 0.0055t$$

Therefore, the optimal solution in this case will be: $n^* = 20$, $k^* = 0.577967$, $TVC(n,k) = 64474.57$ and $T^* = 0.5$year. The number of replenishment, n, inventory system costs and k decrease.

Case (III): Without shortages: If shortages are not allowed, k = 1 can be substituted in expression Eq. 13 and the present value of the total variable cost, TVC(n), can be obtained. The minimum solution of TVC(n) for the discrete variable of n must satisfy the following equation:

$$\Delta TVC(n) \le 0 \le \Delta TVC(n+1) \tag{29}$$

where, $\Delta TVC(n) = TVC(n)-TVC(n-1)$. In the numerical example, using the above inequality, the following solution is obtained: $n^* = 26$, $TVC(n) = 68\ 543.95$ and $T^* = 0.385$year. It shows that n and TVC increase in this case in comparison with the main model.

Case (IV): No deterioration: If there has not been deterioration over time for the available inventory, i.e., $\theta = 0$, the cost function after calculation, will be rewritten as follows:

$$TVC(n,k,\theta=0) = S\sum_{j=0}^{n-1} e^{-(r-a_1)jT} e^{b_1 j^2 T^2}$$
$$+p\left[e^{-(r-a_2-b_2(n-1)T)(n-1)T} I_3((n-1)T) + \sum_{j=1}^{n-1}\left[e^{-(r-a_2-b_2(j-1)T)(j-1)T} I_{1j}((j-1)T) + pe^{-(r-a_2-b_2 jT)jT}\left[-I_{2j}(jT)\right]\right]\right]$$
$$+\sum_{m=1}^{2}\left[c_{1m}\left[\sum_{j=1}^{n-1}\int_{(j-1)T}^{(j-1+k)T} I_{1j}(t_{1j}) e^{-rt_{1j}} e^{(a_m+b_m t_{1j})t_{1j}} dt_{1j} + \int_{(n-1)T}^{nT} I_3(t_3) e^{-rt_3} e^{(a_m+b_m t_3)t_3} dt_3\right]\right]$$
$$+\sum_{m=1}^{2}\sum_{j=1}^{n-1} c_{2m}\left[\int_{(j-1+k)T}^{jT}\left[-I_{2j}(t_{2j})\right] e^{-rt_{2j}} e^{(a_m+b_m t_{2j})t_{2j}} dt_{2j}\right] \tag{30}$$

where,

$$I_{1j}(t_{1j}) = -at_{1j} - \frac{1}{2}bt_{1j}^2 + \frac{b(k+j-1)^2T^2}{2} + a(k+j-1)T \tag{31}$$

$$I_{2j}(t_{2j}) = \left[(k+j-1)T - t_{2j}\right]\left[a + \frac{b\left[(k+j-1)\ T\ + t_{2j}\right]}{2}\right] \tag{32}$$

$$I_3(t_3) = -at_3 - \frac{1}{2}bt_3^2 + anT + \frac{1}{2}bn^2T^2 \tag{33}$$

The total costs of the inventory system can be minimized by the explained methods. The optimal solution is as follow: $n^* = 41$, $k^* = 0.667940$, $TVC(n,k) = 44513.44$ and $T^* = 0.244$year. Thus, n and k are increased and TVC is decreased in comparison with the main model. This is an expected phenomenon, because the goods do not deteriorated and the inventory system manager can increases order quantity, i.e., decreases n and increases the inventory level in warehouse (or increases k).

Case (V): Identical inflation rate, no shortages and no deterioration: Now assume that the internal and external inflation rates be identical to each other, no shortages allowed and $\theta = 0$. This can be solved by using Eq. 30, substituting k = 1 and considering Eq. 28. Therefore, the optimal solution is as follows: $n^* = 25$, $TVC(n,k) = 65\ 271.90$ and $T^* = 0.4$ year. In comparison with the main model, the number of replenishment increases and the total costs decreases.

Case (VI): Constant and identical inflation rate, no shortages and no deterioration: The optimal order policy in this case will be obtained with using Eq. 30, substituting k = 1 and considering Eq. 26 and 28, which is as follow: $n^* = 30$, $TVC(n,k) = 59\ 001.70$ and $T = 0.333$ year. Similarly, the previous case, the number of replenishment increases and the total costs decrease.

CONCLUSIONS

In this study, an inventory model under inflationary conditions with shortages for deteriorating items has been proposed. The internal and external inflation rates are time-dependent. Also, the inflation-proportional demand rate has been considered. The objective is determining the optimal values of the time interval between replenishment and the time occurrence of shortages over the time horizon to minimize the total costs of the inventory systems. A numerical example has been given to illustrate the theoretical results. Finally, six special cases have been discussed. These special cases are compared with the main model through the numerical example.

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