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Radiation Effects on Free Convection Flow Near a Moving Vertical Plate with Newtonian Heating

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Abstract: The influence of radiation on the unsteady free convection flow of a viscous incompressible fluid past a moving vertical plate with Newtonian heating is investigated theoretically. The fluid is gray, absorbing-emitting but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. For a detailed analysis of the problem, three important cases of flow due to: (1) motion with uniform velocity, (2) uniformly accelerated motion and (3) exponentially accelerated motion of the plate have been considered. The dimensionless governing equations are solved analytically using the Laplace transform technique. Numerical results for the temperature, velocity, skin-friction and the Nusselt number are shown graphically. The results show that an increase in the radiation parameter leads to a decrease in the temperature and velocity. Also, an increase in the radiation parameter increases the skin friction and the Nusselt number at the plate.

Key words: Free convection, moving vertical plate, thermal radiation, Newtonian heating

INTRODUCTION

Free convection flow of an incompressible viscous fluid past a moving vertical infinite plate finds applications in many industrial processes. This is particularly important in the design of spaceship, filtration processes, the drying of porous materials in textile industries, solar energy collectors, nuclear reactors, etc. Soundalgekar (1977) studied the free convection flow past an impulsively started infinite vertical plate when it is heated or cooled by free convection currents and he observed that more cooling of the plate by free convection currents may cause separation near the plate in case of air. Soundalgekar and Patil (1980) presented an exact solution to the problem of free convection flow past an impulsively started infinite vertical plate in the presence of constant heat flux. Soundalgekar and Gupta (1980) studied the free convection flow past an accelerated infinite vertical plate. This problem was solved for (1) isothermal plate and (2) constant heat flux at the plate by Laplace transform technique and it is observed that in case of air, separation exists even at small values of Grashof number. Free convection flow of an incompressible and viscous fluid past an exponentially accelerated infinite vertical plate was analyzed by Singh and Kumar (1984). An exact solution of the flow of a viscous incompressible fluid past an impulsively started infinite vertical plate in the presence of foreign mass was presented under the conditions of (1) variable plate temperature and (2) constant heat flux by Soundalgekar *et al*. (1984). An analysis of the effects of free convection currents on the flow of an incompressible viscous fluid past a moving vertical infinite porous plate was presented by Perdikis and Takhar (1986). An analytical study to examine the mass transfer effects on flow past an exponentially accelerated vertical plate was performed by Jha *et al*. (1991). The unsteady free convection flow with constant heat flux and accelerated boundary motion was investigated by Chandran *et al*. (1998). Transient free convection flow past an infinite vertical plate with periodic temperature variation was studied by Das *et al*. (1999). An exact solution to the problem of flow past an impulsively started infinite vertical plate in the presence of variable temperature and mass diffusion was considered by Muthucumaraswamy *et al*. (2000). Theoretical study of unsteady free convection flow past an exponentially accelerated infinite isothermal vertical plate in the presence of variable mass diffusion was presented by Muthucumaraswamy *et al*. (2008). Recently, a theoretical analysis to the problem of free convection flow induced by an infinite moving vertical plate subject to a ramped

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wall temperature with simultaneous mass transfer was presented by Narahari and Dutta (2009).

The usual way in which free convection flows are modeled is to assume that the flow is driven either by a prescribed surface temperature or by a prescribed surface heat flux. Here a somewhat different driving mechanism for free convection flow past a moving vertical plate is considered, in that it is assumed that the flow is set up by a Newtonian heating from the surface. That is, the heat transfer from the surface is taken to be proportional to the local surface temperature. This situation was first considered by Merkin (1994) for the free convection boundary-layer over a vertical flat plate immersed in a viscous fluid. Lesnic *et al.* (1999, 2000) considered free convection boundary-layer flow along vertical and horizontal surfaces in a porous medium with Newtonian heating. The steady free convection boundary-layer along a semi-infinite, slightly inclined to the horizontal plate embedded in a porous medium with the flow generated by Newtonian heating was investigated by Lesnic *et al.* (2004). An exact solution of the unsteady free-convection boundary-layer flow of an incompressible fluid past an infinite vertical plate with flow generated by Newtonian heating and impulsive motion of the plate was presented by Chaudhary and Jain (2007). Recently, an exact solution to the problem of unsteady free convection flow of a viscous incompressible, optically thin radiating fluid past an impulsively started vertical porous plate with Newtonian heating was investigated by Mebine and Adigio (2009). In the present study, an exact solution to the problem of unsteady free convection flow of a viscous incompressible, optically thick radiating fluid past a moving vertical infinite plate with Newtonian heating is derived using Laplace transformation technique and the Rosseland diffusion approximation is used to describe the radiative heat flux in the energy equation.

MATHEMATICAL ANALYSIS

Consider the unsteady free convection flow of a viscous incompressible fluid past a moving vertical plate. The x'- axis is taken along the plate in the vertically upward direction and the y'- axis is normal to the plate. Initially, for time $t' \le 0$, the plate and the fluid are at the same constant temperature T'_∞ in a stationary condition. At time $t'>0$, the plate starts moving with velocity $u_0 f(t')$ in its own plane along the x'- axis against the gravitational field and it is assumed that the rate of heat transfer from the surface is proportional to the local surface temperature T', where u_0 is a constant. Since, the plate is considered infinite in the x'- direction, all the physical variables are functions of y' and t' only. It is also assumed that the radiative heat flux in the x'- direction is negligible as compared to that in the y'- direction. Then, under usual Boussinesq approximation, the free convection flow of a radiating fluid is shown to be governed by the following system of equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{2}$$

The initial and boundary conditions are

$$\left.\begin{array}{ll} t' \le 0: u' = 0,\ T' = T'_\infty & \text{for all } y' \ge 0; \\ t' > 0: u' = u_0 f(t'),\ \dfrac{\partial T'}{\partial y'} = -\dfrac{h}{k}T' & \text{at } y' = 0, \\ u' \to 0,\ T' \to T'_\infty & \text{as } y' \to \infty. \end{array}\right\} \tag{3}$$

where, u' is the velocity in the x' direction, T' the temperature of the fluid, T'_∞ the ambient temperature, g the acceleration due to gravity, β the volumetric coefficient of thermal expansion, ν the kinematic viscosity, ρ the density, C_p the specific heat of the fluid at constant pressure, q_r the radiative heat flux in y'- direction, k the thermal conductivity and h is the heat transfer coefficient.

The radiative heat flux term is simplified by making use of the Rosseland approximation (Siegel and Howell, 2002) as:

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T'^4}{\partial y'} \tag{4}$$

where, σ is the Stefan-Boltzmann constant and k* is the mean absorption coefficient. It should be noted that by using the Rosseland approximation we limit our analysis to optically thick fluids. If temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature, then the Taylor series for T'^4 about T'_∞, after neglecting higher order terms, is given by

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{5}$$

In view of Eqs. 4 and 5, Eq. 2 reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \tag{6}$$

For a detailed analysis of Eq. 1 we have considered three different cases of flow due to: (1) motion with uniform velocity, (2) Uniformly accelerated motion and (3) exponentially accelerated motion of the plate.

Motion with uniform velocity: In this case, f (t') = 1. We introduce the following non-dimensional quantities:

$$\left.\begin{aligned} y=\frac{y'u_0}{\nu},\ t=\frac{t'u_0^2}{\nu},\ u=\frac{u'}{u_0},\ \theta=\frac{T'-T'_\infty}{T'_\infty},\\ \Pr=\frac{\mu C_p}{k},\ Gr=\frac{\nu g\beta T'_\infty}{u_0^3},\ R=\frac{kk^*}{4\sigma T'^3_\infty}\end{aligned}\right\} \tag{7}$$

The corresponding dimensionless forms are:

$$\frac{\partial u}{\partial t}=Gr\,\theta+\frac{\partial^2 u}{\partial y^2} \tag{8}$$

$$3R\Pr\frac{\partial\theta}{\partial t}=(3R+4)\frac{\partial^2\theta}{\partial y^2} \tag{9}$$

The transformed initial and boundary conditions are:

$$\left.\begin{aligned} &t\le 0:\ u=0,\theta=0 && \text{for all } y\ge 0\\ &t>0:\ u=1,\frac{\partial\theta}{\partial y}=-(1+\theta) && \text{at } y=0,\\ &u\to 0,\theta\to 0 && \text{as } y\to\infty.\end{aligned}\right\} \tag{10}$$

where, μ is the coefficient of viscosity, u the dimensionless velocity, θ the dimensionless temperature, Gr the Grashof number, Pr the Prandtl number, R the radiation parameter, t the dimensionless time and y is the dimensionless coordinate axis normal to the plate.

According to the above non-dimensional process, the characteristic velocity u_0 can be defined as:

$$u_0=\frac{h\nu}{k} \tag{11}$$

Equations 8 and 9 subject to the boundary conditions 10 are solved by the usual Laplace transform technique and the solutions are given by

$$\begin{aligned} u(y,t)&=f_1(y,t)\\ &+bN\left[f_1(y\sqrt{N},t)-f_1(y,t)-f_4(y\sqrt{N},t)+f_4(y,t)\right]\\ &+b\sqrt{N}\left[f_2(y\sqrt{N},t)-f_2(y,t)\right]\\ &+b\left[f_3(y\sqrt{N},t)-f_3(y,t)\right]\end{aligned} \tag{12}$$

$$\theta(y,t)=f_4(y\sqrt{N},t)-f_1(y\sqrt{N},t) \tag{13}$$

where:

$$f_1(z,t)=\mathrm{erfc}\left(\frac{z}{2\sqrt{t}}\right)$$

$$f_2(z,t)=2\sqrt{t/\pi}\exp\left(-\frac{z^2}{4t}\right)-z\,\mathrm{erfc}\left(\frac{z}{2\sqrt{t}}\right)$$

$$f_3(z,t)=\left(\frac{z^2}{2}+t\right)\mathrm{erfc}\left(\frac{z}{2\sqrt{t}}\right)-z\sqrt{t/\pi}\exp\left(-\frac{z^2}{4t}\right)$$

$$f_4(z,t)=\exp\left(\frac{t}{N}-\frac{z}{\sqrt{N}}\right)\mathrm{erfc}\left(\frac{z}{2\sqrt{t}}-\sqrt{\frac{t}{N}}\right)$$

$$N=\frac{3R\Pr}{3R+4},\ b=\frac{Gr}{N-1}$$

z is a dummy variable and f_1, f_2, f_3, f_4, f_5 are dummy functions defined above.

From the velocity field, we determine the skin friction, which in dimensionless form is given by:

$$\tau=\frac{\tau'}{\rho u_0^2}=-\left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{14}$$

Substituting Eq. 12 in Eq. 14 we obtain:

$$\begin{aligned}\tau&=\frac{1}{\sqrt{t\pi}}\\ &+\frac{Gr\sqrt{N}}{\sqrt{N}+1}\left[1+2\sqrt{\frac{t}{N\pi}}-\exp\left(\frac{t}{N}\right)\left(1+\mathrm{erf}\left(\sqrt{\frac{t}{N}}\right)\right)\right]\end{aligned} \tag{15}$$

Another important phenomenon in this study is to understand the effects of R, Pr and t on the Nusselt number. The rate of heat transfer is given in the form of Nusselt number as:

$$Nu=-\frac{\nu}{u_0(T'-T'_\infty)}\left.\frac{\partial T'}{\partial y'}\right|_{y'=0}=\frac{1}{\theta(0,t)}+1 \tag{16}$$

Substituting θ (0, t) from Eq. 13 into 16, we obtain:

$$Nu=\frac{1}{\exp(t/N)(1+\mathrm{erf}(\sqrt{t/N}))-1}+1 \tag{17}$$

In the absence of radiation effects, the solutions for the velocity (Pr ≠ 1), temperature, skin-friction and the Nusselt number derived by Chaudhary and Jain (2007) can be deduced from Eq. 12, 13, 15 and 17, respectively, by putting N = Pr (as well as in $f_4(z, t)$ and b).

Uniformly accelerated motion: In this case, f (t') = t'. The velocity field in dimensionless form is given by:

$$\begin{aligned} u(y,t)&=f_3(y,t)\\ &+bN\left[f_1(y\sqrt{N},t)-f_1(y,t)-f_4(y\sqrt{N},t)+f_4(y,t)\right]\\ &+b\sqrt{N}\left[f_2(y\sqrt{N},t)-f_2(y,t)\right]\\ &+b\left[f_3(y\sqrt{N},t)-f_3(y,t)\right]\end{aligned} \tag{18}$$

The non-dimensional quantities introduced in Eq. 18 are defined as follows:

$$y=\frac{y'h}{k}, t=\frac{t'\nu h^2}{k^2}, u=\frac{u'\nu h^2}{u_0 k^2}, \theta=\frac{T'-T'_\infty}{T'_\infty},$$
$$\Pr=\frac{\mu C_p}{k}, Gr=\frac{g\beta T'_\infty}{u_0}, R=\frac{kk^*}{4\sigma T'^3_\infty} \tag{19}$$

Using the expression in Eq. 18, the skin friction in dimensionless form is obtained as:

$$\tau=\frac{\tau' h}{\rho u_0 k}=-\left.\frac{\partial u}{\partial y}\right|_{y=0}=2\sqrt{\frac{t}{\pi}}$$
$$+\frac{Gr\sqrt{N}}{\sqrt{N}+1}\left[1+2\sqrt{\frac{t}{N\pi}}-\exp\left(\frac{t}{N}\right)\left(1+\operatorname{erf}\left(\sqrt{\frac{t}{N}}\right)\right)\right] \tag{20}$$

In the absence of radiation effects, the solutions for the velocity ($\Pr\neq 1$)and skin-friction can be deduced from the Eq. 18 and 20, respectively, by putting N = Pr (as well as in f_4 (z, t) and b).

Exponentially accelerated motion: For this case, f (t') = exp (a' t'). The velocity field in dimensionless form is given by:

$$u(y,t)=\exp(at)F_5(y,t)$$
$$+bN[f_1(y\sqrt{N},t)-f_1(y,t)-f_4(y\sqrt{N},t)+f_4(y,t)]$$
$$+b\sqrt{N}[f_2(y\sqrt{N},t)-f_2(y,t)] \tag{21}$$
$$+b[f_3(y\sqrt{N},t)-f_3(y,t)]$$

Where:

$$f_5(z,t)=\frac{1}{2}\left[\exp(-z\sqrt{a})\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}-\sqrt{at}\right)+\exp(z\sqrt{a})\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}+\sqrt{at}\right)\right]$$

and f_5 is a dummy function of the dummy variable z.

The non-dimensional quantities introduced in Eq. 21 are defined as follows:

$$y=\frac{y'u_0}{\nu}, t=\frac{t'u_0^2}{\nu}, u=\frac{u'}{u_0}, \theta=\frac{T'-T'_\infty}{T'_\infty},$$
$$\Pr=\frac{\mu C_p}{k}, Gr=\frac{\nu g\beta T'_\infty}{u_0^3}, R=\frac{kk^*}{4\sigma T'^3_\infty}, a=\frac{a'\nu}{u_0^2} \tag{22}$$

Using the expression in Eq. 21, the skin-friction in dimensionless form is obtained as:

$$\tau=\frac{\tau'}{\rho u_0^2}=-\left.\frac{\partial u}{\partial y}\right|_{y=0}$$
$$=\frac{1}{\sqrt{t\pi}}+\sqrt{a}\exp(at)\operatorname{erf}\left(\sqrt{at}\right) \tag{23}$$
$$+\frac{Gr\sqrt{N}}{\sqrt{N}+1}\left[1+2\sqrt{\frac{t}{N\pi}}-\exp\left(\frac{t}{N}\right)\left(1+\operatorname{erf}\left(\sqrt{\frac{t}{N}}\right)\right)\right]$$

In the absence of radiation effects, the solutions for the velocity ($\Pr \neq 1$) and skin-friction can be deduced from the Eq. 21 and 23, respectively, by putting N = Pr (as well as in f_4 (z, t) and b). Also, the results of present case includes the corresponding results of the plate with uniform velocity when a = 0.

RESULTS AND DISCUSSION

The effect of thermal radiation and Newtonian heating boundary condition on flow and heat transfer of a viscous incompressible fluid along a moving vertical infinite plate has been considered in the preceding section. The governing coupled linear partial differential Eq. 8 and 9 subject to the initial and boundary conditions corresponding to the three cases of motion of the plate have been solved analytically using Laplace transform technique without any restriction. The numerical results for the temperature, velocity, skin-friction and the Nusselt number are computed to carry out a parametric study showing influences of several system parameters like R, Gr, Pr, a and t.

The temperature profiles for different values of R, Pr and t are shown in Fig. 1. It is observed that the temperature decreases with increasing values of radiation parameter. When radiation is present, the thermal boundary layer always found to thicken, which may be explained by the fact that radiation provides an additional means to diffuse energy. For R = 10 the temperature profile is found to increase 11.22% of the pure convection case (which numerically corresponds to $R\to\infty$) at the plate when t = 0.2 and Pr = 0.71 (air). In the absence of radiation effects, i.e. in the case of pure convection, it is noted that an increase of Prandtl number results in a decreasing the thermal boundary layer thickness and more uniform temperature distribution across the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr. It is also observed that the temperature increases with increasing time in the presence of radiation.

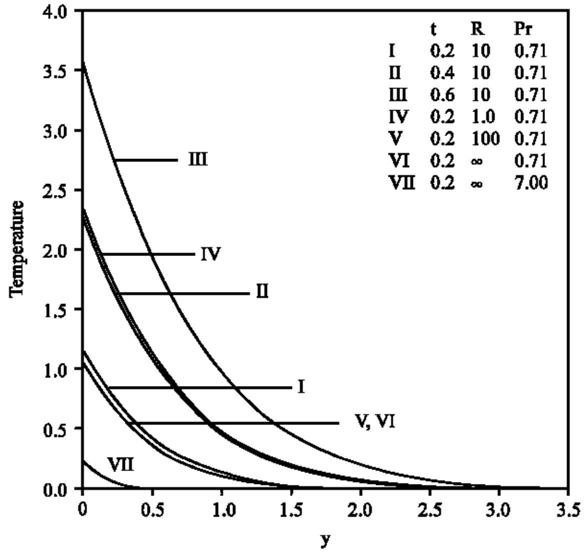

Fig. 1: Temperature profiles for different t, R and Pr

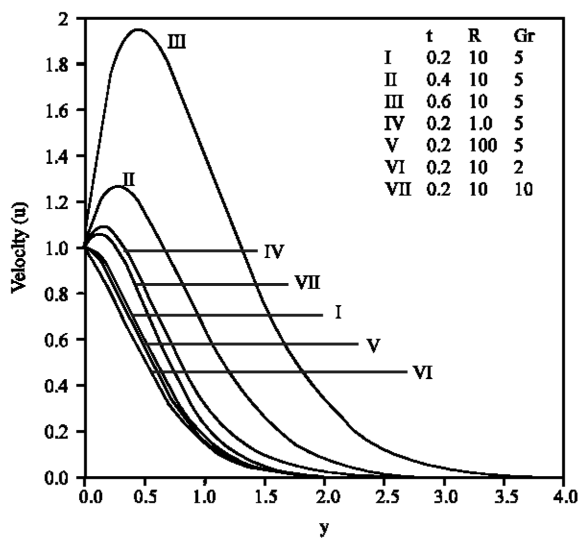

Fig. 2: Velocity profiles for different t, R and Gr when Pr = 0.71 due to uniform velocity of the plate

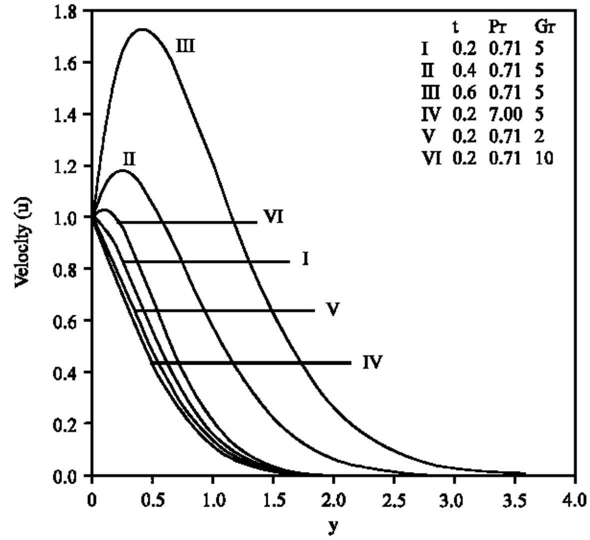

Fig. 3: Velocity profiles for different t, Pr and Gr when $R \to \infty$ due to uniform velocity of the plate

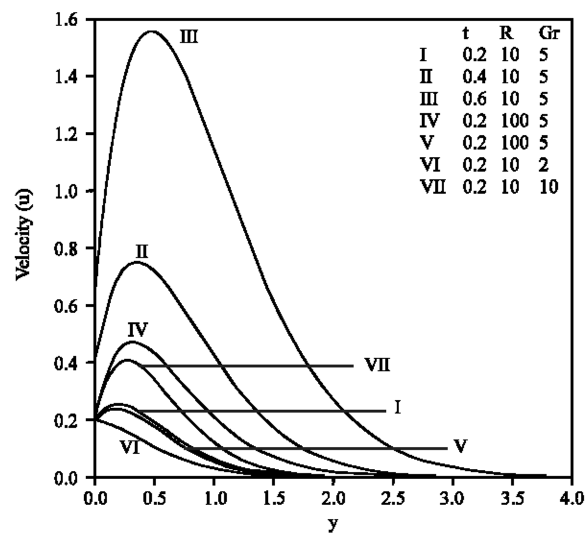

Fig. 4: Velocity profiles for different t, R and Gr when Pr = 0.71 due to uniformly accelerated plate

The velocity profiles are shown in Fig. 2 for different values of t, R and Gr when Pr = 0.71 due to uniform velocity of the plate in the presence of radiation. It is observed that the velocity increases with increasing t and Gr. Physically this is possible because as the Grashof number or time increases, the contribution from the buoyancy force near the plate becomes significant and hence a rise in the velocity near the plate is observed. Moreover, it is seen that the larger the value of radiation parameter, the thinner the momentum boundary layer size. This result may be explained by the fact that an increase in the radiation parameter $R = (k k^*/4\sigma T'^3_\infty)$ for fixed k^* and T'_∞ means an increase in the Rosseland mean absorption coefficient k^*.

The velocity profiles are shown in Fig. 3 for different values of t, Pr and Gr in the pure convection case due to uniform velocity of the plate. It is observed that an increase in t and Gr leads to an increase in the velocity of an air flow due to enhancement in the buoyancy force. Moreover, the velocity of fluid decreases with increasing Prandtl number. This is consistent with the physical observation that the fluids with high Prandtl number have greater viscosity, which makes the fluid thick and hence move slowly.

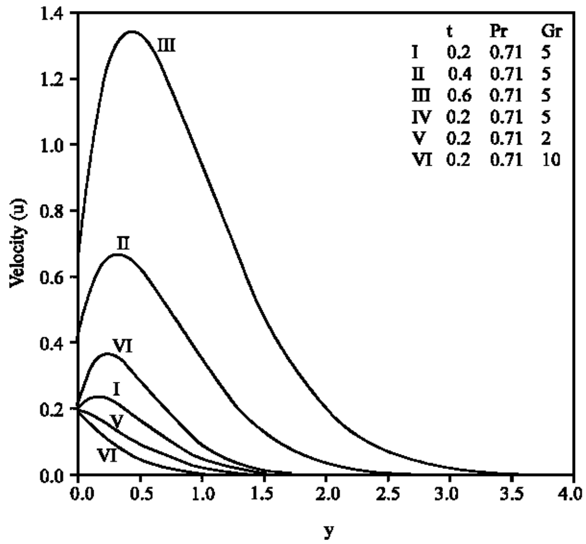

Fig. 5: Velocity profiles for different t, Pr and Gr when $R \to \infty$ due to uniformly accelerated plate

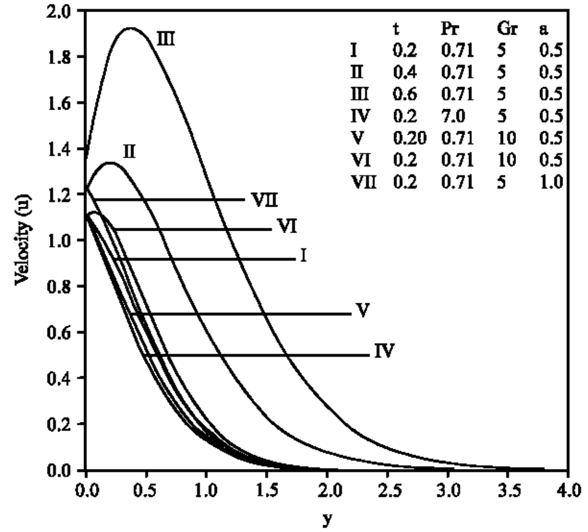

Fig. 7: Velocity profiles for different t, Pr, Gr and a when $R \to \infty$ due to exponentially accelerated plate

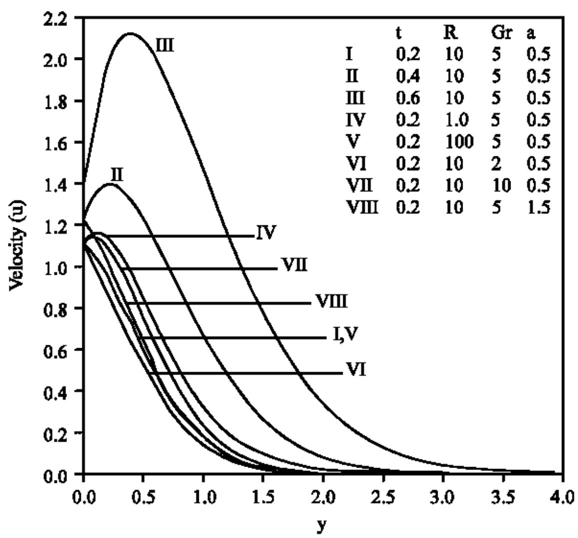

Fig. 6: Velocity profiles for different t, R, Gr and a when Pr = 0.71 due to exponentially accelerated plate

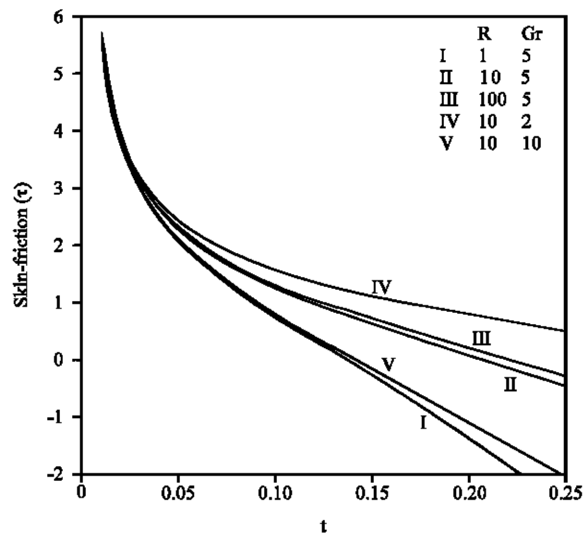

Fig. 8: Skin-friction for different R and Gr when Pr = 0.71 due to uniform velocity of the plate

From Fig. 2 and 3 it is interesting to note that for R = 10 the velocity profile is found to increase 5.95% of the pure convection case near the plate at y = 0.25 when t = 0.4, Pr = 0.71 and Gr = 5. Furthermore, when radiation is present, the momentum boundary layer was found to thicken which is in agreement with the observation made earlier with regard to the temperature variations of air flow.

The velocity profiles are shown in Fig. 4 and 6 for different values of t, R, Gr when Pr = 0.71 and in Fig. 5 and 7 for different values of t, Pr, Gr when $R \to \infty$ for the cases of uniformly accelerated plate and exponentially accelerated plate respectively. It is noted that the parameters in these two cases exhibit similar effects as seen with the case of uniform plate velocity. In addition, the effect of parameter a on the velocity profiles in the presence of radiation and pure convection case for an exponentially accelerated plate is presented in Fig. 6 and 7, respectively. It is observed that the velocity increases with increasing values of a.

From Fig. 4 and 5 it is noted that for R = 10 the velocity profile is found to increase 10.74% of the pure convection case near the uniformly accelerated plate at

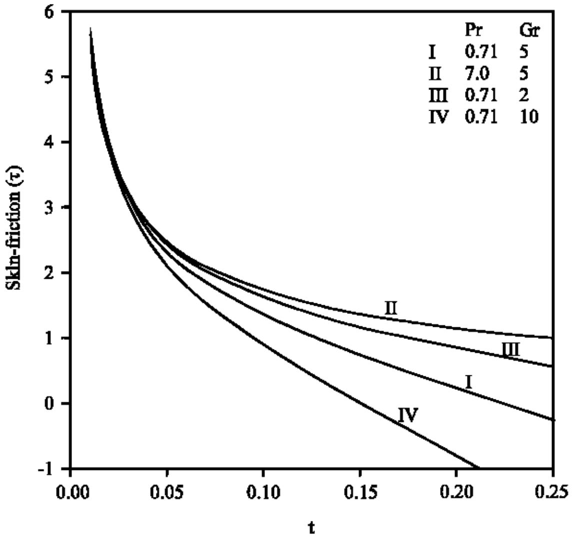

Fig. 9: Skin-friction for different Pr and Gr when R → ∞ due to uniform velocity of the plate

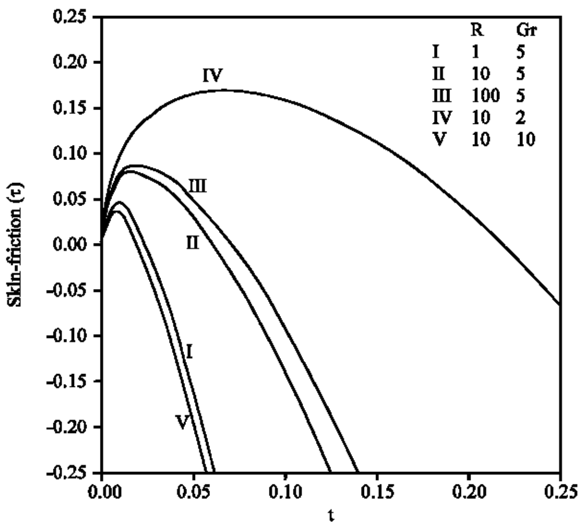

Fig. 10: Skin-friction for different R and Gr when Pr = 0.71 due to uniformly accelerated plate

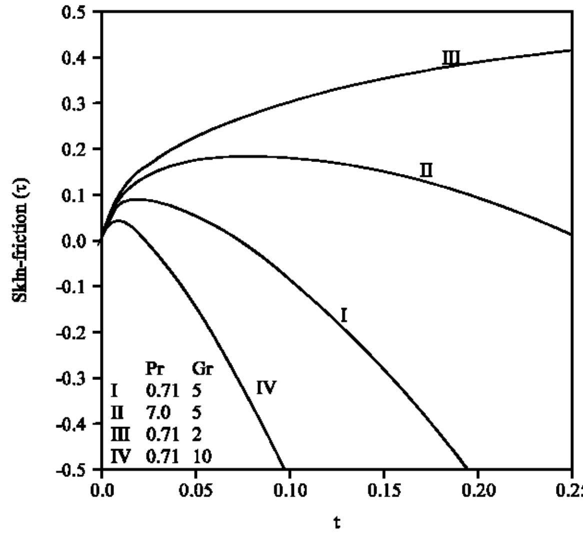

Fig. 11: Skin-friction for different Pr and Gr when R → ∞ due to uniformly accelerated plate

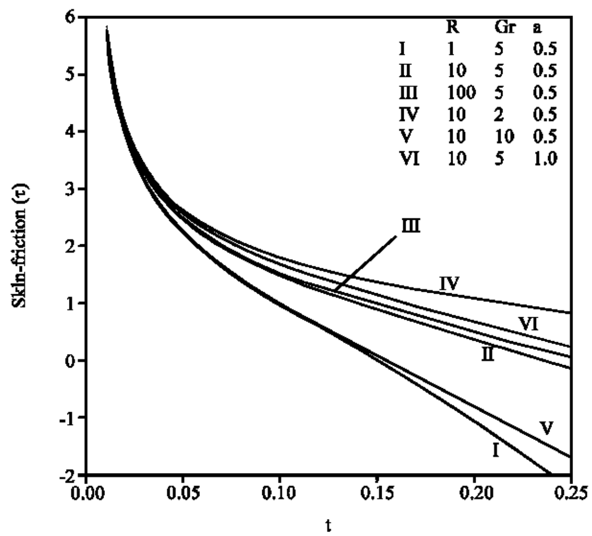

Fig. 12: Skin-friction for different R, Gr and a when Pr = 0.71 due to exponentially accelerated plate

y = 0.25 when t = 0.4, Pr = 0.71 and Gr = 5 whereas it is 5.33% for an exponentially accelerated plate from Fig. 6 and 7.

From Fig. 2 to 7 it is clear that, at low values of t, the velocity distribution is monotonic, but at a higher time it passes through a maximum when the buoyancy effect partly suppresses the inertial effects of the plate velocity. Moreover, close observation of these velocity profiles reveals that, at a higher time, the maximum fluid velocity occurs closer to an exponentially accelerated plate as compared to other two cases. Among the remaining two cases the maximum fluid velocity occurs closer to the plate with uniform velocity than the uniformly accelerated plate.

The skin-friction variation with time is shown in Fig. 8, 10 and 12 for different R and Gr when Pr = 0.71 in all the three cases of plate motion. It is observed that the skin-friction is increasing with increasing R whereas it decreases with increasing value of Gr in the presence of radiation. In the case of pure convection, the variation of skin-friction with time is shown in Fig. 9, 11 and 13 for different values of Pr and Gr in all the three cases of plate motion. It is observed that the skin-friction is increasing

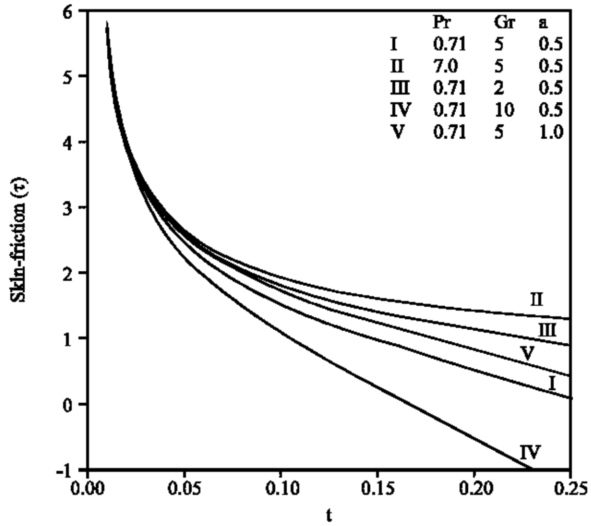

Fig. 13: Skin-friction for different Pr, Gr and a when R → ∞ due to exponentially accelerated plate

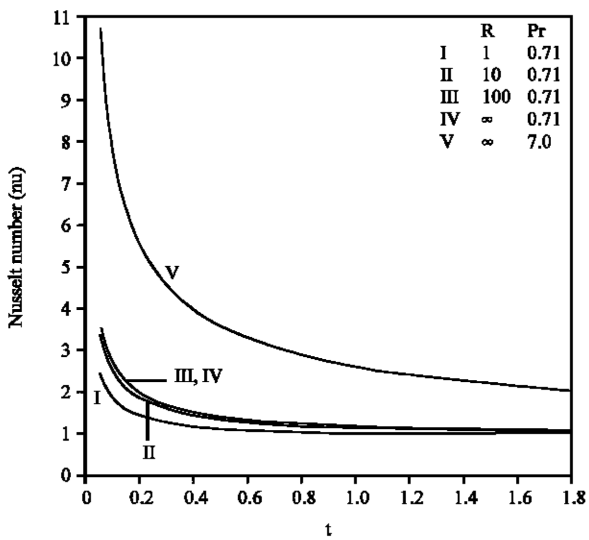

Fig. 14: Nusselt number variation for different R and Pr

with increasing value of Pr whereas it decreases with increasing value of Gr. A large Prandtl number implies more prominent viscous effects causing an enhanced frictional force. In addition, the effect of parameter a on the skin-friction variation with time for an exponentially accelerated plate is shown in Fig. 12 and 13 in the presence of radiation and in the pure convection case respectively. It is seen that the skin-friction increases with an increase in the value of a. It is also interesting to note that these variations are not significant for small values of t.

Moreover, in the case of uniform velocity plate and exponentially accelerated plate, the skin-friction is decreasing with increasing values of t and it is also noted that there is a sharp decrease in the skin-friction at small values of t. In the case of uniformly accelerated plate, there is a sudden increase in the skin-friction for small values of t and it is decreasing gradually with increasing values of t. That is, the skin-friction curves assume parabolic shapes with the variation of time.

From Fig. 8 to 13 it is interesting to note that the value of the skin-friction becomes negative after some time, which means that there occurs a reverse type of flow near the moving plate after some time. Physically this is possible as the motion of the fluid is due to plate motion in the upward direction against the gravitational field. Thus it can be expected that large values of Grashof number may cause separation of the flow even at small time values.

The Nusselt number variation with time is shown in Fig. 14 for various values of R and Pr. It is observed that the Nusselt number increases with increasing R and this variation is significant only for small values of t. In the case of pure convection, it is noted that the Nusselt number increases owing to an increase in the value of Pr. This is evident from the fact that, for increasing values of Pr, frictional forces become dominant and hence yield greater heat transfer. Furthermore, as time advances, the value of the Nusselt number is decreasing and after some time it becomes constant.

CONCLUSIONS

Effects of radiation with Rosseland approximation on free convection boundary layer flow past a moving vertical infinite plate with Newtonian heating have been studied theoretically. Three important cases of flow due to: (1) motion with uniform velocity, (2) uniformly accelerated motion and (3) exponentially accelerated motion of the plate have been considered. The dimensionless governing linear partial differential equations are solved analytically using the Laplace transform technique. Numerical calculations have been carried out for different parameters such as radiation parameter R, Grashof number Gr, Prandtl number Pr, a and time t. The following conclusions are drawn from the present investigation:

- The momentum and thermal boundary layers are found to thicken when the radiation is present
- The temperature of the fluid decreases with increasing value of R and it increases with increasing t in the presence of radiation. In the case of pure

convection, the temperature of the fluid is decreasing with increasing value of Pr

- In all the three cases, the velocity of the fluid decreases with increasing value of R whereas it increases with increasing value of Gr. In the case of pure convection, the velocity decreases with increasing value of Pr
- For an exponentially accelerated plate, the fluid velocity is increasing with increasing value of the parameter a
- In all the three cases, the skin-friction increases with increasing value of R whereas it decreases with increasing value of Gr. In case of pure convection, the skin-friction increases with increasing value of Pr
- For an exponentially accelerated plate, the skin-friction increases with an increase in the value of the parameter a
- The Nusselt number increases with increasing R for small values of *t*. In the case of pure convection, the Nusselt number increases owing to an increase in the value of Pr

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