Optimum Tilt Angle and Orientation of Stand-Alone Photovoltaic Electricity Generation Systems for Rural Electrification

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Abstract: The effect of tilt angle and orientation of Photovoltaic (PV) modules on the performance of Stand-alone Photovoltaic Electricity Generation Systems (SPVEGS) for rural electrification was investigated in this study. In order to have maximum irradiation converted into electricity, both the tilt angle and orientation of the PV surface should be optimum. It is found that the total irradiation captured on a tilted surface mounted in Ipoh, Malaysia can be maximized by positioning the PV modules at the monthly optimum tilt of $\beta_{\text{optimal}} - \phi - \delta$, facing North ($\gamma = 180^\circ$) for the months April to August and facing South ($\gamma = 0^\circ$) for the rest of the months throughout the year. The total irradiation on a tilted surface calculated in this study considered the beam, diffuse and reflected components of the radiation on a tilted surface. The total irradiation incident on surfaces tilted at $\beta - \phi, \beta - 0^\circ$ and $\beta - |\phi| - \delta$ were also investigated. A gain of 6.4 and 6.1%, respectively is achieved by positioning the PV modules at the monthly optimum tilt and orientation as compared to keeping them fixed throughout the year at horizontal ($\beta = 0^\circ$) or at latitude ($\beta = \phi$). PV modules maintained at a monthly tilt of $\beta = |\phi| - \delta$ with South facing orientation at all times accounted for a loss of approximately 5.3% in total tilted irradiation as compared to tilting the PV modules to $\beta_{\text{optimal}}$ with North and South facing surfaces. This indicates that the orientations of PV modules are equally as important as tilt angles.

Key words: Tilt angle, photovoltaic electricity, generation systems, rural electrification, PV module surface

INTRODUCTION

The surface of earth is receiving approximately $10^{24}$ kWh of sunlight every year (Sterling, 2007) but is only using approximately 1/6000 of that amount (World Energy Council, 2007). In Malaysia, the total electrical energy consumption for the year 2005 was approximately 84,517 GWh (http://medis.ptm.org.my/). This figure is expected to increase significantly in years to come as the nation pushes forward towards achieving the status of a developed nation. The predominant method of generating electricity currently, which is by fossil fuel combustion, if pursued to meet the growing energy demand will only worsen the existing climate and environmental problems.

Realizing the need to create a more optimum fuel mix, the Malaysian government has identified renewable energy as the fifth major fuel resource for electricity generation. The government is especially interested in the utilization of solar energy for electricity generation since the energy resource is renewable and abundantly available all throughout the year. Besides that, solar electricity itself is a low-carbon method of electricity generation due to a one-off carbon footprint established only during PV module production and is carbon neutral thereafter. Some of the projects that have been initiated to promote utilization of solar energy are Suria 1000 and Small Renewable Energy Power Programme (SREP).

As there are still large rural areas here in our nation that does not have electricity, a SPVEGS is an extremely relevant option for electrification. The SPVEGS produces power by using PV modules to convert energy from the sunlight into electricity. Figure 1 shows the monthly mean of daily solar irradiation incident in Ipoh sourced from the Malaysian Meteorological Department. This data proves the viability of solar energy as an energy resource for the implementation of an EGS since a good amount of radiation is received all around the year.

Since solar technology at present is quite expensive, it is extremely important to capture the maximum amount of sunlight to operate the SPVEGS efficiently as well as avoid over sizing the system.

In order to have maximum sunlight converted into usable energy, the tilt and orientation angle of the PV surface should be optimum. This is due to the fact that tilt and orientation of the PV surface determines the amount on sunlight incident on the surface and this in turn affects the amount of electricity that can be generated.

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Fig. 1: Monthly mean of daily solar irradiation

The PV surface should be positioned in a way that it is aimed directly perpendicular to the sun’s rays (Messenger and Ventre, 2004). This will capture the maximum amount of sunlight to be converted into electricity. This tilt angle and orientation can be easily achieved using a tracking system that follows the sun’s trajectory at a particular time and day. However, a tracking system is costly and requires high maintenance. Therefore, various studies have been conducted on the optimum tilt and orientation angle for fixed surfaces. There has been some earlier literature that suggested positioning PV modules to face South (\(d = 0^\circ\)) at certain tilt angles for locations in the Northern Hemisphere (Guntherhan and Hepbasli, 2007; Soulaiman, 1991; Gopinathan, 1991). Among the tilt angle recommendations in previous studies are \(\phi\) (Guntherhan and Hepbasli, 2007; Soulaiman, 1991; Gopinathan, 1991), \(\phi \pm 45^\circ\) (Lewis, 1987), \(\phi \pm 15^\circ\) (Lunde, 1980) and \((\phi \pm 15^\circ) \pm 15^\circ\) (Duffie and Beckman, 1991) where, \(\phi\) is the latitude of the location, plus (+) sign is used in winter and minus (-) sign is used in summer. These tilt angle suggestions are mainly for South oriented PV modules at specific locations (\(d = 0^\circ\)) with seasonal climate variation. North oriented PV modules for tropical countries in the Northern hemisphere such as Malaysia require separate consideration due to its geographical location and climate.

Therefore, the goal of this study was to maximize the amount of solar radiation incident on PV modules in Malaysia throughout the year. Theoretical calculations were performed to determine optimum tilt angle, \(\theta_{\text{optimum}}\) and orientation of PV modules, either North (\(d = 180^\circ\)) or South facing (\(d = 0^\circ\)), in particular months.

**THEORY AND METHODOLOGY**

This section describes the process of estimating the total solar irradiation incident on a tilted surface which involves knowledge of solar geometry. Figure 2 shows the overall process workflow for estimating the total solar irradiation incident on a tilted surface.

**Solar geometry:** The position of a PV module is defined by its tilt angle, \(\beta\) and orientation, expressed as the azimuth angle, \(d\). Figure 3 shows a fixed PV module facing due South and tilted at an angle \(\beta\). \(\beta\) is the angle between the tilted surface and the horizontal whereas \(d\) is the angle between the normal of the tilted surface projected on a horizontal surface from the local meridian. A South facing surface has \(d = 0^\circ\) and North facing surface has \(d = 180^\circ\).

The angle of declination, \(\delta\) is the angular displacement of the sun to the centre of the Earth and is given in degrees by:

\[
\delta = 23.45 \sin \left( \frac{360(284 + m)}{365} \right)
\]

where, \(m\) represents a day in a year i.e., \(m = 1\) is 1st of January. \(\delta\) varies between 23.5 to 23.5\(^\circ\) throughout the year as the earth revolves around the sun. \(\delta = 0^\circ\) during an equinox, 23.5\(^\circ\) during the Northern summer solstice and 23.5\(^\circ\) during the Southern summer solstice.

The solar hour angle, \(\omega\), is the angular position of the sun to the east or west of the local meridian. It is positive for time after solar noon and negative for time before solar noon where \(t_e\) is for the \(h\) and \(t_m\) for the min. \(\omega\) is estimated by:

\[
\omega = (0.25t_n + 15t_e - 180)^\circ
\]
The sunset hour angle, $\omega_s$ is the solar angle corresponding to the time when the sun sets that is when $\omega_s$ is 90°. $\omega_s$ is calculated by:

$$\omega_s = \cos^{-1}(-\tan \varphi \tan \delta) \quad (3)$$

The angle of incidence, $\theta$ is the angle between the beam radiation incident on a surface and the normal. For a fixed surface facing North or South, $\theta$ is given by:

$$\theta = \cos^{-1} \left[ a - b + c + d \right] \quad (4)$$

Where,

$$a = \sin \delta \sin \varphi \sin \beta \quad (5)$$

$$b = \sin \delta \cos \varphi \sin \beta \cos \gamma \quad (6)$$

$$c = \cos \delta \cos \varphi \cos \beta \cos \omega \quad (7)$$

$$d = \cos \delta \sin \varphi \sin \beta \cos \gamma \cos \omega \quad (8)$$

Zenith angle, $\theta_z$ is the angular displacement between the line of the sun's rays to the vertical and is given by:

$$\theta_z = \cos^{-1} \left( \cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi \varphi \right) \quad (9)$$

During solar noon, the radiation is at its highest point. At this time, the path length the sun rays travel through the atmosphere is the shortest. Thus, it is desirable to tilt the PV modules at an angle where its plane is perpendicular to the sun at solar noon. This optimum tilt angle, $\beta_{\text{optimum}}$ for the PV modules is found to be $(\phi-\delta)$ which is equal to $\theta_z$ at solar noon. Figure 4 shows that the tilt angle $\beta_{\text{optimum}}$ provides for a plane perpendicular to $\theta_z$ at solar noon (Messenger and Ventre, 2004). The PV modules are oriented to the South $(\gamma = 0^\circ)$ when the $\beta_{\text{optimum}}$ is positive and faced North $(\gamma = 180^\circ)$ when the $\beta_{\text{optimum}}$ is negative.

Irradiation on of a tilted PV module surface: The hourly total radiation incident on a tilted surface consists of three components, i.e., the beam radiation, $I_b$ diffuse radiation, $I_d$ and the solar radiation reflected from the ground $I_r$. Equation 10 is used for calculating the total global solar radiation on a tilted surface, $I_t$.

$$I_t = I_r R_0 + I_d R_d + I_b \left( \frac{1-\cos \beta}{2} \right) \quad (10)$$

$$I_t = I_r$$

The values of $I_b$ and $I_d$ can be calculated by the following formulas:

$$I_b = \tau_r R_b \quad (11)$$

$$I_d = \tau_r R_d \quad (12)$$

$$I_b = I - I_d \quad (13)$$

$$I_t = I_b \quad (14)$$

where, $\tau_r$ is the monthly average daily global radiation and $I$ is the global horizontal hourly irradiation. $\tau_r$ is the ratio of hourly total to daily total global radiation and is estimated by:

$$\tau_r = \frac{\pi}{24} \left( a + b \cos \omega \cos \varphi - \cos \omega \cos \theta \right) \quad (15)$$

Where,
\[ a = 0.409 + 0.5016 \sin \left( \alpha - \frac{\pi}{3} \right) \]  
(16)

\[ b = 0.6609 + 0.4767 \sin \left( \alpha - \frac{\pi}{3} \right) \]  
(17)

where, \( r_d \) is the ratio of hourly total to daily total diffuse radiation and is estimated by:

\[ r_d = \frac{\pi}{24} \sin \alpha - \cos \alpha \]  
(18)

The ground reflectance \( \rho \) of 0.2 is used for the location under study as it falls under category of areas without snow cover and with temperatures of 0°C and above (Liu and Jordan, 1960). The ratio of beam radiation on the tilted surface to that of the horizontal, \( R_b \), and ratio of diffuse radiation on the tilted surface to that of the horizontal, \( R_d \), are given by:

\[ R_b = \frac{\cos \theta}{\cos \alpha} \]  
(19)

\[ I_b = I_s \]  

\[ R_d = \frac{1 + \cos \beta}{2} \]  
(20)

The clearness index, \( \bar{K} \), for locations located between latitude, \( \phi = 0^\circ \) and \( \phi = 8^\circ \) (\( 0^\circ \leq \phi \leq 8^\circ \)) is given by (Balbir, 2004):

\[ \bar{K} = 0.601 - (0.199e^{-0.009\phi - 0.04}) \]  
(21)

The monthly mean daily diffuse radiation, is then estimated using the calculated as the following correlation (Page, 1961):

\[ \bar{H}_d = 0.113 \bar{K} \bar{H} \]  
(22)

Then, the total tilted irradiation for the day, \( \bar{H}_t \), is determined by summing the hourly tilted irradiation for of the day and is given by:

\[ \bar{H}_t = \Sigma \bar{H}_t \]  
(23)

**RESULTS AND DISCUSSION**

The total irradiation incident on a tilted surface of PV module is determined by its tilt angle, \( \beta \) and orientation. It is important to note that the azimuth angle, \( \gamma \) defines the orientation of the PV modules. A positive \( \beta \) indicates that the PV modules are facing South (\( \gamma = 0^\circ \)) and a negative \( \beta \) indicates that the PV modules are facing North (\( \gamma = 180^\circ \)). The location in this study is at latitude, \( \phi = 4.57^\circ \)N (Ipoh, Malaysia).

Figure 5 shows the variation of daily tilt angle plotted for three different cases i.e: \( \beta = 0^\circ \), \( \beta = \phi \) and \( \beta_{optimal} = \phi - \delta \), over the course of one year. The apparent movement of the sun each day gives rise to the daily change in tilt angle for the case \( \beta_{optimal} = \phi - \delta \). The tilt angle is fixed at \( \beta = 0^\circ \) and \( \beta = 4.57^\circ \) for days \( n = 1 \) to \( n = 365 \) for cases \( \beta = 0^\circ \) and \( \beta = \phi \), respectively. For case \( \beta_{optimal} = \phi - \delta \), the tilt angle is negative for days, \( n = 93 \) to \( n = 252 \) indicating the PV modules are facing North. As for the other days around the year, the tilt angles are positive indicating PV modules facing South.

It is rather impractical to change the tilt angle daily as it will incur tedious mechanical work. Therefore, it is better to vary the tilt angle monthly. The variation of monthly tilt angle is shown in Fig. 6 and Table 1 for \( \beta = 0^\circ \), \( \beta = \phi \) and \( \beta_{optimal} = \phi - \delta \). For the case of \( \beta_{optimal} = \phi - \delta \), it is evident that the tilt angle from April to August at Ipoh is negative and this indicates that the PV modules are facing North. For the rest of the months, the module is facing South. The tilt angle for the other cases (i.e. \( \beta = 0^\circ \) and \( \beta = \phi \)) stays fixed throughout the whole year similar to the daily tilt angle variation at \( \beta = 0^\circ \) and \( \beta = 4.57^\circ \).

![Fig. 5: Variation of daily tilt angle](image)

![Fig. 6: Variation of monthly tilt angle](image)
Table 1: Variation of monthly tilt angle

<table>
<thead>
<tr>
<th>Month</th>
<th>$\beta_{\text{optimum}} = d-\delta$</th>
<th>$\beta = \phi$</th>
<th>$\beta = 0^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>25.5</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>Feb.</td>
<td>17.5</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>Mar.</td>
<td>7.0</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>Apr.</td>
<td>-4.8</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>May.</td>
<td>-14.2</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>June</td>
<td>-18.5</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>July</td>
<td>-16.6</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>Aug.</td>
<td>-8.9</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>Sept.</td>
<td>2.4</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>Oct.</td>
<td>14.2</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>Nov.</td>
<td>23.5</td>
<td>4.6</td>
<td>0</td>
</tr>
<tr>
<td>Dec.</td>
<td>27.7</td>
<td>4.6</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Variation of monthly total irradiation on tilted PV surface

<table>
<thead>
<tr>
<th>Month</th>
<th>$I_0$ (kWh m$^{-2}$)</th>
<th>$I_0$ (kWh m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($\beta_{\text{optimum}} = d-\delta$)</td>
<td>($\beta =</td>
</tr>
<tr>
<td>Jan.</td>
<td>5.4</td>
<td>5.4</td>
</tr>
<tr>
<td>Feb.</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Mar.</td>
<td>4.8</td>
<td>4.8</td>
</tr>
<tr>
<td>Apr.</td>
<td>4.8</td>
<td>4.7</td>
</tr>
<tr>
<td>May.</td>
<td>5.0</td>
<td>4.3</td>
</tr>
<tr>
<td>June</td>
<td>5.1</td>
<td>4.0</td>
</tr>
<tr>
<td>July</td>
<td>5.1</td>
<td>4.2</td>
</tr>
<tr>
<td>Aug.</td>
<td>4.8</td>
<td>4.6</td>
</tr>
<tr>
<td>Sept.</td>
<td>4.7</td>
<td>4.7</td>
</tr>
<tr>
<td>Oct.</td>
<td>4.9</td>
<td>4.9</td>
</tr>
<tr>
<td>Nov.</td>
<td>5.3</td>
<td>5.3</td>
</tr>
<tr>
<td>Dec.</td>
<td>5.5</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Fig. 7: Variation of monthly total irradiation on a tilted PV surface

The monthly total irradiation on a tilted surface for $\beta = 0^\circ$, $\beta = \phi$ and $\beta_{\text{optimum}} = |\phi-\delta|$ is shown in Fig. 7 and Table 2. The surface tilted at $\beta_{\text{optimum}}$ provides for the largest amount of irradiation from January to December. PV modules positioned at horizontal ($\beta = 0^\circ$) gives consistent total tilted irradiation for all the months of a year, from January to December. On the other hand, the total tilted irradiation of PV modules positioned at the latitude of location ($\beta = \phi$) suffers a dip from April to August but manages to provide a slightly higher irradiation than tilted to horizontal in other months.

There is a gain of almost 6.4% when PV modules are positioned at the monthly $\beta_{\text{optimum}}$ as compared to positioned at horizontal. It is no doubt easier to position the PV modules at horizontal all throughout the year but it comes at the expense of reduced total irradiation on a tilted surface. Conversely, PV surfaces that were positioned to latitude showed a loss in total tilted irradiation of approximately 6.1% as compared to surfaces that were tilted to monthly $\beta_{\text{optimum}}$.

The results suggest that it is desirable to position the PV modules at $\beta_{\text{optimum}}$ as the surface faces South from January to March and then again from September to December. During April to August, the PV modules should be positioned to the North to maximize the amount of irradiation received on the surface. It is noted here that it is not necessary for all locations in the Northern hemisphere to face South at all times as reported in some literatures. Figure 8 shows the variation of monthly tilt angle between PV surfaces facing South only with $\beta = |\phi-\delta|$ and also facing both North and South with $\beta_{\text{optimum}} = \phi-\delta$.
The variation of monthly total irradiation for cases \( \beta = |\phi - \delta| \) and \( \beta_{\text{optimum}} = \phi - \delta \) is shown in Fig. 9 and tabulated in Table 2. This result further proves the findings of this study whereby PV modules should face North in months April to August and South the rest of the months. PV modules maintained at \( \beta = |\phi - \delta| \) faced South all the time and accounted for a loss of approximately 5.3% in total tilted irradiation as compared to tilting the PV modules to \( \beta_{\text{optimum}} \).

PV modules maintained at \( \beta = |\phi - \delta| \) faced South all the time and accounted for a loss of approximately 5.3% in total tilted irradiation as compared to tilting the PV modules to \( \beta_{\text{optimum}} \). This goes to show that positioning the PV module at the proper tilt angle alone is not sufficient to maximize the irradiation incident on a tilted surface. The orientation of the PV modules is just as important.

CONCLUSION

The amount of electricity generated by the SPVEGS is dependent on the amount of irradiation incident on the surface of the PV modules. The greater the amount of irradiation captured, the greater the electricity generated. In order to have maximum irradiation converted into electricity, the tilt and orientation angle of the PV surface should be optimum. It is found by theoretical calculation, that maximum irradiation can be captured in Ipoh, Malaysia by positioning the PV modules at the monthly optimum tilt of \( \beta_{\text{optimum}} = \phi - \delta \), facing South \( (\gamma = 0^\circ) \) when \( \beta_{\text{optimum}} \) is positive or facing North \( (\gamma = 180^\circ) \) when \( \beta_{\text{optimum}} \) is negative. It is suggested that the PV modules face North for months April to August and South for the other months of the year contrary to conventional literature that suggests location in the Northern hemisphere have South facing PV modules all the time. A gain of 6.4 and 6.1%, respectively is achieved by positioning the PV modules at the monthly optimum tilt and orientation as compared to keeping them fixed throughout the year at horizontal \( (\beta = 0^\circ) \) or at latitude \( (\beta = \phi) \).

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