Electromagnetic Wave Propagation in Microstrip Transmission Lines

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Abstract: We present a novel and accurate approach to compute the loss of electromagnetic waves propagating in a Microstrip line. A set of transcendental equation is derived by matching the tangential fields with the surface impedance at the dielectric-air and dielectric-conductor interfaces. The propagation constant is obtained by numerically solving for the root of the equation and substituting the values into the dispersion relation. We found that the loss predicted by our method, though appears to be somewhat higher, is nevertheless still considered to be in agreement with those from the quasi-static method. In our analysis, we also showed that the phase velocity varies with frequencies indicating dispersive effect in the microstrip lines. Since the quasi-static method assumes pure TEM mode of propagation, while our method takes into consideration the coexistence of TE and TM modes, we attribute the higher loss as due to the presence of the longitudinal fields and dispersive effect in a lossy microstrip line.

Key words: Microstrip line, propagation constant, surface impedance, phase velocity, dispersive effect

INTRODUCTION

Microstrip transmission lines have been widely used in Microwave Integrated Circuits (MIC), such as filters (Hsu et al., 2005; Ahn et al., 2001; Hong and Lancaster, 1997), couplers (Brenner, 1967; Brenner, 1967; Campbell et al., 2003) and mixers (Wengler, 1992; Tuckman and Feldman, 1985), etc. At low frequencies where the dimensions of the microstrip structure are much smaller than the wavelengths of the signals, the fundamental HE, mode resembles closely a TEM wave (Zysman and Varon, 1969). Thus, electrostatic approximations such as the conformal mapping technique (Wheeler, 1964; Wheeler, 1965; Astradouri and Rimai, 1952; Pueel et al., 1968; Pueel et al., 1968a) have been commonly used to analyze the propagation of waves in the structure. As experimentally verified in (Grunberger et al., 1970; Grunberger and Meine, 1971), however, the solutions of these approximation methods deviate from the measurement results at high frequencies. This is because, in reality, the nature of wave propagation is a superposition of both TE and TM modes and the presence of the longitudinal fields cannot be neglected at high frequencies.

Although Mittra and Itoh have considered the co-existence of the hybrid modes using the Spectral Domain Approach (SDA) (Mittra and Itoh, 1971; Itoh and Mittra, 1973, 1974), they have assumed the thickness of the strip to be infinitesimally thin. Hence, their method is only applicable in cases where the thickness of the strip (l0) is much smaller than the height of the dielectric substrate (s).

In this study, we present a rigorous analysis which incorporates the finite thickness of the strip and ground plane of a microstrip structure. In our method, the superposition of both TE and TM modes are taken into account during formulation. A set of transcendental equation is derived by matching the tangential fields with the surface impedance at the boundaries of the structure. By solving for the root of the equation and substituting the values into the dispersion relation, we are able to compute the attenuation constant of the propagating wave.
wave. We will demonstrate that our method gives more realistic results as it incorporates the non-TEM characteristics and dispersive nature of the propagating mode.

FORMATION

**Fields in the substrate:** As shown in Fig. 1, the microstrip structure that we analyze here is assumed to be enclosed by a pair of perfectly conducting walls at both ends of the substrate at \( x = a/2 \) and \(-a/2\). The width of the substrate \( a \) is taken to approach infinity so that the fields localized at the strip will not be perturbed by the wall and, thus, the strip conductor resembles closely to that of an open microstrip structure.

In a lossless microstrip line, the boundary condition requires that the tangential electric fields \( E_v \) and the normal derivative of the tangential magnetic fields \( \delta H_v / \delta a \) to vanish at the boundary of the conductor. Here, \( \alpha \) is the normal direction to the conductor wall. Due to the finite conductivities of the strip and groundplane, however, both \( E_v \) and \( \delta H_v / \delta a \) do not decay to zero at the boundary. However, \( E_v \) and \( \delta H_v / \delta a \) at the boundary of a highly conducting strip and groundplane are very small and are only slightly perturbed from the lossless solution. For a microstrip structure having equivalent surface impedance at the boundary of the strip-substrate and groundplane-substrate interfaces, respectively, the skin depth of the fields penetrating into the conductor are the same. Hence, applying the boundary conditions for the fundamental HE mode of the microstrip line at the substrate-conductor interface and solving Helmholtz's homogeneous equation in Cartesian coordinate (Pozar, 2005), the longitudinal fields can be derived as:

\[
E_v = E_0 \cos \left( \frac{\pi x}{a} \right) \cos(k_y y) \quad (1)
\]

\[
H_v = H_0 \sin \left( \frac{\pi x}{a} \right) \sin(k_y y) \quad (2)
\]

where, \( E_0 \) and \( H_0 \) are constant coefficients of the fields; while \( k_y \) is the transverse wavenumber in the \( y \) direction. The usual wave factor in the form of \( \exp[j(\omega t - k_z z)] \) is omitted. Here, \( \omega \) is the angular frequency and \( k_z \) is the propagation constant. \( k_z \) is a complex variable which comprises a phase constant \( \beta_z \) and an attenuation constant \( \alpha_z \) and can be expressed as:

\[
K_z = \beta_z - j\alpha_z \quad (3)
\]

![Fig. 1: The cross section of a microstrip structure](image)

The transverse field components \( E_x \) and \( H_z \) can be derived by substituting the longitudinal fields into Maxwell's source-free curl equations:

\[
\nabla \times E = -j\omega \mu E \quad (4)
\]

\[
\nabla \times H = j\omega \epsilon E \quad (5)
\]

where, \( \mu \) and \( \epsilon \) are the permeability and permittivity of the dielectric substrate, respectively. Hence, substituting Eq 1 and 2 into 4 and 5 and expressing the transverse field components in terms of \( E_x \) and \( H_z \) we obtain:

\[
H_z = \frac{j(\epsilon k_y E_v - k_z k_h H_z)}{h^2} \cos \left( \frac{\pi x}{a} \right) \cos(k_y y) \quad (6)
\]

\[
E_x = \frac{j(k_z k_h E_z + \epsilon k_z H_z)}{h^2} \sin \left( \frac{\pi x}{a} \right) \sin(k_y y) \quad (7)
\]

where, \( h^2 = k_x^2 + k_z^2 \).

**Derivation of the transcendental equation:** At the boundary of the conductors, the tangential fields are related through the surface impedance \( Z_s \) by (Tham et al., 2003; Yeap et al., 2009):

\[
Z_s = \frac{-E_v}{(a_x \times H_z)} \quad (8)
\]

For the strip and groundplane fabricated using the same material and having the same thicknesses, the surface impedance are equivalent. Hence, the surface impedance \( Z_s \) can be expressed in terms of the constitutive relations as (Kerr, 1999):

\[
Z_s = \frac{jk_z}{\sigma_z} \left[ \frac{\exp(jk_z t_x) + \sigma Z_n - jk_z \exp(-jk_z t_x)}{\sigma Z_n + jk_z} - \frac{\exp(jk_z t_x) + \sigma Z_n - jk_z \exp(-jk_z t_x)}{\sigma Z_n + jk_z} \right] \quad (9)
\]
where, \( Z_a \) is the intrinsic impedance of free space, \( \sigma \) and \( \sigma_0 \) the conductivities, \( t \) and \( t_a \) the thicknesses, and \( k \) and \( k_0 \) the wavenumbers in the strip and groundplane, respectively.

The total surface impedance \( Z_s \) of the microstrip structure can be computed by integrating Eq 8 from \( x = a/2 \) to \(-a/2\) at \( y = s/2 \) and \(-s/2\), respectively. From Eq 8, \( Z_s = -E_z/H_z = E_z/H_z \) at \( y = \pm s/2 \). Thus, we have:

\[
\begin{align*}
\int_{-a/2}^{a/2} \frac{-Ex(y = s/2)}{Hz(y = s/2)} dx + \int_{-a/2}^{a/2} \frac{Ex(y = -s/2)}{Hz(y = -s/2)} dx \\
= 2 \int_{-a/2}^{a/2} Z_w(y = s/2) dx + 2 \int_{-a/2}^{a/2} Z_w(y = -s/2) dx
\end{align*}
\]

(11)

\[
\begin{align*}
\int_{-a/2}^{a/2} \frac{-H_z(y = s/2)}{E_z(y = s/2)} dx + \int_{-a/2}^{a/2} \frac{-H_z(y = -s/2)}{E_z(y = -s/2)} dx \\
= 2 \int_{-a/2}^{a/2} \frac{Z_w(y = s/2)}{Z_w(y = -s/2)} dx + 2 \int_{-a/2}^{a/2} \frac{1}{Z_w(y = s/2)} dx + 2 \int_{-a/2}^{a/2} \frac{1}{Z_w(y = -s/2)} dx
\end{align*}
\]

(12)

Here, we assume that the tangential fields at the air region decay almost instantaneously. Thus, \( Z_s = Z_w \) in Eq. 8 at the substrate-air boundary.

Substituting the field equations Eq. 1, 2, 6 and 7 into 11 and 12, we obtain:

\[
\begin{align*}
\left[ \frac{2\pi}{h} k \tan \left( \frac{k_s s}{2} \right) \right] E_s = \\
\left[ (w-a)Z_a - aZ_e - wZ_m - \frac{2\pi}{h^2} \sigma_0 k \tan \left( \frac{k_s s}{2} \right) \right] H_s
\end{align*}
\]

(13)

\[
\begin{align*}
\left[ \frac{2\pi}{h} k \cot \left( \frac{k_e s}{2} \right) \right] H_s = \\
\left[ (w-a)Z_n - wZ_e - \frac{2\pi}{h} \sigma k_0 \cot \left( \frac{k_e s}{2} \right) \right] E_s
\end{align*}
\]

(14)

Equations 13 and 14 admit nontrivial solution only in case where the determinant is zero. Thus, by letting the determinant of the coefficients \( E_s \) and \( H_s \) vanish, we obtain the following transcendental equation:

\[
\begin{align*}
\left[ \frac{(a-w)}{aZ_n} + \frac{1}{aZ_m} - \frac{w}{a} - \frac{j2\omega \sigma k_0}{h^2 \tan \left( \frac{k_s s}{2} \right)} \right] \left[ 1 - \frac{w}{a} \right] Z_s + \\
\frac{wZ_m + \frac{j2\omega \sigma k_0 \tan \left( \frac{k_s s}{2} \right)}{h^2}}{a} \left[ \frac{k_s s}{2} \right] = \left[ \frac{j2\omega \sigma k_0}{a^2} \right] \left[ \frac{k_s s}{2} \right]
\end{align*}
\]

(15)

In the transcendental equation, \( k_0 \) is the unknown to be numerically solved for, since \( k_0 \) can be expressed in term of \( k_s \) using the dispersion relation in Eq. 16 given below:

\[
k_s = \sqrt{k_0^2 - \left( \frac{\pi}{a} \right)^2 - k_0^1}
\]

(16)

where, \( k_s \) is the wavenumber in the dielectric substrate.

To compute our results, the Powell Hybrid root-searching algorithm in a NAG routine is used to find the root of \( k_s \). The returned values of \( k_s \) depend entirely on the values of the initial guesses given for the search.

Since the fundamental mode of the microstrip line is the HE_00 mode, suitable guesses for \( k_s \) are clearly values close to zero. It is worthwhile noting that the solution did not always converge to the required mode. It was often necessary to refine initial values slightly in order to force convergence to the correct mode. The attenuation constant \( \alpha \) can be obtained by substituting \( k_s \) into Eq. 16 and extracting the imaginary part of \( k_s \) in Eq. 3.

**RESULTS AND DISCUSSION**

In order to validate our formulation, we have calculated the attenuation constant using the transcendental equation in Eq. 15 based on two sets of microstrip parameters arbitrarily chosen from the results by Pucel et al. (1968). Both the strip and groundplane of the microstrip line is made of copper. The attenuation curve as a function of frequency \( f \) for rutile substrate with a dielectric constant \( \epsilon_r = 105 \) is depicted in Fig. 2 and for alumina substrate with \( \epsilon_r = 9.35 \) in Fig. 3. For the rutile substrate, we have taken \( w = s = 508.0 \mu m \), and \( t_b = t_g = 8.382 \mu m \). For the alumina substrate, we have taken \( w = 3.048 \mu m \), \( s = 1.27 \mu m \), and \( t_b = t_g = 990.6 \mu m \). The attenuation constants are compared with those obtained by Pucel et al. (1968) (PMH), derived using the quasi-static methods (Wheeler, 1965; Wheeler, 1964; Assadourian and Ramai, 1952) and Wheeler’s incremental inductance method (Wheeler, 1942). As illustrated in Fig. 2 and 3, the attenuation curves predicted by our
results suggest strongly that our method gives more accurate prediction of loss.

Next, we have also computed the phase velocity \( v_p = \frac{c}{\sqrt{\varepsilon_r}} \) for the microstrip lines with rutile and alumina substrates, respectively. As can be clearly seen in Fig. 4 and 5, the phase velocities vary with frequencies, indicating that the lossy microstrip line is dispersive in nature. In the electrostatic solutions, the phase velocity is approximated as (Pozar, 2005):

\[
v_p = \frac{c}{\sqrt{\varepsilon_r}}
\]

(17)

where, \( c \) is the velocity of light in free space and \( \varepsilon_r \) is a constant variable known as the effective dielectric constant. It is apparent that \( v_p \) in Eq. 17 is independent of the variation in frequency since both \( c \) and \( \varepsilon_r \) are constant variables. Thus, the dispersive effect fails to be accounted for using the quasi-static methods.

**CONCLUSION**

As a conclusion, we have presented a new and fundamental method to compute the propagation constant of waves in a microstrip transmission line. A set of transcendental equation is derived by integrating the total surface impedance at the boundary of the substrate. The phase and attenuation constants can be calculated by numerically solving for the root of the equation and substituting the value into the dispersion relation.

We have validated our results by comparing with those obtained using quasi-static PMH’s equation. Although considered to be in agreement with PMH’s results, we observe that the attenuation constants predicted by our method are somewhat higher. Since our method incorporates the superposition of hybrid modes,
we attribute such discrepancies to the existence of the longitudinal fields and dispersive effect in our results.

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REFERENCES


