Performance Evaluation of Stratified TES using Sigmoid Dose Response Function

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Abstract: Temperature distribution on the operating stratified Thermal Energy Storage (TES) is commonly available as discrete data. Due to this, it is difficult to conduct performance evaluation based on thermocline profile, because limit points could not be determined accurately. This paper discusses a practical method in determining the performance parameters of stratified TES based on thermocline profile. Non linear regression fitting was adopted to identify the function which could represent temperature distribution profile. The function was used to define performance parameters namely limit points of thermocline, thermocline thickness, lost capacity, integrated capacity, theoretical capacity as well as half cycle of Figure of Merit. Results identified a function which could represent S-curve of temperature distribution, namely Sigmoid Dose Response (SDR) equation. The function was observed to fit the temperature distributions having coefficient determination more than 0.99. Based on evaluations the method was capable to be utilized for evaluation of the performance of the stratified TES. The methods offer an advantage to obtain an exact value of performance parameters.

Key words: Temperature distribution profile, stratified TES, performance parameter, sigmoid dose response

INTRODUCTION

Thermal Energy Storage (TES) system offers a load management technology with great potential to shift load from peak to off-peak demand. This is achieved by charging the TES tank during the off-peak hours and discharging it later during the peak hours. During the cycle, water temperature distribution inside the tank changes with respect to time which reflects changing of cooling energy stored in TES.

Many researches related to performance evaluation of stratified TES have been undertaken, both for full scale TES (Musser and Bahnfleth, 1999; Bahnfleth and Musser, 1998; Musser, 1998; Musser and Bahnfleth, 1998; Caldwell and Bahnfleth, 1998) and experimental study (Karim, 2009; Nelson et al., 1999; Walmsley et al., 2009). One method to measure the performance of the stratified TES due to the presence of mixing and conduction of the water is lost capacity (Musser and Bahnfleth, 1999; Musser, 1998). The lost capacity associated with a cycle is that capacity that can not be removed from the tank due to the occurrence of thermocline region between cool and warm water in the tank. Lost capacity in relation to stratified TES tank performance was estimated based on captured continuous profile of water temperature distribution (Musser and Bahnfleth, 1999; Bahnfleth and Musser, 1998; Musser, 1998). Using continuous profile, thermocline region is identified as asymptote regions with limit points located on the edge of profiles. The difficulties of identifying the thermocline region arise if the temperature is available as a discrete data of hourly temperature records. The uncontinuous profiles that are formed could not be used to determine the thermocline thickness due to its ambiguity in defining the limit points. A method to determine performance parameters from discrete data of temperature distribution is investigated in this study. This study focuses on developing an approach of establishing the profile by adopting fitting method and using the profile to determine the parameters based on functional relationship. Data acquired from an operating TES tank equipped with long interval-distance sensors recorded based on hourly basis, was used in this research.

Basic concepts: Water temperature distribution in the stratified TES tank reflects a region of warm water at the top of the tank while cool water region laid in the bottom, with a thermocline region forms in the middle. An S-Curve water temperature profile with respect to TES height is shown in Fig. 1.

Figure 1 shows that, cool water exist at average cool water temperature $T_c$ whereas warm water has average warm water temperature $T_w$. Position of the thermocline is defined as midpoint of thermocline height designated as $C$. The $C$ parameters also define the boundary line of cool and warm water in the tank. Parameter $S$ is slope gradient
water. Thermocline thickness as a function of $\Theta$ and $S$, is expressed by Eq. 3:

$$W_{\infty} = f(\Theta, S)$$  \hspace{1cm} (3)$$

**Lost Capacity ($C_{\text{lost}}$):** Lost capacity is described as a region with boundary line of bottom limit of the thermocline region to the mid point of thermocline. This is represented as the shaded region in Fig. 1. This region can be evaluated using Eq. 4:

$$C_{\text{lost}} = \int_{T_{c}}^{T_{w}} \rho A c_p (T(X) - T_c) \, dx$$  \hspace{1cm} (4)$$

where, $A$ is the area of the tank ($m^2$), $\rho$ is density ($kg \, m^{-3}$) and $c_p$ is specific heat of the water ($kJ \, kg^{-1} \, ^{\circ}C$).

**Integrated Capacity ($C_{\text{int}}$):** Integrated capacity is defined as a useful cooling energy stored in the storage Musser (1998), indicated as hatched area in Fig. 1. The integrated capacity is calculated as temperature difference of warm water temperature to the temperature distribution. This is expressed by Eq. 5:

$$C_{\text{int}} = \int_{T_{c}}^{T_{w}} \rho A c_p (T(X) - T_c) \, dx$$  \hspace{1cm} (5)$$

**Theoretical Capacity ($C_{\text{theo}}$):** The theoretical capacity defines the capacity of storing cooling energy in the absence of mixing and conduction losses Musser (1998). The theoretical capacity is proportional to the mass of water contained, average of warm water temperature and cool water temperature. The theoretical capacity is expressed by Eq. 6:

$$C_{\text{theo}} = \rho A C c_p (T_1 - T_c)$$  \hspace{1cm} (6)$$

The theoretical capacity is also defined as a total summation of integrated and lost capacity as per Eq. 7:

$$C_{\text{max}} = C_{\text{lost}} + C_{\text{int}}$$  \hspace{1cm} (7)$$

**Half-cycle Figure of Merit ($\text{FoM}_{1/2}$):** Half-cycle Figure of Merit reflects the ratio of integrated capacity over the theoretical capacity (Bahnfleth and Musser, 1998). Therefore $\text{FoM}_{1/2}$ can be defined related to integrated and lost capacity, as per Eq. 8:

$$\text{FoM}_{1/2} = 1 - \frac{C_{\text{lost}}}{C_{\text{max}}}$$  \hspace{1cm} (8)$$

**METHODOLOGY**

The temperature distribution in Eq. 1 was used as a basis for determining performance parameters in this
The analysis procedure for determining the performance equations are as follow:

- Acquiring temperature data from an operating TES
- Observation of functions that could represent the S-curve temperature distribution profile. The criteria for selection were based on Eq. 1
- Identifying and fitting the temperature profile function using non linear regression fitting
- Determining the temperature parameters in the function
- Establishing performance equations by solving Eq. 2 to 8
- Evaluation of the performance equations
- Implementation for performance evaluation

Temperature data of the TES system from a co-generated district cooling plant were acquired for this study. The TES system consists of two 1,250 tons of Refrigeration (RT) of Steam Absorption Chillers (SACs) and four 325 RT Electric Chillers (ECs) and one 5,400 m³ storage TES tank with designed capacity of 10,000 Rth. Inlet nozzle is made from 20 NPS located at elevation 3.4 m height, while outlet nozzle is of 12 NPS at elevation end-connection in the storage tank. Overflow line is connected at elevation of 14.025 m. The entire tank is 12.3 m. Both nozzles are provided with diffuser on its externally insulated. The tank is equipped with 14 temperature sensors, installed at approximately 1 m vertical interval, to measure the water temperatures. The lowest temperature sensor is located at 0.51 m height. All temperatures are hourly recorded using acquisition data system. The schematic flow diagram of the system is shown in Fig. 2.

The data acquired in this study were obtained from charging cycle on September 9, 2008. The charging was conducted continuously from 18.00 to 03.00 in the following day. Selection of the observed data was based on constant flow rate supply of 393 m³ h⁻¹ within the charging period.

The plot of temperature distribution is shown in Fig. 3. The hourly temperature distribution within charging periods are presented with respect to sensor elevation in TES tank. Each hourly charging course form a continue S-curve profile. This profile move upward from the initial condition at 18.00 h.

**RESULTS AND DISCUSSION**

**Temperature distribution function:** The plot of observed data as shown in Fig. 3 were used to select temperature distribution function as described in Eq. 1. Selection was performed utilizing commercial software of SigmaPlot (Systat Software Inc., 2008) based on non linear regression fitting. A function which was identified that could represent the temperature distribution profile was Sigmoid Dose Response (SDR) function. The function was formed as a modification from dose response (variable slope) function. The SDR function is represented by Eq. 9:

\[
T = T_o + \frac{T_a - T}{1 + 10^{(c-x)/h}}
\]  

(9)

The SDR function relates temperature distribution to variable of X and parameters of \(T_o\), \(T_a\), C and S. Parameters of \(T_o\) and \(T_a\) are cool and warm temperatures (°C). X variable expresses the dimensionless elevation (x.N H⁻¹), where x is the elevation of the temperature sensors (m), H is effective height of the tank content of water (m) and N is number of stratified layers. Parameter C is a dimensionless elevation unit and S is constant parameter related to slope gradient of the function.

Fig. 2: Schematic flow diagram of charging cycle

Fig. 3: Plotting of observed temperature distribution data
in the Table 1. The values of coefficient of determination, $R^2$, for evaluation the goodness of fitting are also provided in the Table 1.

As shown in Table 1, the values of $T_b$ and $T_c$ decrease with respect to time. The decreased in values of $T_b$ was due to incoming supply of cooler water at the lower section of the tank from the ECs. The decreased in values of $T_b$ was due to conduction across thermocline region. The values of $C$ as shown in the Table 1, increase with charging time. The increasing of $C$ due to increased cool water depth as a result of more cool water generated within charging time. These trends were also noted by Nelson et al. (1999) and Karim (2009) through their experimental investigations on stratified tanks.

**Performance equations:** Determination of the performance parameters were conducted based on SDR function as shown in Eq. 9. Equation 2 and 3 were solved by rearranging of Eq. 9 to express parameter of bottom limit points and thermocline thickness. In addition, solving of Eq. 4 and 6 was conducted by replacing term of $T(X)$ in Eq. 9. Therefore, Eq. 4 to 6 emerges as integral formula with variable of $X$. The solutions are presented as Eq. 10 to 13 in the Table 2.

**Evaluation of the performance equations:** Temperature parameters obtained from the fitting, as shown in Table 1, were used to calculate bottom limit point of thermocline ($B$), thermocline thickness ($W_{TC}$), lost capacity ($C_{Loss}$), integrated capacity ($C_{int}$), theoretical capacity ($C_{theo}$), as well as half-cycle Figure of Merit (FoMR). Evaluation used a pre-determined value of dimensionless cut-off ratio. The dimensionless cut-off ratio values can be selected in the range of 0 to 0.5. The $\Theta$ at 0 value indicates maximum limit point, where bottom limit located at $T_b$ and $\Theta$ at 0.5 indicates the upper bottom limit is located in the position of thermocline ($C$).

Calculation was conducted utilizing Eq. 6 to 8 and Eq. 10 to 13 using data from Table 1. The calculation using $\Theta$ of 0.0001 with values of $A = 390.37$ m², $\rho = 1000$ kg m⁻³, $c_p = 4.192$ kJ kg⁻¹. The value $\Theta = 0.0001$, was chosen as minimum as possible to reach the real capacity of each hourly temperature distribution.

Table 3 shows the results obtained from the calculation. Further evaluation was carried out by comparing the theoretical capacity ($C_{theo}$) obtained from Eq 7 with independent formulae of Eq. 6. Equation 7 was used to represent the summation of integral formula Eq. 12 and 13. Referring to Table 3, it is noted that the calculated values of Eq. 6 and 7 are equal with second decimal difference. The deviations between two values are below 0.0004%. This indicates that the obtained equations are reliable.
Table 3: Calculation of the parameters

<table>
<thead>
<tr>
<th>θ</th>
<th>B (m)</th>
<th>Wc (m)</th>
<th>C_{\text{net}} (RTh)</th>
<th>C_{\text{p}} (RTh)</th>
<th>C_{\text{w}} (RTh)</th>
<th>C_{\text{m}} (RTh)</th>
<th>R = (12) + (13)</th>
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<td>2321.27</td>
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<td>4578.72</td>
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<td>9689.89</td>
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Fig. 5: Thermocline thickness growth

y = -0.0204x^2 + 0.0532x + 5.3985
R^2 = 0.7633

Fig. 6: Lost capacity growth

y = -0.6664x^2 + 1.8243x + 174.07
R^2 = 0.7656

Fig. 7: Percentage of C_{\text{lost}} growth

y = 0.0733x^2 - 104166x + 8.298
R^2 = 0.9988

Fig. 8: FoM_{1/2} growth

Implementation to the performance equations:
Performance evaluation was carried out utilizing result values of Table 3 and position of thermocline (C) in Table 1. Observations were focused in term of parameters growth of thermocline thickness, lost capacity and half-cycle figure of Merit within charging period. Thermocline thickness growth within charging cycle is presented in Fig. 5.

From Fig. 5, it can be seen that W_{Tc} varies with respect to time. W_{Tc} increased from initial condition to 19.00 and decreased from 19.00 to 03.00. The highest W_{Tc}
occurred at 19:00 when the positions of thermocline reach at 3.64 m as presented in Table 1. The occurrence of thicker thermocline at the lower section of the storage indicated significant contribution of mixing inflow nearby the inlet diffuser.

Figure 6 shows the lost capacity growth with respect to time. It shows similar trends as variations of thermocline thickness, whereby thicker thermoline lead to higher capacity lost. The occurrence of higher lost capacity at the lesser of C, indicated that inflow mixing from inlet diffuser plays a significant role to the losses.

Percentage of lost capacity was also evaluated as a ratio of lost capacity to theoretical capacity obtained using Eq. 12 and 6. Percentage lost capacity growth during charging cycle is presented in Fig. 7. It is noted that percentage of lost capacity decreases from the lower to higher section of the TES. This is due to increase of theoretical capacity during the charging hours.

The growth of half-cycle Figure of Merit (FoM$_{0.5}$) is shown in Fig. 8. The parameter was calculated using Eq. 8. Figure of Merit (FoM$_{0.5}$) increases in value with increased C. FoM$_{0.5}$ during the charging periods has value of 93.06 to 98.60%.

**CONCLUSION**

S-curve of temperature distribution of stratified TES was approached using fitting function. Performance parameter was determined using the fitted function. Results indicate that Sigmoid Dose Response (SDR) enabled to Figure out the S-curve of temperature distribution profile with coefficient of determination, $R^2$, more than 0.99. Based on the function, parameters for performance evaluation of stratified TES were determined namely limit points of thermocline profile, thermocline thickness, lost capacity, integrated capacity, theoretical capacity, as well as half-cycle Figure of Merit. The implementation to the observed temperature distribution showed that this method capable to be utilized for performance evaluation of TES. The approach could be used to obtain exact values of TES performance parameters.

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