Capacitance of Metallic Plates Forming a Corner

1D. Mehta Prarhan and 2S.B. Chakrabarty
1Department of Electronics and Communication Engg.,
Faculty of Technology, Dharmsinh Desai University, Nadiad, India
2Space Applications Centre, Ahmedabad, India

Abstract: The capacitance of metallic tilted plates forming a corner is analyzed using Method of Moments based on pulse basis functions and point matching. The charge distributions on the conductor surfaces and capacitance of metallic tilted plates are found by solving the integral equation relating the potential on the conductor surface and unknown charge distribution by applying Method of Moments. Numerical data on capacitance and charge distribution are presented. The validity of the analysis was carried out by considering the tilt angle as 180° which is a case of rectangular plate and for which the data on capacitance is available in the literature.

Keywords: Capacitance, charge distribution, pulse basis function, point matching, rectangular metallic plates, corner

INTRODUCTION

The spacecraft bodies in orbits experience significant charge buildup on its’ surfaces during inconsistent magnetic storm periods caused by the emission of charged particles from the sun. Rapid change in the accumulated charge distribution on satellite bodies produces current pulse, the consequence of which is the generation of electromagnetic pulse causing electromagnetic interference (EMI). This type of EMI may be very damaging to spacecraft electronic sub-systems. In order to study the phenomenon of EMI in space-craft bodies, the estimation of equivalent circuit parameters of exposed bodies of space-craft of different geometrical shapes is imperative. Capacitance is a very important circuit parameter needed for the analysis of Electrostatic Discharge (ESD) which causes EMI. Electrostatic modeling for the evaluation of capacitance and charge distribution of finite metallic bodies of different geometrical shapes like rectangular plate, cylinder, truncated cone, paraboloid, hemisphere, cube etc., which are widely used in orbiting spacecraft are presented by Ouda (2010), Wei et al. (2010), Azimi and Golnabi (2009), Ghosh and Chakrabarty (2008), Hwang and Michael (2004), Bai and Lumgren (2002, 2003, 2004), Das and Chakrabarty (1997a, 1998), Adams (1972) and Harrington (1968). In space-craft bodies, a number of sharp corners are present. Thus the evaluation of capacitance and charge distribution of corner formed by two rectangular metallic plates is of interest. The analysis of two metallic plates forming an arbitrary angle corner is not available in the open literature to the best of authors’ knowledge.

In this study, the evaluation of capacitance and charge distribution of metallic structure in the form of two plates forming a corner is presented using Method of Moment (MoM). In the present case, a set of two integral equations is formed by relating the unknown charge distribution on the two metallic plates forming the corner to the known potential on the corresponding metallic surface. In order to solve the integral equations by MoM, the surfaces of conductors are divided into rectangular subsections. The unknown charge densities are expanded using pulse basis functions and delta functions are used as testing functions for generating the set of simultaneous equations (Ouda, 2010; Wei et al., 2010; Azimi and Golnabi, 2009; Ghosh and Chakrabarty, 2008; Adams, 1972). The unknown charge densities are found from the inversion of the matrix formed from the set of simultaneous equations. The numerical data of the capacitances as well as charge distributions on the surface of the metallic plates are presented.

ANALYSIS

Two metallic rectangular plates of size 2a×2b and 2a×2e, forming a corner of angle θ, with the co-ordinate system is shown in Fig. 1.

For the plate 1 which is oriented in the X-Y plane with Z-axis perpendicular to the surface; the origin of the co-ordinate system is located at the geometrical centre
In order to find the capacitance of the structure shown in Fig. 1, the knowledge of unknown charge distribution on the metallic surfaces should be known. Assuming the value of potential on both the metallic surfaces as $V_1$ and $V_2$, the following integral equations can be obtained from Eq. 2:

$$V_1 = \frac{1}{4\pi\varepsilon} \left[ \int_{\text{bounds}} \rho_s(t_1) ds + \int_{\text{bounds}} \rho_i(t_1') ds \right]$$

(3)

$$V_2 = \frac{1}{4\pi\varepsilon} \left[ \int_{\text{bounds}} \rho_s(t_1) ds + \int_{\text{bounds}} \rho_i(t_1') ds \right]$$

(4)

The capacitance and unknown charge distribution on the metallic surfaces of the structure shown in Fig. 1 may be found by solving the integral Eq. 3 and 4 by method of moments. In order to apply the method of moments, the surfaces of metallic plates 1, 2 are divided into $M_1 \times N_1$, and $M_2 \times N_2$ equal sub-areas of the size $2g_s' \times 2g_i'$, and $2g_s' \times 2g_i'$, respectively.

The unknown charge distributions appearing in the integral equations are expressed in terms of known basis function (Azimi and Gholnabi, 2009; Ghosh and Chakrabarty, 2008; Adams, 1972) as:

$$\rho_i(t_1) = \sum_{m=0}^{\infty} \alpha_{mk} f_k(t_1) = \sum_{m=0}^{\infty} \alpha_{mk} f_k(t_1)$$

(5)

where, $f_k$ are the pulse basis functions given by:

$$f_k = \begin{cases} 1, & \text{on } \Delta s_k \\ 0, & \text{on any other } \Delta s_k \\ \end{cases}$$

In order to simplify the analytical complexity, it is assumed that the charge on each rectangular sub-section is concentrated at its center (Ghosh and Chakrabarty, 2008; Harrington, 1968). Substituting Eq. 5 in Eq. 3 and 4 and matching the boundary conditions for the potential at the center point of each subsection and following the procedure of Ghosh and Chakrabarty (2008) and Das and Chakrabarty (1997c), the equations reduce to the form:

$$V_1 = \sum_{m=0}^{\infty} \alpha_{mk} b_{mk} + \sum_{m=0}^{\infty} \alpha_{mk} b_{mk}, \ m = 1, 2, 3, \ldots, M_1 N_1$$

(6)

$$V_2 = \sum_{m=0}^{\infty} \alpha_{mk} b_{mk} + \sum_{m=0}^{\infty} \alpha_{mk} b_{mk}, \ m = M_1 N_1 + 1, \ldots, M_1 N_1 + M_2 N_2$$

(7)

The set of simultaneous equations, resulting from Eq. 6 and 7 lead to matrix equation of the form:
In Eq. 8 the non-diagonal elements of \( l_{11} \) and \( l_{22} \) can be written as (Adams, 1972):

\[
l_{1mn} = \frac{\Delta x_n}{4 \pi \varepsilon_0 R} \frac{g_{1f}}{\pi \varepsilon_0 \sqrt{(x_{m} - x_{n})^2 + (y_{m} - y_{n})^2}}
\]

\[
l_{2mn} = \frac{\Delta x_n}{4 \pi \varepsilon_0 R} \frac{g_{1f}}{\pi \varepsilon_0 \sqrt{(y_{m} - y_{n})^2 + (z_{m} - z_{n})^2}}
\]

The diagonal elements of \( l_{11} \) and \( l_{22} \) can be written following the procedure suggested in (Adams, 1972):

\[
l_{11n} = \frac{1}{2 \pi \varepsilon_0} \left[ f_{1n} \ln \sqrt{\frac{z_{1}^2 + z_{n}^2 + z_{1'}^2}{z_{1}^2 + z_{n}^2 + z_{1'}^2}} + f_{1n} \ln \sqrt{\frac{z_{1}^2 + z_{n}^2 + z_{1'}^2}{z_{1}^2 + z_{n}^2 + z_{1'}^2}} \right]
\]

\[
l_{22n} = \frac{1}{2 \pi \varepsilon_0} \left[ f_{2n} \ln \sqrt{\frac{z_{2}^2 + z_{n}^2 + z_{2'}^2}{z_{2}^2 + z_{n}^2 + z_{2'}^2}} + f_{2n} \ln \sqrt{\frac{z_{2}^2 + z_{n}^2 + z_{2'}^2}{z_{2}^2 + z_{n}^2 + z_{2'}^2}} \right]
\]

Using the method for the inversion of a matrix by partitioning it into sub-matrices, the solution of Eq. 8 is obtained as:

\[
\begin{bmatrix}
[\alpha_1] \\
[\alpha_2]
\end{bmatrix} =
\begin{bmatrix}
[l_{11} & l_{12}]
[l_{21} & l_{22}]
\end{bmatrix}^{-1}
\begin{bmatrix}
[V_1] \\
[V_2]
\end{bmatrix}
\]

where, \([\alpha_{mn}]\) are the sub-matrices of the inverted co-efficient matrix as given by Eq. 8.

The charge distributions on the metallic as well as dielectric faces are obtained from the unknown coefficients of \( \alpha \)'s obtained by solving the above matrix Eq.

The total charge on the surface of plate-1 is:

\[
Q_1 = \sum_{n=1}^{N} \alpha_{mn} \frac{4\pi b}{M_{1}N_{1}}
\]

Similarly, the charge on the surface of plate-2 is:

\[
Q_2 = \sum_{n=1}^{N} \alpha_{mn} \frac{4\pi b}{M_{2}N_{2}}
\]

The total charge on the conductor is:

\[
Q = Q_1 + Q_2
\]

The capacitance of the structure shown in Fig. 1 assuming \( V_1 = V_2 = V \) is:

\[
C = \frac{Q}{V}
\]

**RESULTS AND DISCUSSION**

Using the Eq. 9 through Eq. 19, the capacitance per unit length of the metallic tilted plates for the geometry shown in Fig. 1 is evaluated with \( a/b, a/c \) and \( \theta \) as parameters. The convergence of the numerical data on the capacitance per unit length is checked for each parameter. The convergence data is presented in Fig. 3. The variation of normalized capacitance as a function \( \theta \), with \( a/b \) and \( a/c \) as parameters has been computed and is shown in Fig. 4. Figure 5 shows the normalized capacitance per unit length for the metallic tilted plates, for different values of \( a/c \). The charge distribution along the XY plane for the surface of plate-1 and the charge distribution along the XoYo plane for the surface of plate-2 are shown in Fig. 6 and 7, respectively for parameters \( a/b = a/c = 2 \) and \( \theta = 135^\circ \). Table 1 shows the comparison of the data for the capacitance of a square plate presented by Harrington (1968) with that of the present configuration for \( \theta = 180^\circ \), \( a/b = a/c = 2 \) which is the case of a square metallic plate.

The data of capacitance and charge distribution for the geometry shown in Fig. 1 are not available in open literature and hence the validation of the results of...
Fig. 4: The capacitance per unit length as a function of $\theta$, with $a/b$ and $a/c$ as parameters

Fig. 5: The capacitance per unit length as a function of $a/c$, with $a/b$ as parameters for $\theta = 135^\circ$

Fig. 6: Charge distribution on the surface of plate-1 along the XY-plane

Fig. 7: Charge distribution on the surface of plate-2 along the $X_0Y_0$-plane

Table 1: Comparison of the method presented by Harrington (1968) and the present method

<table>
<thead>
<tr>
<th>No. of subsections</th>
<th>$C/2a$ (pF/m) present method for $\theta = 180^\circ$</th>
<th>$C/2a$ (pF/m) method presented by Harrington (1968)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>38.150</td>
<td>38.2</td>
</tr>
<tr>
<td>36</td>
<td>39.189</td>
<td>39.2</td>
</tr>
<tr>
<td>100</td>
<td>39.832</td>
<td>39.8</td>
</tr>
</tbody>
</table>

(1998) and Harrington (1968). The numerical data for the parameters, $\theta = 180^\circ$, $a/b = a/c = 2$, were compared with the results obtained for the square metallic plate presented by Harrington (1968). The close agreement between the data on capacitance as seen from Table 1 justifies the validity of the analysis. The validity of data was also checked by comparing the data on capacitance available in Ghosh and Chakrabarty (2008) with that computed by the present method. The data on capacitance/length ($C/2a$) computed by the present method for $a/b = a/c = 2$, $\theta = 180^\circ$ and $N = 32$ is 39 pF/m which closely matches with the $C/2L = 38.69$ pF/m in Ghosh and Chakrabarty (2008) for the same parameters ($L/W = 1$ and $N = 32$). The capacitance data of a rectangular plate coated with dielectric was reported by Das and Chakrabarty (1998). They presented data on capacitance for very low thickness of dielectric coating. The value of the capacitance for very low value of dielectric constant should be comparable to the value of capacitance without dielectric coating. The value of capacitance presented by Das and Chakrabarty (1998) for a square plate of dielectric thickness/length = 0.001 is 39.53 pF closely matches with that given in Table 1. This also justifies the validity of the analysis. Convergence plot for the capacitance of the structure per unit length is shown in Fig. 3. Figure 5 re-authenticate the method used in this study by showing the convergence of capacitance for very large value of $a/c$ ratio of the structure shown in Fig. 1, to an equivalent capacitance of unit square plate. It is apparent from Fig. 6 and 7 that the peak of charge.
distributions along the surfaces connected to the surfaces of another metallic plate is lower because the two plates are connected.

REFERENCES